Physics Letters B 760 (2016) 736–741

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

The width of the Roper resonance in baryon chiral perturbation theory

Jambul Gegelia [a,b], Ulf-G. Meißner [a,c], De-Liang Yao [a,*]

[a] Institute for Advanced Simulation, Institut für Kernphysik and Jülich Center for Hadron Physics, Forschungszentrum Jülich, D-52425 Jülich, Germany
[b] Tbilisi State University, 0186 Tbilisi, Georgia
[c] Helmholtz Institut für Strahlen- und Kernphysik and Bethe Center for Theoretical Physics, Universität Bonn, D-53115 Bonn, Germany

ARTICLE INFO

Article history:
Received 17 June 2016
Received in revised form 26 July 2016
Accepted 26 July 2016
Available online 29 July 2016
Editor: W. Haxton

ABSTRACT

We calculate the width of the Roper resonance at next-to-leading order in a systematic expansion of baryon chiral perturbation theory with pions, nucleons, and the delta and Roper resonances as dynamical degrees of freedom. Three unknown low-energy constants contribute up to the given order. One of them can be fixed by reproducing the empirical value for the width of the Roper decay into a pion and a nucleon. Assuming that the remaining two couplings of the Roper interaction take values equal to those of the nucleon, the result for the width of the Roper decaying into a nucleon and two pions is consistent with the experimental value.

1. Introduction

At low energies, chiral perturbation theory [1,2] provides a successful description of the Goldstone boson sector of QCD. It turns out that a systematic expansion of loop diagrams in terms of small parameters in effective field theories (EFTs) with heavy degrees of freedom is a rather complicated issue. The problem of power counting in baryon chiral perturbation theory [3] may be solved by using the heavy-baryon approach [4–6] or by choosing a suitable renormalization scheme [7–10]. The Δ resonance and (axial) vector mesons can also be included in EFT (see e.g. Refs. [11–20]). On the other hand, the inclusion of heavier baryons such as the Roper resonance is more complicated.

Despite the fact that the Roper resonance was found a long time ago in a partial wave analysis of pion–nucleon scattering data [21], a satisfactory theory of this state is still missing. The Roper is particularly interesting as it is the first nucleon resonance that exhibits a decay mode into a nucleon and two pions, besides the decay into a nucleon and a pion. Also, the Roper appears unexpectedly low in the spectrum, below the first negative parity nucleon resonance, the $S_{11}(1535)$. It is therefore timely to address this state in a chiral EFT. First steps in this direction have been made in Refs. [22–26]. In particular, the pion mass dependence of the pole mass and the width of the Roper resonance has been studied in Refs. [22,23] and the magnetic moment was investigated in Ref. [24]. The authors of Ref. [25] studied the impact of the explicit inclusion of the Roper resonance in chiral EFT on the P_{11} pion–nucleon scattering phase shift, and Ref. [26] presented new ideas on the extension of the range of applicability of chiral EFT beyond the low-energy region.

In this work we calculate the width of the Roper resonance in a systematic expansion in the framework of baryon chiral perturbation theory with pions, nucleons, the delta and Roper resonances as explicit degrees of freedom.

The paper is organized as follows: in Section 2 we specify the effective Lagrangian, in Section 3 the pole mass and the width of the Roper resonance are defined and the perturbative calculation of the width is outlined in Section 4. In Section 5 we discuss the renormalization and the power counting applied to the decay amplitude of the Roper resonance, while Section 6 contains the numerical results. We briefly summarize in Section 7.

2. Effective Lagrangian

We start by specifying the elements of the chiral effective Lagrangian which are relevant for the calculation of the width of the Roper at next-to-leading order in the power counting specified below. We consider pions, nucleons, the delta and Roper resonances as dynamical degrees of freedom. The corresponding most general effective Lagrangian can be written as

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\pi\pi} + \mathcal{L}_{\pi N} + \mathcal{L}_{\pi\Delta} + \mathcal{L}_{\pi R} + \mathcal{L}_{\pi N\Delta} + \mathcal{L}_{\pi NR} + \mathcal{L}_{\pi\Delta R}, \quad (1)$$

where the subscripts indicate the dynamical fields contributing to a given term. From the purely mesonic sector we need the following structures [2,27]

* Corresponding author.
E-mail addresses: j.gegelia@fz-juelich.de (J. Gegelia), meissner@hiskp.uni-bonn.de (U.-G. Meißner), d.yao@fz-juelich.de, yaodeliang@pku.edu.cn (D.-L. Yao).

http://dx.doi.org/10.1016/j.physletb.2016.07.068

$$\mathcal{L}_{\pi\pi}^{(2)} = \frac{F^2}{4}\langle \partial_\mu U \partial^\mu U^\dagger \rangle + \frac{F^2 M^2}{4}\langle U^\dagger + U \rangle,$$
$$\mathcal{L}_{\pi\pi}^{(4)} = \frac{1}{8} l_4 \langle u^\mu u_\mu \rangle \langle \chi_+ \rangle + \frac{1}{16}(l_3 + l_4)\langle \chi_+ \rangle^2, \tag{2}$$

where $\langle\ \rangle$ denotes the trace in flavor space, F is the pion decay constant in the chiral limit and M is the leading term in the quark mass expansion of the pion mass [2]. The pion fields are contained in the unimodular unitary 2×2 matrix U, with $u = \sqrt{U}$ and

$$u_\mu = i\left[u^\dagger \partial_\mu u - u \partial_\mu u^\dagger\right],$$
$$\chi^+ = u^\dagger \chi u^\dagger + u \chi^\dagger u\,, \quad \chi = \begin{bmatrix} M^2 & 0 \\ 0 & M^2 \end{bmatrix}. \tag{3}$$

The terms of the Lagrangian with pions and baryons contributing to our calculation read:

$$\mathcal{L}_{\pi N}^{(1)} = \bar\Psi_N \left\{ i\not{D} - m + \frac{1}{2} g \not{u} \gamma^5 \right\} \Psi_N\,,$$
$$\mathcal{L}_{\pi R}^{(1)} = \bar\Psi_R \left\{ i\not{D} - m_R + \frac{1}{2} g_R \not{u} \gamma^5 \right\} \Psi_R\,,$$
$$\mathcal{L}_{\pi R}^{(2)} = \bar\Psi_R \left\{ c_1^R \langle \chi^+ \rangle \right\} \Psi_R\,,$$
$$\mathcal{L}_{\pi NR}^{(1)} = \bar\Psi_R \left\{ \frac{g_{\pi NR}}{2} \gamma^\mu \gamma_5 u_\mu \right\} \Psi_N + \text{h.c.}\,,$$
$$\begin{aligned}\mathcal{L}_{\pi\Delta}^{(1)} = -\bar\Psi_\mu^i \xi_{ij}^{\frac{3}{2}} \Big\{ &\left(i\not{D}^{jk} - m_\Delta \delta^{jk} \right) g^{\mu\nu} \\ &- i\left(\gamma^\mu D^{\nu,jk} + \gamma^\nu D^{\mu,jk} \right) + i\gamma^\mu \not{D}^{jk} \gamma^\nu + m_\Delta \delta^{jk} \gamma^\mu \gamma^\nu \\ &+ \frac{g_1}{2} \not{u}^{jk} \gamma_5 g^{\mu\nu} + \frac{g_2}{2}(\gamma^\mu u^{\nu,jk} + u^{\nu,jk}\gamma^\mu)\gamma_5 \\ &+ \frac{g_3}{2} \gamma^\mu \not{u}^{jk} \gamma_5 \gamma^\nu \Big\} \xi_{kl}^{\frac{3}{2}} \Psi_\nu^l\,,\end{aligned}$$
$$\mathcal{L}_{\pi N\Delta}^{(1)} = h\, \bar\Psi_\mu^i \xi_{ij}^{\frac{3}{2}} \Theta^{\mu\alpha}(z_1)\, \omega_\alpha^j \Psi_N + \text{h.c.}\,,$$
$$\mathcal{L}_{\Delta R}^{(1)} = h_R\, \bar\Psi_\mu^i \xi_{ij}^{\frac{3}{2}} \Theta^{\mu\alpha}(\tilde z)\, \omega_\alpha^j \Psi_R + \text{h.c.}\,, \tag{4}$$

where Ψ_N and Ψ_R are isospin doublet fields with bare masses m_{N0} and m_{R0}, corresponding to the nucleon and the Roper resonance, respectively. The vector–spinor isovector–isospinor Rarita–Schwinger field Ψ_ν represents the Δ resonance [28] with bare mass $m_{\Delta 0}$, $\xi^{\frac{3}{2}}$ is the isospin-3/2 projector, $\omega_\alpha^i = \frac{1}{2}\langle \tau^i u_\alpha \rangle$ and $\Theta^{\mu\alpha}(z) = g^{\mu\alpha} + z\gamma^\mu\gamma^\alpha$, where z is a so-called off-shell parameter. We fix the off-shell structure of the interactions involving the delta by adopting $g_2 = g_3 = 0$ and $z_1 = \tilde z = 0$. Note that these off-shell parameters can be absorbed in LECs and are thus redundant [29–31]. Leaving out the external sources, the covariant derivatives are defined as follows:

$$D_\mu \Psi_{N/R} = \left(\partial_\mu + \Gamma_\mu\right)\Psi_{N/R}\,,$$
$$\left(D_\mu \Psi\right)_{\nu,i} = \partial_\mu \Psi_{\nu,i} - 2\,i\,\epsilon_{ijk}\Gamma_{\mu,k}\Psi_{\nu,j} + \Gamma_\mu \Psi_{\nu,i}\,,$$
$$\Gamma_\mu = \frac{1}{2}\left[u^\dagger \partial_\mu u + u \partial_\mu u^\dagger\right] = \tau_k \Gamma_{\mu,k}\,. \tag{5}$$

Note that a mixing kinetic term of the form $i\lambda_1 \bar\Psi_R \gamma_\mu D^\mu \Psi_N - \lambda_2 \bar\Psi_R \Psi_N + \text{h.c.}$ can be dropped, since, using field transformations and diagonalizing the nucleon-Roper mass matrix, it can be reduced to the form of operators of the Lagrangian presented above [22].

3. The pole mass and the width of the Roper resonance

The dressed propagator of the Roper resonance can be written as

$$iS_R(p) = \frac{i}{\not{p} - m_{R0} - \Sigma_R(\not{p})}\,, \tag{6}$$

where $-i\,\Sigma_R(\not{p})$ is the self-energy, i.e. the sum of all one-particle-irreducible diagrams contributing to the two-point function of the Roper resonance. The pole of the dressed propagator S_R is obtained by solving the equation

$$S_R^{-1}(z) \equiv z - m_{R0} - \Sigma_R(z) = 0\,. \tag{7}$$

We define the physical mass and the width of the Roper resonance by relating them to the real and imaginary parts of the pole

$$z = m_R - i\,\frac{\Gamma_R}{2}\,. \tag{8}$$

The pertinent topologies of the one- and two-loop diagrams contributing to the self-energy of the Roper resonance are shown in Fig. 1. We use BPHZ renormalization by subtracting real parts of loop diagrams in their chiral limit and replacing the parameters of the Lagrangian by their renormalized values. Imaginary parts of loop diagrams remain untouched. All counter terms responsible for these subtractions are generated by the effective Lagrangian at given order, however we do not show them explicitly.

We solve Eq. (7) perturbatively order by order in the loop expansion. We parameterize the pole as

$$z = m_2 + \hbar\delta z_1 + \hbar^2 \delta z_2 + \mathcal{O}(\hbar^3), \tag{9}$$

where $m_2 = m_R^0 + 4c_1^R M^2$, with m_R^0 the physical Roper mass in the chiral limit, and substitute in Eq. (7) in which we write the self-energy as an expansion in the number of loops

$$\Sigma_R = \hbar\Sigma_1 + \hbar^2\Sigma_2 + \mathcal{O}(\hbar^3)\,. \tag{10}$$

By expanding in powers of $\hbar$, we get

$$\begin{aligned}&\hbar\delta z_1 + \hbar^2\delta z_2 - \hbar\Sigma_1(m_2) - \hbar^2 \delta z_1 \Sigma_1'(m_2) \\ &\quad - \hbar^2\Sigma_2(m_2) + \mathcal{O}(\hbar^3) = 0\,.\end{aligned} \tag{11}$$

Solving Eq. (11) we obtain

$$\delta z_1 = \Sigma_1(m_2),$$
$$\delta z_2 = \Sigma_1(m_2)\,\Sigma_1'(m_2) + \Sigma_2(m_2). \tag{12}$$

Equations (8), (9) and (12) lead to the following expression for the width

$$\begin{aligned}\Gamma_R = \hbar\, &2i\,\mathrm{Im}\left[\Sigma_1(m_2)\right] \\ &+ \hbar^2\, 2i\left\{\mathrm{Im}\left[\Sigma_1(m_2)\right]\mathrm{Re}\left[\Sigma_1'(m_2)\right]\right. \\ &\left. + \mathrm{Re}\left[\Sigma_1(m_2)\right]\mathrm{Im}\left[\Sigma_1'(m_2)\right]\right\} \\ &+ \hbar^2\, 2i\,\mathrm{Im}\left[\Sigma_2(m_2)\right] + \mathcal{O}(\hbar^3).\end{aligned} \tag{13}$$

Using the power counting specified in section 5, it turns out that the contribution of the second term in Eq. (13) is of an order higher than the accuracy of our calculation, which is δ^5 (where δ is a small expansion parameter). In particular, $\mathrm{Im}\left[\Sigma_1(m_2)\right]$ is of order δ^3, $\mathrm{Re}\left[\Sigma_1'(m_2)\right]$ is of order δ^4, $\mathrm{Re}\left[\Sigma_1(m_2)\right]$ is of order δ^6 and $\mathrm{Im}\left[\Sigma_1'(m_2)\right]$ is of order δ^2. Also, modulo higher order corrections, we can replace m_2 by the physical mass m_R. To calculate