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Basic oscillation measurables in the neutrino pair beam

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ABSTRACT

It was recently shown that the neutrino-pair emission may occur with large rates, their energy being extended to GeV region, if appropriate heavy ions are circulated in a quantum state of mixture. In the present work it is further demonstrated that the vector current contribution of neutrino interaction with electrons in ion, not necessarily suppressed in high atomic number ions, gives rise to the oscillating component, even when a single neutrino is detected alone. On the other hand, the single neutrino detection in Z-boson decay does not show the oscillating component, as known for some time. CP violation measurements in the neutrino pair beam may become a possibility, along with determination of mass hierarchical patterns.

1. Introduction

A new strong source of neutrinos consisting of all flavor pairs of ν_a and $\bar{\nu}_a$ $(a = e, \mu, \tau)$ was recently proposed [1]. The proposal uses, as the emitting source, circulating heavy ions in a quantum state of two-state mixture, $\sin\theta_c e^{-i\epsilon_{eg}t/\gamma}|e\rangle + \cos\theta_c|g\rangle$ ($|e\rangle$ is a suitable excited state, while $|g\rangle$ is the ground state of ion) with a mixing angle θ_c. A high coherence quantified by a large value of $\sin(2\theta_c)/2$ produces high energy neutrino-pair with its energy sum extending up to $2\gamma\epsilon_{eg}$ where $\gamma = 1/\sqrt{1-\beta^2}$ is the boost factor of circulating ions and ϵ_{eg} is the level energy difference of the two states. Produced pair-beam is well collimated to the angular region $1/\gamma$ from the tangential direction of circulating ion. Moreover, produced neutrinos in the pair are in a highly entangled quantum state. These features of the neutrino-pair beam make it worthwhile to investigate the new possibility of neutrino oscillation experiments.

It was however shown in [2] that the neutrino oscillation can be measured only when both neutrinos in the pair are detected, which is experimentally difficult due to the smallness of the double detection rate. This disappearance of oscillation for a single neutrino detection is based on (i) the unitarity of the 3×3 neutrino mixing matrix and (ii) the equality of pair emission amplitude squared that holds for the dominant axial vector contribution of light ions.

In the present work we show that the second condition, the equality of pair emission amplitude squared, does not hold in the vector current contribution (sub-dominant, when ionic electrons move with non-relativistic velocities, but may be comparable to the axial vector contribution in heavy ions) of pair emission amplitude, hence the emergence of oscillation patterns occurs from the vector contribution. When neutrino oscillation is made possible this way, the CP violating (CPV) parameter determination (the CPV phase δ common to both Dirac and Majorana cases) becomes possible.

We derive basic formulas for the three-flavor neutrino scheme including the earth matter effect and present numerical outputs of quantities for new experiments using the neutrino-pair beam. It is found that oscillation patterns appear in all $\nu_a, \bar{\nu}_a$, $a = e, \mu, \tau$, but determination of CPV parameter is possible only by detection of $\nu_\mu, \bar{\nu}_\mu$ and tau-neutrinos. Electron neutrinos do not allow CPV determination.

One of the most important advantages of the neutrino-pair beam is that it is a CP even beam consisting of equal mixture of neutrinos and anti-neutrinos. This is the reason why the direct measurement of CP-odd quantity, showing genuine CP violation effects, is possible in our neutrino-pair beam. CPV parameter determination is also possible by measuring CP-even observables alone. Simultaneous measurements of CP-odd and CP-even observables should be of great help for determination of CPV parameter.

Moreover, we would like to show that the neutrino pair beam offers experiments for other interesting neutrino physics. One example is the determination of the mass hierarchy of neutrinos by using the oscillation patterns in the ν_e $(\bar{\nu}_e)$ and ν_μ $(\bar{\nu}_\mu)$ ap-

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pearance probabilities. Another interesting topic is a possibility to probe the deep interior of earth by exploiting a large matter effect for sufficiently long baseline experiments of length $\gg 1000$ km.

Our estimate of the feasibility of high energy neutrino-pair is admittedly crude, but results thus obtained appear promising for further studies.

In the rest of paper we first explain under what conditions the disappearance of oscillation pattern occurs when a single neutrino is detected and how this can be evaded. We shall then proceed to calculation of oscillation effects.

Throughout this work we use the natural unit of $\hbar = c = 1$.

2. How oscillation pattern appears in single neutrino detection

We shall recapitulate how neutrino oscillation may arise in the case of pair-beam following notations of [2]. The probability amplitude of the entire process consists of three parts: the production, the propagation, and the detection due to charged current interaction (neutral current interaction is much smaller, hence not considered here), each to be multiplied at the amplitude level. Thus, one may write the probability for the ν_a neutrino quasi-elastic scattering (with J^α the nucleon weak current) as

$$\sum_c \left(\frac{G_F}{\sqrt{2}}\right)^2 \bar{\nu}_a\gamma_\alpha(1-\gamma_5)l_a J^\alpha \bar{l}_a\gamma_\beta(1-\gamma_5)\nu_a(J^\beta)^\dagger$$
$$\times \left|\sum_b \langle\bar{c}|e^{-i\bar{H}\bar{L}}|\bar{b}\rangle\langle a|e^{-iHL}|b\rangle\mathcal{M}_{\bar{b}b}(1,2)\right|^2, \quad (1)$$

where H ($\bar{H}$) is the hamiltonian for propagation of neutrino (anti-neutrino) including earth-induced matter effect [3–5], which is in the flavor basis

$$H = U^* \begin{pmatrix} \frac{m_1^2}{2E} & 0 & 0 \\ 0 & \frac{m_2^2}{2E} & 0 \\ 0 & 0 & \frac{m_3^2}{2E} \end{pmatrix} U^T + \sqrt{2}G_F n_e \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad (2)$$

where U_{ai} is the neutrino mixing matrix with $|a\rangle = \sum_i U_{ai}^*|i\rangle$, $a = e, \mu, \tau$, $i = 1, 2, 3$, and n_e is the number density of electrons in the earth. $\bar{H}$ can be obtained by replacing $U \to U^*$ and changing the sign in the second term $\propto G_F$.

We shall denote three eigenvalues by λ_i for neutrinos, and $\bar{\lambda}_i$ for anti-neutrinos. Let $V(\sim U)$ and $\bar{V}$ are unitary 3×3 matrices that diagonalize the hamiltonian H for neutrino and $\bar{H}$ for anti-neutrino, including the earth matter effect. The propagation amplitude is then

$$\langle a|e^{-iHL}|b\rangle = \sum_i V_{ai}V_{bi}^* e^{-i\lambda_i L},$$
$$\langle\bar{c}|e^{-i\bar{H}\bar{L}}|\bar{b}\rangle = \sum_i \bar{V}_{ci}^*\bar{V}_{bi}e^{-i\bar{\lambda}_i\bar{L}}, \quad (3)$$
$$\sum_b \langle\bar{c}|e^{-i\bar{H}L}|\bar{b}\rangle\langle a|e^{-iHL}|b\rangle c_b = \sum_{ij} V_{ai}\bar{V}_{cj}^*\xi_{ij}\exp[-i(\lambda_i L + \bar{\lambda}_j\bar{L})],$$
$$\xi_{ij} = \sum_b c_b V_{bi}^*\bar{V}_{bj}. \quad (4)$$

The factor c_b arises from the production amplitude $\mathcal{M}_{\bar{b}b}(1,2)$ and it is $(c_b^A) = \frac{1}{2}(1,-1,-1)$ for the axial vector contribution and for the vector contribution,

$$(c_b^V) = \left(\frac{1}{2}(1+4\sin^2\theta_w), -\frac{1}{2}(1-4\sin^2\theta_w), -\frac{1}{2}(1-4\sin^2\theta_w)\right), \quad (5)$$

with the weak mixing angle θ_w. The precise relation between neutrino and anti-neutrino eigenvalue problem is given by

$$\bar{\lambda}(G_F) = \lambda(-G_F), \quad \bar{V}_{ai}(G_F) = V_{ai}(-G_F). \quad (6)$$

The rate of neutrino ν_a detected and $\bar{\nu}_c$ undetected contains the squared propagation factor,

$$\sum_c \left|\sum_{ij} V_{ai}\bar{V}_{cj}^*\xi_{ij}\exp[-i(\lambda_i L + \bar{\lambda}_j\bar{L})]\right|^2$$
$$= \sum_{ijkl}\sum_c V_{ai}V_{ak}^*\bar{V}_{cj}^*\bar{V}_{cl}\xi_{ij}\xi_{kl}^*\exp[-i(\lambda_i-\lambda_k)L]\exp[-i(\bar{\lambda}_j-\bar{\lambda}_l)\bar{L}]$$
$$= \sum_{ik} V_{ai}V_{ak}^* p_{ik}\exp[-i(\lambda_i-\lambda_k)L], \quad p_{ik} = \sum_j \xi_{ij}\xi_{kj}^*. \quad (7)$$

When $(|c_b^A|^2) = (1,1,1)/4 \propto 1$ for the axial vector contribution, $p_{jl} = \delta_{jl}/4$ and the detection probability becomes 1/4, hence no oscillation pattern exits.

The relevant weak amplitude for the vector part gives oscillating components. Candidate ions for circulation that contribute to the vector current interaction are Be-like heavy ions of $2p2s\,{}^3P_1^-$ and Ne-like heavy ions of $2p^+3s\,{}^3P_1^-$ (electron–hole system).

3. Basic measurable quantities in neutrino pair beam

We first note

$$p_{ik} = \sum_j \xi_{ij}\xi_{kj}^* = \sum_b |c_b^V|^2 V_{bi}^* V_{bk}$$
$$= \frac{1}{4}(1+4\sin^2\theta_w)^2\, V_{ei}^* V_{ek}$$
$$+ \frac{1}{4}(1-4\sin^2\theta_w)^2\,(V_{\mu i}^* V_{\mu k} + V_{\tau i}^* V_{\tau k})$$
$$= \frac{1}{4}(1-4\sin^2\theta_w)^2\delta_{ik} + 4\sin^2\theta_W\, V_{ei}^* V_{ek}. \quad (8)$$

The detection probability of ν_a (when the other neutrino of the pair is undetected) is given by the oscillation formula based on the vector part of weak current,

$$P_a(E, L; m_i, \delta) \equiv \frac{1}{3(1-4\sin^2\theta_w)^2/4 + 4\sin^2\theta_w}$$
$$\times \left(\frac{1}{4}(1-4\sin^2\theta_w)^2 + 4\sin^2\theta_w \left|\sum_j U_{ej}^* U_{aj}\exp\left(-i\frac{m_j^2 L}{2E}\right)\right|^2\right), \quad (9)$$

with $\sin^2\theta_w \sim 0.231$. The formula (9) is valid when the earth matter effect is neglected. When the earth matter effect is included, one replaces $U \to V$, $m_j^2/2E \to \lambda_j$. The quantity $P_a(E, L; m_i, \delta)$ is the normalized probability: $\sum_a P_a(E, L; m_i, \delta) = 1$. The oscillating component in eq. (9) is equivalent to the $\nu_e \to \nu_a$, $a = \mu, \tau$ appearance probability multiplied by

$$\frac{4\sin^2\theta_w}{3(1-4\sin^2\theta_w)^2/4 + 4\sin^2\theta_w} \sim 0.995. \quad (10)$$

Thus, the constant off-set term $\propto (1-4\sin^2\theta_w)^2$ in eq. (9) is very small. In the limit of $\sin^2\theta_w = 1/4$ there is no contribution to the