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Horizon of quantum black holes in various dimensions

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ABSTRACT

We adapt the horizon wave-function formalism to describe massive static spherically symmetric sources in a general $(1 + D)$-dimensional space-time, for $D > 3$ and including the $D = 1$ case. We find that the probability P_{BH} that such objects are (quantum) black holes behaves similarly to the probability in the $(3 + 1)$ framework for $D > 3$. In fact, for $D \geq 3$, the probability increases towards unity as the mass grows above the relevant D-dimensional Planck scale m_D. At fixed mass, however, P_{BH} decreases with increasing D, so that a particle with mass $m \simeq m_D$ has just about 10% probability to be a black hole in $D = 5$, and smaller for larger D. This result has a potentially strong impact on estimates of black hole production in colliders. In contrast, for $D = 1$, we find the probability is comparably larger for smaller masses, but $P_{BH} < 0.5$, suggesting that such lower dimensional black holes are purely quantum and not classical objects. This result is consistent with recent observations that sub-Planckian black holes are governed by an effective two-dimensional gravitation theory. Lastly, we derive Generalised Uncertainty Principle relations for the black holes under consideration, and find a minimum length corresponding to a characteristic energy scale of the order of the fundamental gravitational mass m_D in $D > 3$. For $D = 1$ we instead find the uncertainty due to the horizon fluctuations has the same form as the usual Heisenberg contribution, and therefore no fundamental scale exists.

1. Introduction

Unusual causal structures like trapping surfaces and horizons can only occur in strongly gravitating systems, such as astrophysical objects that collapse and possibly form black holes. One might argue that for a large black hole, gravity should appear "locally weak" at the horizon, since tidal forces look small to a freely falling observer (their magnitude being roughly controlled by the surface gravity, which is inversely proportional to the horizon radius). Like any other classical signal, light is confined inside the horizon no matter how weak such forces may appear to a local observer. This can be taken as the definition of a "globally strong" interaction.

As the black hole's mass approaches the Planck scale, tidal forces become strong both in the local and global sense, thus granting such an energy scale a remarkable role in the search for a quantum theory of gravity. It is indeed not surprising that modifications to the standard commutators of quantum mechanics and Generalised Uncertainty Principles (GUPs) have been proposed, essentially in order to account for the possible existence of small black holes around the Planck scale, and the ensuing minimum measurable length [1]. Unfortunately, that regime is presently well beyond our experimental capabilities, at least if one takes the Planck scale at face value,[1] $m_p \simeq 10^{16}\,$TeV (corresponding to a length scale $\ell_p = \hbar/m_p = m_p\, G_N \simeq 10^{-35}\,$m). Nonetheless, there is the possibility that the low energy theory still retains some signature features that could be accessed in the near future (see, for example, Ref. [2]).

1.1. Gravitational radius and horizon wave-function

Before we start calculating phenomenological predictions, it is of the foremost importance that we clarify the possible conceptual issues arising from the use of arguments and observables that we know work at our every-day scales. One of such key concepts is

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[1] We use units where $c = 1$ and $\hbar = \ell_p\, m_p = \ell_D\, m_D$.

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the gravitational radius of a self-gravitating source, which can be used to assess the existence of trapping surfaces, at least in spherically symmetric systems. As it is very well known, the location of a trapping surface is determined by the equation

$$g^{ij}\nabla_i r \nabla_j r = 0\,, \tag{1.1}$$

where $\nabla_i r$ is perpendicular to surfaces of constant area $\mathcal{A} = 4\pi r^2$. If we set $x^1 = t$ and $x^2 = r$, and denote the matter density as $\rho = \rho(r,t)$, the Einstein field equations tell us that

$$g^{rr} = 1 - \frac{2\ell_p(m/m_p)}{r}\,, \tag{1.2}$$

where the Misner–Sharp mass is given by

$$m(r,t) = 4\pi \int_0^r \rho(\bar{r},t)\bar{r}^2\,\mathrm{d}\bar{r}\,, \tag{1.3}$$

as if the space inside the sphere were flat. A trapping surface then exists if there are values of r and t such that the gravitational radius $R_S = 2\,\ell_p\, m/m_p$, satisfies

$$R_S(r,t) \geq r\,. \tag{1.4}$$

When the above relation holds in the vacuum outside the region where the source is located, R_S becomes the usual Schwarzschild radius, and the above argument gives a mathematical foundation to Thorne's hoop conjecture [3], which (roughly) states that a black hole forms when the impact parameter b of two colliding small objects is shorter than the Schwarzschild radius of the system, that is for $b \lesssim 2\,\ell_p\, E/m_p$ where E is the total energy in the centre-of mass frame.

If we consider a spin-less point-particle of mass m, the Heisenberg principle of quantum mechanics introduces an uncertainty in the particle's spatial localisation of the order of the Compton scale $\lambda_m \simeq \ell_p\, m_p/m$.[2] Since quantum physics is a more refined description of reality, we could argue that R_S only makes sense if[3]

$$R_S \gtrsim \lambda_m \quad \Longrightarrow \quad m \gtrsim m_p\,, \tag{1.5}$$

which brings us to face the conceptual challenge of describing quantum mechanical systems whose classical horizon would be smaller than the size of the uncertainty in their position. In Refs. [6], a proposal was put forward in order to describe the "fuzzy" Schwarzschild (or gravitational) radius of a localised but likewise fuzzy quantum source. One starts from the spectral decomposition of the spherically symmetric wave-function

$$|\psi_S\rangle = \sum_E C(E)\,|\psi_E\rangle\,, \tag{1.6}$$

with the usual constraint

$$\hat{H}\,|\psi_E\rangle = E\,|\psi_E\rangle\,, \tag{1.7}$$

and associates to each energy level $|\psi_E\rangle$ a probability amplitude $\psi_H(R_S) \simeq C(E)$, where $R_S = 2\,\ell_p\, E/m_p$. From this Horizon Wave-Function (HWF), a GUP and minimum measurable length were derived [7], as well as corrections to the classical hoop conjecture [8], and a modified time evolution proposed [9]. The same approach was generalised to electrically charged sources [10], and used to show that Bose–Einstein condensate models of black holes [11–15] actually possess a horizon with a proper semiclassical limit [16].

It is important to emphasise that the HWF approach differs from most previous attempts in which the gravitational degrees of freedom of the horizon, or of the black hole metric, are quantised independently of the nature and state of the source (for some bibliography, see, e.g., Ref. [17]). In our case, the gravitational radius is instead quantised along with the matter source that produces it, somewhat more in line with the highly non-linear general relativistic description of the gravitational interaction. However, having given a practical tool for describing the gravitational radius of a generic quantum system is just the starting point. In fact, when the probability that the source is localised within its gravitational radius is significant, the system should show (some of) the properties ascribed to a black hole in general relativity. These properties, the fact in particular that no signal can escape from the interior, only become relevant once we consider how the overall system evolves.

1.2. Higher and lower dimensional models

Extra-dimensions have been proposed as a possible explanation for some of the incongruences affecting particle physics, such as the hierarchy problem between fundamental interactions. In $(1+D)$-dimensional space-times, with $D \geq 4$, gravity shows its true quantum nature at a scale m_D (possibly much) lower than the Planck mass m_p. Such scenarios have been extensively studied after the well known ADD [18] and Randall–Sundrum [19] models were proposed (see Ref. [20] for a comprehensive review). However our purpose is not to study any model in particular, but to see how the probability of a microscopic black hole formation could be affected by assuming the existence of extra dimensions. We shall therefore just consider black holes in the ADD scenario with a horizon radius significantly shorter than the size of the extra dimensions. It is then important to recall that in these models the Newton constant is replaced by the gravitational constant

$$G_D = \frac{\ell_D^{D-2}}{m_D}\,, \tag{1.8}$$

where $\ell_D = \hbar/m_D \gg \ell_p$ is the new gravitational length scale.

On the other hand, gravitational theories become much simpler in space-times with fewer than 3 spatial dimensions, where corresponding quantum theories are exactly solvable [21]. Such theories have been revisited in recent years, motivated by model-independent evidence that the number of space-time dimensions decreases as the Planck length is approached. Such formalisms – known generically as "spontaneous dimension reduction" mechanisms – have been studied in various contexts, mostly focusing on the energy-dependence of the space-time spectral dimension, including causal dynamical triangulations [22] and non-commutative geometry inspired mechanisms [23–26]. An alternative approach suggests the effective dimensionality of space-time increases as the ambient energy scale drops [27–30].

Given these arguments, we will generealize the results of Ref. [9] in an arbitrary number of spatial dimensions. In Section 2 we will introduce the concept horizon wave-function and we will apply it to a system described by a gaussian wave-packet. Subsequently, we will compute the probability that the system is a black hole in Section 3 and obtain a Generalised Uncertainty Principle in Section 4. Finally we will give some conclusions and possible outlook about the obtained results in Section 5.

[2] Strictly speaking, this bound holds in the non-relativistic limit $E \lesssim 2m$ [4], but we shall employ it in this work since we always consider particles and black holes in their rest frame.

[3] One could also derive this condition from the famous Buchdahl's inequality [5], which is however a result of classical general relativity, whose validity in the quantum domain we cannot take for granted.