Physics Letters B 760 (2016) 605–610

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

# Bouncing cosmologies in geometries with positively curved spatial sections

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ARTICLE INFO

*Article history:*
Received 5 April 2016
Accepted 20 July 2016
Available online 22 July 2016
Editor: M. Trodden

ABSTRACT

Background bouncing cosmologies, driven by a single scalar field, having a quasi-matter domination period during the contracting phase, i.e., depicting the so-called Matter Bounce Scenario, are reconstructed for cosmologies with spatial positive curvature. These cosmologies lead to a nearly flat power spectrum of the fluctuation curvature in co-moving coordinates for modes that leave the Hubble radius during this quasi-matter domination period, and whose spectral index and its running, which are related with the effective Equation of State parameter given by the quotient of the pressure over the energy density, are compatible with experimental data.



## 1. Introduction

Bouncing cosmologies (see [1] for a review) do not have the horizon problem that appears in Big Bang cosmology [2] and, when the bounce is symmetric, improve the flatness problem (where spatial flatness is an unstable fixed point and fine tuning of initial conditions is required), because the contribution of the spatial curvature decreases in the contracting phase at the same rate as it increases in the expanding one (see for instance [3]). Therefore they could, in principle, be a viable alternative to the inflationary paradigm [4]. On the other hand, it is well known that when the background is the Friedmann–Lemaître–Robertson–Walker (FLRW) geometry and one has a single scalar field filling the Universe, within General Relativity (GR), only geometries with positive spatial curvature could lead to bounces. However, the most usual way to obtain bounces is to work in the flat FLRW space-time and to introduce nonconventional matter fields [5] in order to break down the weak energy condition $\rho + P > 0$ (being $\rho$ the energy density and $P$ the pressure), or to go beyond GR and to deal with theories such as Loop Quantum Cosmology (LQC), where holonomy corrections introduce a quadratic correction in the Friedmann equation leading to a Big Bounce that replaces the Big Bang singularity (see, for instance, [6]), modified $F(R)$ gravity [7] or teleparallel $F(T)$ theories [2,8].

Once one has a bouncing background, the next step is to deal with cosmological perturbations. There is a well-known duality between a matter domination epoch in the contracting phase and the de Sitter regime in the expanding one [9], thus a quasi-matter dominated Universe when modes leave the Hubble radius in the contracting phase would produce the same kind of power spectrum as a quasi-de Sitter Universe (an inflationary Universe) when modes leave the Hubble radius in the expanding phase. In fact, it has been shown that bouncing cosmologies in the flat FLRW space-time produce a nearly flat power spectrum [10], as in inflation.

The main goal of the present work is to provide, for the $\mathcal{K} = 1$-FLRW metric and using a single scalar field, background bouncing cosmologies in the framework of GR, and calculate the corresponding power spectrum of the curvature fluctuations in co-moving coordinates.

These backgrounds cannot come from a field mimicking a fluid with Equation of State (EoS) $P = w\rho$ as in holonomy corrected LQC [11], because when the spatial curvature is positive, a linear EoS produces cosmologies with a Big Bang and a Big Crush. Then, the way to obtain bouncing cosmologies is to choose some particular bouncing backgrounds, for instance $a(t) = (\rho_c t^2 + 1)^n$, in our case bouncing symmetric backgrounds that have a quasi-matter domination (see equation (14) which is our main model), that is, we choose some Matter Bouncing Scenarios (see [12] for a recent review), and apply the reconstruction techniques to obtain a potential and the corresponding conservation equation (a second order differential equation) whose solutions lead to different cosmologies. In general it is impossible to calculate analytically that potential, and thus, numerical calculations are needed to recover it. Once the potential has been calculated, one can calculate numerically the different backgrounds, and for each one of them the corresponding relevant terms of the power spectrum such as the spectral index and its running, coming from the Mukhanov–

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http://dx.doi.org/10.1016/j.physletb.2016.07.053

Sasaki equation for geometries with positively curved space sections [13–15]. However, these numerical calculations are very involved and need future investigation, for this reason here we will only calculate the spectral index and its running for our main background (14), and we will indicate how they will be for the other backgrounds.

The units used throughout the work are $\hbar = c = 8\pi G = 1$.

## 2. Potential reconstruction for a single scalar field

When one deals with a single scalar field, in the $\mathcal{K} = 1$-FLRW geometry, the Raychaudhuri equation in cosmic time becomes [13] (see also equation (6) of [14] where it appears in conformal time)

$$\dot{H} = -\frac{\dot{\varphi}^2}{2} + \frac{1}{a^2}. \tag{1}$$

Then, given some background, i.e. a scale factor $a(t)$, and thus the Hubble parameter $H(t) = \frac{\dot{a}(t)}{a(t)}$ and its derivative, from the equation

$$\varphi(t) = \int_{\tilde{t}_0}^{t} \sqrt{-2\left(\dot{H}(s) - \frac{1}{a^2(s)}\right)}ds, \tag{2}$$

where $\tilde{t}_0$ is an arbitrary constant, we obtain the relation between the scalar field and the cosmic time, namely $\varphi = g(t)$.

On the other hand, using the Friedmann equation [16],

$$H^2 = \frac{\rho}{3} - \frac{1}{a^2}, \tag{3}$$

we obtain the potential as a function of time

$$\bar{V}(t) = 3H^2(t) + \dot{H}(t) + \frac{2}{a^2(t)}, \tag{4}$$

and making the replacement $t = g^{-1}(\varphi)$ (note that $g$ is always an inversible function because $\dot{g} \geq 0$ for all cosmic time $t$), one finally obtains the corresponding potential

$$V(\varphi) \equiv \bar{V}(g^{-1}(\varphi)). \tag{5}$$

In general, it is impossible to find analytically the function $g^{-1}$, and thus, the potential must be obtained numerically, but there are some cases where an analytic calculation is allowed.

**Example 2.1.** As an academic example we choose the scale factor $a(t) = a_0(\rho_c t^2 + 1)$ with $4a_0^2\rho_c = 1$. One easily obtains $\dot{H} - \frac{1}{a} = -\frac{2\rho_c}{\rho_c t^2+1}$, then taking $\tilde{t}_0 = 0$ in (2) one gets

$$\varphi(t) = 2\ln(\sqrt{\rho_c}t + \sqrt{\rho_c t^2 + 1}) \Longleftrightarrow \rho_c t^2 + 1 = \frac{(e^{\varphi}+1)^2}{4e^{\varphi}}. \tag{6}$$

Finally, inserting this last expression in (4) one will get the symmetric potential

$$V(\varphi) = \frac{40\rho_c e^{\varphi}}{(1+e^{\varphi})^2}, \tag{7}$$

which has the same shape as the potential [11]

$$V(\varphi) = \frac{2\rho_c(1-w)e^{\sqrt{3(1+w)}\varphi}}{\left(1+e^{\sqrt{3(1+w)}\varphi}\right)^2} \tag{8}$$

used in holonomy corrected LQC to mimic a hydrodynamical fluid with EoS $P = w\rho$ (for the potential (7) one has to choose $w = -\frac{2}{3}$). Of course, in geometries with positively curved spatial sections the potential (7) does not mimic any hydrodynamical fluid with a linear EoS, but it depicts some bouncing backgrounds (see Fig. 1).

Once we have reconstructed the potential, the dynamics is given by the following autonomous system

$$\begin{cases} \dot{\varphi} = \psi \\ \dot{\psi} + 3H_{\pm}(\varphi,\psi,a)\psi + V_{\varphi} = 0 \\ \dot{a} = H_{\pm}(\varphi,\psi,a)a, \end{cases} \tag{9}$$

where

$$H_{\pm}(\varphi,\psi,a) = \pm\sqrt{\frac{\psi^2}{6} + \frac{V(\varphi)}{3} - \frac{1}{a^2}}. \tag{10}$$

Note that equation (9) is a first order system of three differential equations, so apart from the originally chosen background it leads to infinitely many new, different ones.

In fact, we have integrated numerically the equation (9) for the potential given in Example 2.1, obtaining a set of measure no zero in the ensemble of initial conditions $(\varphi_0, \psi_0, a_0)$ that leads to backgrounds with only one bounce, that is, depicting at very early times (resp. late times) a universe in the contracting (resp. expanding) phase (see Fig. 1). As we have already explained this potential is a particular case of the potentials used in holonomy corrected LQC to mimic a hydrodynamical fluid with linear EoS. This opens the possibility to study these potentials in the context of $\mathcal{K} = 1$-FLRW geometry, and to obtain new bouncing backgrounds solving numerically the equation (9).

Coming back to the reconstruction method, note that the condition to reconstruct the potential is that $\dot{H}(t) - \frac{1}{a^2(t)}$ must be negative for all cosmic time. This places constrains on the backgrounds, for example dealing with the simplest bouncing scale factor $a(t) = a_0(\rho_c t^2 + 1)^{\frac{\alpha}{2}}$, only could be reconstructed when $\alpha a_0^2\rho_c \leq 1$. In fact, it is easy to show that the condition

$$\alpha a_0^2\rho_c \leq 1, \quad 1 \leq \alpha \leq 2, \tag{11}$$

is enough to reconstruct the potential corresponding to the simple background $a(t) = a_0(\rho_c t^2 + 1)^{\frac{\alpha}{2}}$.

On the other hand, for these backgrounds the effective EoS parameter given by the ratio of the pressure to the energy density is given by

$$w \equiv \frac{P}{\rho} = -1 - \frac{2(a^2\dot{H} - 1)}{3(a^2H^2 + 1)}$$
$$= -1 - \frac{2}{3}\left(\frac{\alpha\rho_c a_0^2 x^{\alpha}\left(\frac{2}{x^2} - \frac{1}{x}\right) - 1}{\alpha^2\rho_c a_0^2 x^{\alpha}\left(\frac{1}{x} - \frac{1}{x^2}\right) + 1}\right), \tag{12}$$

where $x \equiv \rho_c t^2 + 1$. When $x \gg 1$, i.e. far away from the bounce, it becomes

$$w = -1 + \frac{2}{3}\left(\frac{\alpha\rho_c a_0^2 x^{\alpha-1} - 1}{\alpha^2\rho_c a_0^2 x^{\alpha-1} + 1}\right). \tag{13}$$

Then, for $0 < \alpha < 1$, when $\alpha\rho_c a_0^2 \leq 1$ one has $w = -\frac{1}{3}$ and when $\alpha\rho_c a_0^2 \gg x^{1-\alpha} \gg 1$ one has $w = -1 + \frac{2}{3\alpha}$ which is nearly zero, and thus defines a quasi-matter dominated Universe, only when $\alpha \cong \frac{2}{3}$. On the other hand, for $\alpha \geq 1$ its impossible to have, far away to the bounce, a quasi-matter domination, because in that case one always has $w \leq -1 + \frac{2}{3} = -\frac{1}{3}$.

This result means that one cannot reconstruct, using a single scalar field, a bouncing cosmology with the simplest scale factor $a(t) = a_0(\rho_c t^2 + 1)^{\frac{\alpha}{2}}$ $(\alpha > 0)$ that has a matter domination period, because as we have already seen, the reconstruction only holds for $\alpha\rho_c a_0^2 \leq 1$ and matter domination requires, in that case, $\alpha\rho_c a_0^2 \gg 1$. For this reason, in the framework of GR, if one wants to