Physics Letters B 760 (2016) 617–626

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Beyond generalized Proca theories

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ARTICLE INFO

Article history:
Received 18 May 2016
Received in revised form 18 July 2016
Accepted 19 July 2016
Available online 21 July 2016
Editor: M. Trodden

ABSTRACT

We consider higher-order derivative interactions beyond second-order generalized Proca theories that propagate only the three desired polarizations of a massive vector field besides the two tensor polarizations from gravity. These new interactions follow the similar construction criteria to those arising in the extension of scalar–tensor Horndeski theories to Gleyzes–Langlois–Piazza–Vernizzi (GLPV) theories. On the isotropic cosmological background, we show the existence of a constraint with a vanishing Hamiltonian that removes the would-be Ostrogradski ghost. We study the behavior of linear perturbations on top of the isotropic cosmological background in the presence of a matter perfect fluid and find the same number of propagating degrees of freedom as in generalized Proca theories (two tensor polarizations, two transverse vector modes, and two scalar modes). Moreover, we obtain the conditions for the avoidance of ghosts and Laplacian instabilities of tensor, vector, and scalar perturbations. We observe key differences in the scalar sound speed, which is mixed with the matter sound speed outside the domain of generalized Proca theories.

1. Introduction

General Relativity (GR) is still the fundamental theory for describing the gravitational interactions even after a century. Cosmological observations [1–3] led to the standard model yielding an accelerated expansion of the late Universe driven by the cosmological constant. The standard model of particle physics describes the strong and electro-weak interactions with an exquisite experimental success marking the milestone in high-energy physics. It is still a big challenge to unify gravity with the known forces in Nature and to merge these two standard models into a single theory. Moreover, employing the usual techniques of quantum field theory, we are not able to explain the small observed value of the cosmological constant. On the other hand, this has motivated to consider infra-red modifications of gravity which could account for an appropriate screening of the cosmological constant. On a similar footing, one can also consider infra-red gravitational modifications to realize an effective negative pressure against gravity in form of dark energy [4].

The simplest and mostly studied large-distance modification of gravity is attributed to an additional scalar field beyond the standard model of particle physics, e.g., the DGP braneworld [5], Galileons [6], and massive gravity [7]. The scalar field arising in such theories can have non-trivial self-interactions but also it can be generally coupled to gravity [8,9]. These interactions have to be constructed with great caution to guarantee the absence of ghost-like Ostrogradski instability [10], which otherwise would yield an unbounded Hamiltonian from below.

It is well known that matter fields have to be coupled to the Lovelock invariants or to the divergence-free tensors constructed from the Lovelock invariants. Hence they can for instance couple to the volume element $\sqrt{-g}$ and to the Ricci scalar R which are the only two non-trivial Lovelock invariants, since the Gauss–Bonnet term is topological in four dimensions. Furthermore, they can couple to the divergence-free metric $g_{\mu\nu}$, Einstein tensor $G_{\mu\nu}$, and the double dual Riemann tensor $L_{\mu\nu\alpha\beta}$. In flat space–time the ghost-free scalar interactions with derivatives acting on them are known as the Galileon interactions [6]. If one would naively promote the partial derivatives to covariant derivatives, this procedure would yield the equations of motion higher than second order [10]. The appearance of higher-order derivative terms can be avoided by introducing non-minimal couplings to gravity through the Lovelock invariants or the divergence-free tensors.

Horndeski theories [11] constitute the most general scalar–tensor interactions with second-order equations of motion. In these theories there is only one scalar degree of freedom (DOF)

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http://dx.doi.org/10.1016/j.physletb.2016.07.052

besides two graviton polarizations without having the Ostrogradski instability [12]. It is a natural question to ask whether abandoning the requirement of second-order equations of motion inevitably alters the propagating DOF. Allowing interactions beyond the Horndeski domain will introduce derivative interactions higher than second order. However, this does not necessarily mean that the number of propagating DOF increases. Exactly this spirit was followed in GLPV theories [13], where they expressed the Horndeski Lagrangian in terms of the $3+1$ Arnowitt–Deser–Misner (ADM) decomposition of space–time in the unitary gauge [14] and did not impose the two conditions that Horndeski theories obey. The Hamiltonian analysis in the unitary gauge revealed that there is still only one scalar DOF [15]. The cosmology and the spherically symmetric solutions in GLPV theories have been extensively studied in Refs. [16–19]. The ghost freedom beyond the unitary gauge and beyond a conformal and disformal transformation is still an ongoing research investigation in the literature [20–25].

Even if the large-distance modifications of gravity through a scalar field are simpler, considerations in form of a vector field can yield interesting phenomenology for the cosmic expansion and growth of large-scale structures. Furthermore, the presence of the vector field might explain the anomalies reported in CMB observations [26]. For a gauge-invariant vector field, the only new interaction is via a coupling of the field strength tensor to the double dual Riemann tensor. Unfortunately, the existence of derivative self-interactions similar to those arising for covariant Galileons is forbidden for a massless, Lorentz-invariant vector field coupled to gravity [27].

However, this negative result does not apply to massive vector fields, for which one can successfully construct derivative self-interactions due to the broken $U(1)$ symmetry. The idea was to construct interactions with only three propagating degrees of freedom, out of which two would correspond to the transverse and one to the longitudinal mode of the vector field. This was systematically constructed in Ref. [28] together with the Hessian and Hamiltonian analysis. The key point is the requirement that the longitudinal mode belongs to the class of Galileon/Horndeski theories. This constitutes the generalized Proca theories up to the quintic Lagrangian on curved space–time with second-order equations of motion, which is guaranteed by the presence of non-minimal couplings to the Lovelock invariants in the same spirit as in the scalar Horndeski theories [28–31].

One can also construct the sixth-order derivative interactions, if one allows for trivial interaction terms for the longitudinal mode [30,31]. Its generalization to curved space–time contains the double dual Riemann tensor, which keeps the equations of motion up to second order [31]. In fact, this sixth-order Lagrangian accommodates similar vector–tensor theories constructed by Horndeski in 1976 [32]. We refer the reader to Refs. [33–41] for related works. The second-order massive vector theories up to the sixth-order Lagrangian studied in Refs. [31,40,41] constitute the generalized Proca theories.

It is a natural follow-up question to ask whether or not the extension of generalized Proca theories is possible in such a way that there are still three propagating vector DOF even with derivatives higher than second order. In the GLPV extension of Horndeski theories, the Lagrangians of two additional scalar derivative interactions can be expressed in terms of the anti-symmetric Levi-Civita tensor. Outside the domain of generalized Proca theories, one can also construct generalized Lagrangians by using the Levi-Civita tensor. It is then expected that, in beyond-generalized Proca theories, the longitudinal vector mode would have some correspondence with the scalar mode in GLPV theories, but there will be also new interactions corresponding to the purely intrinsic vector modes.

In this Letter, we will propose candidates for new beyond-generalized Proca Lagrangians in Sec. 2 to study the possibility of the healthy extension of generalized Proca theories. In Sec. 3 we derive the background equations of motion on the flat Friedmann–Lemaître–Robertson–Walker (FLRW) background and the associated Hamiltonian $\mathcal{H}$. We see that, even in the presence of these new interactions, there exists a second class constraint ($\mathcal{H}=0$) that removes the Ostrogradski ghost. In Sec. 4 we consider linear cosmological perturbations on the flat FLRW background and show that the number of DOF in beyond-generalized Proca theories is not altered relative to that in generalized Proca theories. We also study what kinds of differences arise for the stability of perturbations by extending generalized Proca theories to beyond-generalized Proca theories. Sec. 5 is devoted to conclusions and future outlook.

2. Extension of generalized Proca theories to beyond-generalized Proca theories

The generalized Proca theories are characterized by second-order interactions with two transverse and one longitudinal polarizations of a vector field A^μ coupled to gravity. Introducing the field tensor $F_{\mu\nu}=\nabla_\mu A_\nu-\nabla_\nu A_\mu$, where ∇_μ is the covariant derivative operator, the four-dimensional action of generalized Proca theories is given by

$$S_{\rm gen.Proca}=\int d^4x\sqrt{-g}\sum_{i=2}^{6}\mathcal{L}_i\,, \tag{2.1}$$

where g is the determinant of the metric tensor $g_{\mu\nu}$, and

$$\mathcal{L}_2=G_2(X,F,Y)\,, \tag{2.2}$$

$$\mathcal{L}_3=G_3\nabla_\mu A^\mu\,, \tag{2.3}$$

$$\mathcal{L}_4=G_4R+G_{4,X}\left[(\nabla_\mu A^\mu)^2-\nabla_\rho A_\sigma\nabla^\sigma A^\rho\right], \tag{2.4}$$

$$\begin{aligned}\mathcal{L}_5=&\,G_5G_{\mu\nu}\nabla^\mu A^\nu-\frac{1}{6}G_{5,X}[(\nabla_\mu A^\mu)^3\\&-3\nabla_\mu A^\mu\nabla_\rho A_\sigma\nabla^\sigma A^\rho+2\nabla_\rho A_\sigma\nabla^\gamma A^\rho\nabla^\sigma A_\gamma]\\&-g_5\tilde{F}^{\alpha\mu}\tilde{F}^\beta{}_\mu\nabla_\alpha A_\beta\,,\end{aligned} \tag{2.5}$$

$$\begin{aligned}\mathcal{L}_6=&\,G_6L^{\mu\nu\alpha\beta}\nabla_\mu A_\nu\nabla_\alpha A_\beta\\&+\frac{1}{2}G_{6,X}\tilde{F}^{\alpha\beta}\tilde{F}^{\mu\nu}\nabla_\alpha A_\mu\nabla_\beta A_\nu\,.\end{aligned} \tag{2.6}$$

The function G_2 depends on the following three quantities

$$X=-\frac{A_\mu A^\mu}{2}\,,\quad F=-\frac{F_{\mu\nu}F^{\mu\nu}}{4}\,,\quad Y=A^\mu A^\nu F_\mu{}^\alpha F_{\nu\alpha}\,, \tag{2.7}$$

while $G_{3,4,5,6}$ and g_5 are arbitrary functions of X with the notation $G_{i,X}\equiv\partial G_i/\partial X$. The vector field is coupled to the Ricci scalar R and the Einstein tensor $G_{\mu\nu}$ through the functions $G_4(X)$ and $G_5(X)$. The $L^{\mu\nu\alpha\beta}$ and $\tilde{F}^{\mu\nu}$ are the double dual Riemann tensor and the dual strength tensor defined, respectively, by

$$L^{\mu\nu\alpha\beta}=\frac{1}{4}\mathcal{E}^{\mu\nu\rho\sigma}\mathcal{E}^{\alpha\beta\gamma\delta}R_{\rho\sigma\gamma\delta}\,,\quad \tilde{F}^{\mu\nu}=\frac{1}{2}\mathcal{E}^{\mu\nu\alpha\beta}F_{\alpha\beta}\,, \tag{2.8}$$

where $R_{\rho\delta\gamma\delta}$ is the Riemann tensor and $\mathcal{E}^{\mu\nu\rho\sigma}$ is the Levi-Civita tensor obeying the normalization $\mathcal{E}^{\mu\nu\rho\sigma}\mathcal{E}_{\mu\nu\rho\sigma}=-4!$. We can potentially include the dependence of the quantity $F^{\mu\nu}\tilde{F}_{\mu\nu}$ in the function G_2 [28,36]. If we impose the parity invariance, however, it does not contribute to the perturbations at linear order, so we do not take into account such dependence in G_2.

The action (2.1) was constructed to keep the equations of motion up to second order to avoid the appearance of an extra DOF