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A Lagrangian for mass dimension one fermionic dark matter

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ABSTRACT

The mass dimension one fermionic field associated with Elko satisfies the Klein–Gordon but not the Dirac equation. However, its propagator is not a Green's function of the Klein–Gordon operator. We propose an infinitesimal deformation to the propagator such that it admits an operator in which the deformed propagator is a Green's function. The field is still of mass dimension one, but the resulting Lagrangian is modified in accordance with the operator.

1. Introduction

The theoretical discovery of Elko and the associated mass dimension one fermions by [2,3] is a radical departure from the Standard Model (SM). These fermions have renormalizable self-interactions and only interact with the SM particles through gravity and the Higgs boson. These properties make them natural dark matter candidates.

Since their conceptions, Elko and its fermionic fields have been studied in many disciplines. The graviational interactions of Elko have received much attention [8,9,11–18,20,23,29,36,46,47,49] while its mathematical properties have been investigated by da Rocha and collaborators [10,22,24–28,38]. These works established Elko as an inflaton candidate and that it is a flagpole spinor of the Lounesto classification [42] thus making them fundamentally different from the Dirac spinor. In particle physics, the signatures of these mass dimension one fermions at the Large Hardon Collider have been studied [7,30]. In quantum field theory, much of the attention is focused on the foundations of the construction [5,6,19,32–34,41,43–45]. Their supersymmetric and higher-spin extensions have also been carried out by [40,50]. An important result is that the fermionic field and its higher-spin generalization violate Lorentz symmetry due to the existence of a preferred direction. This led Ahluwalia and Horvath to suggest that the fermionic field satisfies the symmetry of very special relativity [4,21].

One question remains unanswered in the literature. What is the correct Lagrangian of the mass dimension one fermion? Since the field is constructed using Elko as expansion coefficients which satisfy the Klein–Gordon equation, the naive answer would be the Klein–Gordon Lagrangian. But this has two unsatisfactory aspects. Firstly, the resulting field-momentum anti-commutator is not given by the Dirac-delta function. Secondly, the propagator is not a Green's function of the Klein–Gordon operator.

We propose an infinitesimally deformed propagator such that it is an Green's function to an operator. The resulting Lagrangian determined from the operator does not have the above mentioned problems and is still of mass dimension one.

2. The Elko construct

We briefly review the construction of Elko and its fermionic field. For more details, please refer to the review article [1]. Elko is a German acronym for **E**igenspinoren des **L**adungs**k**onjugations**o**perators. They are a complete set of eigenspinors of the charge-conjugation operator of the $(\frac{1}{2},0)\oplus(0,\frac{1}{2})$ representation of the Lorentz group. The charge-conjugation operator is defined as

$$\mathcal{C}=\begin{pmatrix} O & -i\Theta^{-1} \\ -i\Theta & O \end{pmatrix} K \tag{1}$$

where K complex conjugates anything to its right and Θ is the spin-half Wigner time-reversal matrix

$$\Theta=\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}. \tag{2}$$

Its action on the Pauli matrices $\sigma=(\sigma_1,\sigma_2,\sigma_3)$ is

$$\Theta\sigma\Theta^{-1}=-\sigma^*. \tag{3}$$

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The complete set of Elko is constructed from a four-component spinor of the form

$$\chi(\mathbf{p},\alpha)=\begin{pmatrix}\vartheta\Theta\phi^*(\mathbf{p},\sigma)\\ \phi(\mathbf{p},\sigma)\end{pmatrix} \tag{4}$$

where $\phi(\epsilon,\sigma)$ is a left-handed Weyl spinor in the helicity basis with $\epsilon=\lim_{|\mathbf{p}|\to\mathbf{0}}\hat{\mathbf{p}}$ and

$$\frac{1}{2}\sigma\cdot\hat{\mathbf{p}}\,\phi(\epsilon,\sigma)=\sigma\phi(\epsilon,\sigma) \tag{5}$$

so that $\sigma=\pm\frac{1}{2}$ denotes the helicity. Here $\alpha=\mp\sigma$ denotes the dual-helicity nature of the spinor with the top and bottom signs denoting the helicity of the right- and left-handed Weyl spinors respectively. The spinor $\chi(\mathbf{p},\alpha)$ becomes the eigenspinor of the charge-conjugation operator $\mathcal{C}$ with the following choice of phases

$$\mathcal{C}\chi(\mathbf{p},\alpha)|_{\vartheta=\pm i}=\pm\chi(\mathbf{p},\alpha)|_{\vartheta=\pm i} \tag{6}$$

thus giving us four Elkos. Spinors with the positive and negative eigenvalues are called the self-conjugate and anti-self-conjugate spinors. They are denoted as

$$\mathcal{C}\xi(\mathbf{p},\alpha)=\xi(\mathbf{p},\alpha), \tag{7a}$$

$$\mathcal{C}\zeta(\mathbf{p},\alpha)=-\zeta(\mathbf{p},\alpha). \tag{7b}$$

There are subtleties involved in choosing the labellings and phases for the self-conjugate and anti-self-conjugate spinors. The details, including the solutions of the spinors can be found in [6, sec. II.A].

The Elko dual which yields the invariant inner-product is defined as [1,48]

$$\bar{\xi}(\mathbf{p},\alpha)=[\Xi(\mathbf{p})\xi(\mathbf{p},\alpha)]^\dagger\Gamma, \tag{8a}$$

$$\bar{\zeta}(\mathbf{p},\alpha)=[\Xi(\mathbf{p})\zeta(\mathbf{p},\alpha)]^\dagger\Gamma \tag{8b}$$

where $\dagger$ represents Hermitian conjugation and Γ is a block-off-diagonal matrix comprised of 2×2 identity matrix

$$\Gamma=\begin{pmatrix}O & I\\ I & O\end{pmatrix}. \tag{9}$$

The matrix $\Xi(\mathbf{p})$ is defined as

$$\Xi(\mathbf{p})=\frac{1}{2m}\sum_\alpha\left[\xi(\mathbf{p},\alpha)\bar{\xi}(\mathbf{p},\alpha)-\zeta(\mathbf{p},\alpha)\bar{\zeta}(\mathbf{p},\alpha)\right]. \tag{10}$$

The bar over the spinors denotes the Dirac dual. The dual ensures that the Elko norms are orthonormal

$$\bar{\xi}(\mathbf{p},\alpha)\xi(\mathbf{p},\alpha')=-\bar{\zeta}(\mathbf{p},\alpha)\zeta(\mathbf{p},\alpha')=2m\delta_{\alpha\alpha'} \tag{11}$$

and their spin-sums read

$$\sum_\alpha\xi(\mathbf{p},\alpha)\bar{\xi}(\mathbf{p},\alpha)=m[\mathcal{G}(\phi)+I], \tag{12a}$$

$$\sum_\alpha\zeta(\mathbf{p},\alpha)\bar{\zeta}(\mathbf{p},\alpha)=m[\mathcal{G}(\phi)-I] \tag{12b}$$

where $\mathcal{G}(\phi)$ is an off-diagonal matrix

$$\mathcal{G}(\phi)=i\begin{pmatrix}0 & 0 & 0 & -e^{-i\phi}\\ 0 & 0 & e^{i\phi} & 0\\ 0 & -e^{-i\phi} & 0 & 0\\ e^{i\phi} & 0 & 0 & 0\end{pmatrix}. \tag{13}$$

The angle ϕ is defined via the following parametrization of the momentum

$$\mathbf{p}=|\mathbf{p}|(\sin\theta\cos\phi,\sin\theta\sin\phi,\cos\theta) \tag{14}$$

where $0\le\theta\le\pi$ and $0\le\phi<2\pi$. Multiply eqs. (12a) and (12b) with $\xi(\mathbf{p},\alpha')$ and $\zeta(\mathbf{p},\alpha')$ from the right and apply the orthonormal relations, we obtain

$$[\mathcal{G}(\phi)-I]\xi(\mathbf{p},\alpha)=0, \tag{15a}$$

$$[\mathcal{G}(\phi)+I]\zeta(\mathbf{p},\alpha)=0. \tag{15b}$$

Since these identities have no explicit energy dependence, the corresponding equation in the configuration space has no dynamics and therefore cannot be the field equation for the mass dimension one fermions. Nevertheless, writing the above identities in the configuration space for $\lambda(x)$ is non-trivial and is a task that must be accomplished in order to derive the Hamiltonian. This issue is addressed in the next section.

Identifying the self-conjugate and anti-self-conjugate spinors with the expansion coefficients for particles and anti-particles, the two mass dimension one fermionic fields and their adjoints, with the appropriate normalization are

$$\Lambda(x)=(2\pi)^{-3/2}\int\frac{d^3p}{\sqrt{2mE_\mathbf{p}}}\sum_\alpha[e^{-ip\cdot x}\xi(\mathbf{p},\alpha)a(\mathbf{p},\alpha)+e^{ip\cdot x}\zeta(\mathbf{p},\alpha)b^\ddagger(\mathbf{p},\alpha)], \tag{16a}$$

$$\bar{\Lambda}(x)=(2\pi)^{-3/2}\int\frac{d^3p}{\sqrt{2mE_\mathbf{p}}}\sum_\alpha[e^{ip\cdot x}\bar{\xi}(\mathbf{p},\alpha)a^\ddagger(\mathbf{p},\alpha)+e^{-ip\cdot x}\bar{\zeta}(\mathbf{p},\alpha)b(\mathbf{p},\alpha)], \tag{16b}$$

$$\lambda(x)=\Lambda(x)|_{b^\ddagger=a^\ddagger}, \tag{16c}$$

$$\bar{\lambda}(x)=\bar{\Lambda}(x)|_{b^\ddagger=a^\ddagger}. \tag{16d}$$

Here $a(\mathbf{p},\alpha)$ and $b^\ddagger(\mathbf{p},\alpha)$ are the annihilation and creation operators for particles and anti-particles. They satisfy the standard anti-commutation relations

$$\{a(\mathbf{p}',\alpha'),a^\ddagger(\mathbf{p},\alpha)\}=\{b(\mathbf{p}',\alpha'),b^\ddagger(\mathbf{p},\alpha)\}=\delta_{\alpha'\alpha}\delta^3(\mathbf{p}'-\mathbf{p}). \tag{17}$$

Note that for the creation operators, we have introduced a new operator $\ddagger$ in place of the usual Hermitian conjugation $\dagger$. This follows from the observation that since the Dirac and Elko dual are different, it suggests that the corresponding adjoints for the respective particle states may be different. Assuming they are different, it may then become necessary to develop a new formalism for particles states with the new $\ddagger$ adjoint in parallel to [31]. This is an important issue that deserves further study but since it does not affect our objective of deriving the Lagrangian, we shall leave it for future investigation.

3. The Lagrangian: defining the problem

There are two reasons why the Klein–Gordon Lagrangian are unsatisfactory for the mass dimension one fermions. Firstly, the field does not satisfy the canonical anti-commutation relations (CARs) since the field-momentum anti-commutator is not equal to $i\delta^3(\mathbf{x}-\mathbf{y})I$. Instead, it is given by[1]

$$\{\lambda(t,\mathbf{x}),\pi_{kg}(t,\mathbf{y})\}=i\int\frac{d^3p}{(2\pi)^3}e^{-i\mathbf{p}\cdot(\mathbf{x}-\mathbf{y})}[I+\mathcal{G}(\phi)] \tag{18}$$

[1] For the rest of the paper, we will be working with $\lambda(x)$, but the results hold for $\Lambda(x)$ also.