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Hierarchical majorana neutrinos from democratic mass matrices

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ABSTRACT

In this paper, we obtain the light neutrino masses and mixings consistent with the experiments, in the democratic texture approach. The essential ansatz is that ν_{Ri} are assumed to transform as "right-handed fields" $\mathbf{2_R} + \mathbf{1_R}$ under the $S_{3L} \times S_{3R}$ symmetry. The symmetry breaking terms are assumed to be diagonal and hierarchical. This setup only allows the normal hierarchy of the neutrino mass, and excludes both of inverted hierarchical and degenerated neutrinos.

Although the neutrino sector has nine free parameters, several predictions are obtained at the leading order. When we neglect the smallest parameters ζ_ν and ζ_R, all components of the mixing matrix $U_{\rm PMNS}$ are expressed by the masses of light neutrinos and charged leptons. From the consistency between predicted and observed $U_{\rm PMNS}$, we obtain the lightest neutrino masses $m_1 = (1.1 \to 1.4)$ meV, and the effective mass for the double beta decay $\langle m_{ee} \rangle \simeq 4.5$ meV.

1. Introduction

The observation of the neutrino oscillation [1,2] clarified finite masses of the neutrinos and lepton flavor nonconservation. Furthermore, the Daya Bay and RENO experiments [3,4] discovered that U_{e3} is nonzero and relatively large. However, these experiments shed us a further mysteries, *e.g.*, dozen of unexplained parameters, and the origin of the flavor. In particular, the lepton mixing matrix $U_{\rm PMNS}$ [5,6] is remarkably different from the quark mixing matrix $U_{\rm CKM}$ [7,8].

Innumerable models have been proposed so far, to explain the mysterious flavor structures of the standard model. As representative approaches, researchers explore the continuous or discrete flavor symmetries [9–11], and specific flavor textures [12,13]. In the texture approach, the democratic texture [14–23], realized by the $S_{3L} \times S_{3R}$ symmetry is widely studied. It assumes that the Yukawa interactions of the fermions $f = u, d, e$ have the "democratic matrix" in Eq. (1). In particular, Fujii, Hamaguchi and Yanagida [24] has derived the large mixing angles of light neutrinos by the seesaw mechanism [25], assuming almost degenerated neutrino Yukawa matrix $Y_\nu \sim c_\nu \, {\rm diag}(1,1,1)$. This degenerated Y_ν is aesthetically unsatisfactory, because it is realized by assuming that the right-handed neutrinos ν_{Ri} transform as "left-handed fields" $\mathbf{2_L} + \mathbf{1_L}$ under the $S_{3L} \times S_{3R}$ symmetry. Furthermore, the degenerated Y_ν is undesirable in viewpoints of grand unified theory (GUT). A part of previous authors also have considered the democratic matrices in SU(5) GUT [26]. However, degenerated Y_ν can not be unified to other Yukawa matrices.

Then, in this paper, ν_{Ri} are assumed to transform as "right-handed fields" $\mathbf{2_R} + \mathbf{1_R}$ under the $S_{3L} \times S_{3R}$ symmetry. The symmetry breaking terms are assumed to be diagonal and hierarchical, which is basically same as the previous studies. These assumptions realize hierarchical Y_ν and forbid degenerated Y_ν. It enables us to treat quarks and leptons uniformly under a simple framework. By the seesaw mechanism, we obtain the light neutrino masses and mixings consistent with the experiments. This setup only allows the normal hierarchy of the neutrino masses, and excludes both of inverted hierarchical and degenerated neutrinos.

Although the neutrino sector has nine free parameters, several predictions are obtained at the leading order. When we neglect the smallest parameters ζ_ν and ζ_R, the resulting neutrino matrix m_ν has only three parameters and then determined from the neutrino masses m_i. Therefore, all components of the mixing matrix $U_{\rm PMNS}$ are expressed by the masses of light neutrinos and charged leptons. From the consistency between predicted and observed $U_{\rm PMNS}$, we obtain the lightest neutrino masses $m_1 = (1.1 \to 1.4)$ meV, and the effective mass for the double beta decay $\langle m_{ee} \rangle \simeq 4.5$ meV.

In the second-order perturbation, the predictability becomes little lower. However, the hierarchical Y_ν can be unified to other Yukawa interactions in SO(10) GUT or Pati–Salam models. Relating Y_ν and Y_u in some manner, several free parameters in the neutrino sector are expected to be removed. Meanwhile, the derivation in this paper remains only at tree level. The radiative corrections [35–37] and threshold correction [38] will modify the results. We leave it for our future work.

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This paper is organized as follows. In the next section, we review the Yukawa matrices with democratic texture. In Sect. 3 and 4, we present the parameter analysis of the light neutrino mass. Section 5 is devoted to conclusions and discussions.

2. Yukawa matrices with democratic texture

In the democratic mass matrix approach [14–23], the Yukawa matrices are assumed to be the following texture:

$$Y_f = \frac{K_f}{3}\begin{pmatrix}1&1&1\\1&1&1\\1&1&1\end{pmatrix} + \begin{pmatrix}\zeta_f&0&0\\0&\epsilon_f&0\\0&0&\delta_f\end{pmatrix}, \tag{1}$$

where f is the SM fermions $f = u, d, e$. The first term (often called "democratic" mass matrix [17,18]) is realized by assigning fermions $f_{L,R}$ as $\mathbf{1_{L,R}} + \mathbf{2_{L,R}}$ under the $S_{3L} \times S_{3R}$ symmetry;

$$f'_{(L,R)i} = S^{(abc)}_{(L,R)ij} f_{(L,R)j}. \tag{2}$$

For example, right-handed fields are explicitly written as

$$u_{Ri} = \begin{pmatrix}u_R\\c_R\\t_R\end{pmatrix},\quad d_{Ri} = \begin{pmatrix}d_R\\s_R\\b_R\end{pmatrix},\quad e_{Ri} = \begin{pmatrix}e_R\\\mu_R\\\tau_R\end{pmatrix}, \tag{3}$$

and the left-handed fermions are written as similar way. The representation of $S^{(abc)}_{ij}$ is

$$S^{(123)}_{(L,R)} = \begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix},\ S^{(213)}_{(L,R)} = \begin{pmatrix}0&1&0\\1&0&0\\0&0&1\end{pmatrix},$$
$$S^{(132)}_{(L,R)} = \begin{pmatrix}1&0&0\\0&0&1\\0&1&0\end{pmatrix}, \tag{4}$$

$$S^{(321)}_{(L,R)} = \begin{pmatrix}0&0&1\\0&1&0\\1&0&0\end{pmatrix},\ S^{(312)}_{(L,R)} = \begin{pmatrix}0&0&1\\1&0&0\\0&1&0\end{pmatrix},$$
$$S^{(231)}_{(L,R)} = \begin{pmatrix}0&1&0\\0&0&1\\1&0&0\end{pmatrix}. \tag{5}$$

The second term in Eq. (1) breaks the permutation symmetry slightly [15,16]. Here, the hierarchical relation

$$K_f \gg \delta_f \gg \epsilon_f \gg \zeta_f, \tag{6}$$

is assumed. For the sake of simplicity of the discussion, we assume all breaking parameters are real. The discussion on the CP violation is given later.

The previous study by Fujii, Hamaguchi, and Yanagida [24] has derived the large mixing angles of light neutrinos by the seesaw mechanism, assuming almost degenerated neutrino Yukawa matrix $Y_\nu \sim c_\nu\,\mathrm{diag}(1,1,1)$. This degenerated Y_ν is aesthetically unsatisfactory, because it is realized by assuming that the right-handed neutrinos ν_{Ri} transform as "left-handed fields" $\mathbf{2_L} + \mathbf{1_L}$ under the $S_{3L} \times S_{3R}$ symmetry. Furthermore, the degenerated Y_ν is undesirable in viewpoints of grand unified theory (GUT).

Then, in this paper, ν_{Ri} are assumed to transform as "right-handed fields" $\mathbf{2_R} + \mathbf{1_R}$ under the $S_{3L} \times S_{3R}$ symmetry. The charge assignments of the leptons are shown in the Table 1. The symmetry breaking terms are assumed to be diagonal and hierarchical, which is basically same as the previous studies. These assumptions realize hierarchical Y_ν and forbid degenerated Y_ν. The texture of Yukawa matrices are determined as Eq. (1) for all SM leptons $f = \nu, e$.

Table 1
The charge assignments of the leptons under the discrete symmetries.

	S_{3L}	S_{3R}
l_{Li}	$\mathbf{1_L} + \mathbf{2_L}$	$\mathbf{1_R}$
ν_{Ri}, e_{Ri}	$\mathbf{1_L}$	$\mathbf{1_R} + \mathbf{2_R}$

Due to the charge assignment, the majorana mass term of ν_{Ri} invariant under the S_{3R} symmetry is found to be

$$M_R = m_R\left[\frac{K_R}{3}\begin{pmatrix}1&1&1\\1&1&1\\1&1&1\end{pmatrix} + c_R\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix} + \begin{pmatrix}\zeta_R&0&0\\0&\epsilon_R&0\\0&0&\delta_R\end{pmatrix}\right]. \tag{7}$$

Here, we assume the symmetry breaking term to M_R is also the diagonal. The term proportional to c_R is forbidden for the Y_ν by the assignment. In order to cancel out the hierarchy of Y_ν in the seesaw mechanism, the mass matrix (7) should be strongly hierarchical. Then, $K_R \gg c_R$ is required. Since the term $c_R\,\mathrm{diag}\,(1,1,1)$ is symmetric under $S_{3L} \times S_{3R}$, the parameter c_R need not necessarily be small parameter. Then, we assume

$$K_R \gg c_R \gg \delta_R \gg \epsilon_R \gg \zeta_R. \tag{8}$$

When we analyze the matrices (1), at first the democratic matrix is diagonalized by the following unitary matrix:

$$U_{\mathrm{DC}} = \begin{pmatrix}\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}}\\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}}\\ 0 & -\frac{\sqrt{2}}{\sqrt{3}} & \frac{1}{\sqrt{3}}\end{pmatrix}. \tag{9}$$

It is explicitly written as,

$$U^\dagger_{\mathrm{DC}} Y_f U_{\mathrm{DC}} \tag{10}$$

$$= \begin{pmatrix}\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0\\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & -\frac{\sqrt{2}}{\sqrt{3}}\\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}}\end{pmatrix}\left[\frac{K_f}{3}\begin{pmatrix}1&1&1\\1&1&1\\1&1&1\end{pmatrix} + \begin{pmatrix}\zeta_f&0&0\\0&\epsilon_f&0\\0&0&\delta_f\end{pmatrix}\right] \times \begin{pmatrix}\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}}\\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}}\\ 0 & -\frac{\sqrt{2}}{\sqrt{3}} & \frac{1}{\sqrt{3}}\end{pmatrix} \tag{11}$$

$$= K_f\begin{pmatrix}0&0&0\\0&0&0\\0&0&1\end{pmatrix} + \begin{pmatrix}\frac{1}{2}(\zeta_f+\epsilon_f) & \frac{1}{2\sqrt{3}}(\zeta_f-\epsilon_f) & \frac{1}{\sqrt{6}}(\zeta_f-\epsilon_f)\\ \frac{1}{2\sqrt{3}}(\zeta_f-\epsilon_f) & \frac{1}{6}(\zeta_f+\epsilon_f+4\delta_f) & \frac{1}{3\sqrt{2}}(\zeta_f+\epsilon_f-2\delta_f)\\ \frac{1}{\sqrt{6}}(\zeta_f-\epsilon_f) & \frac{1}{3\sqrt{2}}(\zeta_f+\epsilon_f-2\delta_f) & \frac{1}{3}(\zeta_f+\epsilon_f+\delta_f)\end{pmatrix}. \tag{12}$$

From the hierarchical relation (6), approximate form of this matrix found to be the "cascade texture" [27]

$$U^\dagger_{\mathrm{DC}} Y_f U_{\mathrm{DC}} \cong \frac{1}{6}\begin{pmatrix}3\epsilon_f & -\sqrt{3}\epsilon_f & -\sqrt{6}\epsilon_f\\ -\sqrt{3}\epsilon_f & 4\delta_f & -2\sqrt{2}\delta_f\\ -\sqrt{6}\epsilon_f & -2\sqrt{2}\delta_f & 6K_f\end{pmatrix}. \tag{13}$$

If we assign $\zeta_f = -\epsilon_f$, it leads to the zero texture $(U^\dagger_{\mathrm{DC}} Y_f U_{\mathrm{DC}})_{11} = 0$ [15,16,19,28], that corresponds the "hybrid texture" in Ref. [27].