Physics Letters B 760 (2016) 244–248

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Quantum dress for a naked singularity

Marc Casals [a,b,*], Alessandro Fabbri [c,d,e,f], Cristián Martínez [g], Jorge Zanelli [g]

[a] Centro Brasileiro de Pesquisas Físicas (CBPF), Rio de Janeiro, CEP 22290-180, Brazil
[b] School of Mathematical Sciences and Complex & Adaptive Systems Laboratory, University College Dublin, Belfield, Dublin 4, Ireland
[c] Centro Studi e Ricerche E. Fermi, Piazza del Viminale 1, 00184 Roma, Italy
[d] Dipartimento di Fisica dell'Università di Bologna and INFN sezione di Bologna, Via Irnerio 46, 40126 Bologna, Italy
[e] Laboratoire de Physique Théorique, CNRS UMR 8627, Bât. 210, Université Paris-Sud 11, Univ. Paris-Saclay, 91405 Orsay Cedex, France
[f] Departamento de Física Teórica and IFIC, Universidad de Valencia-CSIC, C. Dr. Moliner 50, 46100 Burjassot, Spain
[g] Centro de Estudios Científicos (CECs), Av. Arturo Prat 514, Valdivia, Chile

A R T I C L E I N F O

Article history:
Received 27 May 2016
Accepted 20 June 2016
Available online 23 June 2016
Editor: M. Cvetič

Keywords:
Semiclassical gravity
Quantum backreaction
Cosmic censorship
Black holes
Naked singularities
BTZ

A B S T R A C T

We investigate semiclassical backreaction on a conical naked singularity space–time with a negative cosmological constant in $(2+1)$-dimensions. In particular, we calculate the renormalized quantum stress–energy tensor for a conformally coupled scalar field on such naked singularity space–time. We then obtain the backreacted metric via the semiclassical Einstein equations. We show that, in the regime where the semiclassical approximation can be trusted, backreaction dresses the naked singularity with an event horizon, thus enforcing (weak) cosmic censorship.

1. Introduction

Naked Singularities (NSs) in gravitation theory are irksome: the curvature tensor and the energy density can 'blow up'; the space–time fabric may fail to resemble a smooth manifold and it may not be possible to continue geodesics past them; the laws of physics and standard features like causality may be violated [1]. If singularities are hidden behind an event horizon, however, one can safely ignore the problem because no causal signal can reach an outside observer from the troublesome region. It is in the spirit of Penrose's Cosmic Censorship hypothesis that NSs do not occur in nature [2]. In its weak version, this hypothesis essentially states that, generically, no 'naked' (i.e., without an event horizon) space–time singularities can form in Nature. NSs have been seen to form in some settings in $(3+1)$-dimensions, e.g., [3,4], although how 'natural' and 'generic' these settings are may be a matter of debate; in higher dimensions, NSs have been seen to form in [5]. However, in none of these works, quantum effects were taken into account. That naturally leads to the question of whether NSs are stable under quantum effects or else, for example, these effects lead to the formation of an event horizon.

Quantum effects on a curved background space–time, however, are notoriously difficult to calculate. One way to incorporate quantum effects is to include them in the energy momentum tensor and to solve the 'semiclassical' Einstein equations (Eq. (2) below) for the backreaction on the metric. The quantized stress–energy tensor for matter fields suffers from well-known ultraviolet divergences and so it must be appropriately renormalized (see, e.g., [6]). Such renormalization and obtention of the corresponding backreacted gravitational field, however, is very hard to perform in practice in $(3+1)$-dimensions unless the background space–time is highly-symmetric –such as pure de Sitter or pure anti-de Sitter, (A)dS, space–times–, which is not the case for a black hole or NS space–times in $(3+1)$-dimensions. On the other hand, in a space–time with one dimension less it is possible to make significant analytical progress while the results still yield an important insight into the physical processes that take place and into what one might expect there to happen in similar settings in $(3+1)$-dimensions.

In this paper we will investigate conical defects/excesses in $(2+1)$-AdS space–time. These are a particular class of NSs that do

* Corresponding author.
E-mail addresses: mcasals@cbpf.br, marc.casals@ucd.ie (M. Casals), afabbri@ific.uv.es (A. Fabbri), martinez@cecs.cl (C. Martínez), z@cecs.cl (J. Zanelli).

http://dx.doi.org/10.1016/j.physletb.2016.06.044
0370-2693/

not seem to give rise to catastrophic phenomena. Like in an ordinary cone, the curvature singularity is a Dirac delta distribution at the tip. The source that produces this curvature can be identified with a point particle, which can also be understood as a removed point from the manifold [7]. The geometry with the conical singularity is obtained by identification under a Killing vector in the universal covering of anti-de Sitter space–time, $CAdS_3$, in complete analogy with the $2+1$ (BTZ) black hole [8,9]. Since the identification does not change the local geometry, the conical singularity is a locally AdS space–time.

The static circularly symmetric metric in Schwarzschild-like coordinates, $-\infty < t < \infty$, $0 \le r < \infty$, $0 \le \theta \le 2\pi \approx 0$, is given by

$$ds^2 = -\left(\frac{r^2}{\ell^2} - M\right) dt^2 + \left(\frac{r^2}{\ell^2} - M\right)^{-1} dr^2 + r^2 d\theta^2, \tag{1}$$

where the mass M is an integration constant and the cosmological constant is given by $\Lambda = -\ell^{-2}$ [8]. This metric corresponds to a family of extrema of the vacuum Einstein–Hilbert action in $(2+1)$-dimensions. In three dimensions, black holes and conical singularities are just different parts of the spectrum of pure gravity, with black holes occupying the mass range $M > 0$ and naked conical singularities corresponding to $0 > M \neq -1$. The case $M = -1$ corresponds to AdS_3.

The naked singularity is of a conical type at $r = 0$, with deficit/excess angle $\Delta \equiv 2\pi(1 - \sqrt{-M})$: for $0 > M > -1$ there is an angular defect, while for $-1 > M$ there is an angular excess. For $M \to 0^-$ the conical deficit approaches 2π and the NS undergoes a topological transition: the cone becomes a cylinder. On the other side of the transition there is a black hole of vanishing mass $M = 0^+$. As shown in [10], conical singularities can also carry angular momentum J, with $M \le -|J|$. In the extreme case $M = -|J|$, these spinning particles, like the extreme black holes counterparts ($M = |J|$), are BPS states, admitting a supersymmetric extension and enjoying perturbative stability [11].

The identification vector ξ in $CAdS_3$ that produces the black hole has norm $\xi \cdot \xi = r^2$. Thus, the region where ξ is spacelike ($r^2 > 0$) is defined as the BTZ space–time. The region where the vector is timelike is excised in order to avoid the closed timelike curves produced by the identification, generating a causal boundary at $r = 0$. On the other hand, the conical singularity is produced by identifying with a rotation Killing vector $\eta \equiv \Delta\partial_\theta$, in AdS_3, where θ is the azimuthal angle and Δ is the conical deficit around $r = 0$. This Killing vector is spacelike everywhere and therefore does not produce closed timelike curves. However, the identification gives rise to a conical singularity at $r = 0$, the fixed point of η. The opposite of a conical defect, an angular excess, is also a NS with a "negative angular deficit", which is not produced by an identification, but by an insertion of an angular wedge.

These features make conical singularities in AdS_3 as acceptable as black holes. The question we wish to address, then, is, what happens in the geometry of a conical singularity when one includes vacuum fluctuations of some matter field: does the conical defect of the NS grow? What is the fate of the singularity? In this paper we investigate precisely this issue on a non-rotating, naked conical singularity space–time in $(2+1)$-dimensions with a negative cosmological constant and find that quantum effects create an event horizon surrounding a curvature singularity.

This paper is organized as follows. In Sec. 2, we calculate the renormalized expectation value of the stress–energy tensor for a scalar field in a NS space–time after reviewing the corresponding literature result in a black hole space–time. In Sec. 2.2 we analytically calculate the quantum-backreacted metric. We finish in Sec. 4 with a discussion of our results. We use units $c = 1$, $G = 1/8$ throughout.

2. Renormalized stress–energy tensor

In [12] it was shown that quantum fluctuations of a scalar field with periodic boundary conditions around a black hole make its horizon radius grow.[1] Hence, a black hole will remain a black hole if the quantum fluctuations are included. Here we explore the effect of quantum fluctuations on a conical singularity: does the conical singularity remain naked, or do the quantum corrections dress this singularity with an event horizon? Our analysis shows that the latter is the case. A similar question in flat (zero cosmological constant) $(2+1)$-dimensional space–time was raised by Souradeep and Sahni [15] and by Soleng [16], who showed that quantum effects on a conical singularity in flat space turn it into a $(2+1)$-dimensional 'Schwarzschild-like' space–time with gravitational attraction. In flat $2+1$ space–time, an analogous question was also addressed in the context of an accelerated C-metric in [17].

In order to address the above question, we consider the semi-classical Einstein equations

$$G_{\mu\nu} - \ell^{-2} g_{\mu\nu} = \kappa \left\langle \hat{T}_{\mu\nu} \right\rangle_{\rm ren}, \tag{2}$$

where $G_{\mu\nu}$ is the Einstein tensor for the metric $g_{\mu\nu}$ and $\kappa = 8\pi G$. These equations determine the perturbed metric via the renormalized expectation value of the stress–energy tensor (RSET), $\left\langle \hat{T}_{\mu\nu} \right\rangle_{\rm ren}$, of the matter field in some quantum state. We consider as quantum source a conformally coupled scalar field without a mass parameter, whose (unrenormalized) expectation value of the stress–energy tensor is given by [6,20]:

$$\langle \hat{T}_{\mu\nu}(x) \rangle = \lim_{x' \to x} \frac{\hbar}{4} \left[3\nabla^x_\mu \nabla^{x'}_\nu - g_{\mu\nu} g^{\alpha\beta} \nabla^x_\alpha \nabla^{x'}_\beta - \nabla^x_\mu \nabla^x_\nu - \frac{1}{4\ell^2} g_{\mu\nu} \right] G(x, x'), \tag{3}$$

where x and x' are space–time points. Here, $G(x, x')$ is Hadamard's elementary two-point function, i.e., the anticommutator $\langle \{\hat{\Phi}(x), \hat{\Phi}(x')\} \rangle$, where $\hat{\Phi}(x)$ is the quantum scalar field. The quantum state of the field where the expectation values of the stress–energy tensor and of the two-point function are evaluated is determined by imposing boundary conditions on the solutions of the field equations. In the present analysis, we choose for the two-point function $G(x, x')$ for the scalar field to satisfy 'transparent' boundary conditions [12]. Imposing transparent boundary conditions corresponds to quantizing the scalar field using modes which are smooth on the entire Einstein static universe [18,19].

We first review the calculation of the RSET existing in the literature in the case of the black hole and afterwards we derive our new results in the case of the NS.

2.1. Black hole case ($M > 0$)

In the BTZ black hole case, the RSET in Eq. (3) when the scalar field satisfies 'transparent' boundary conditions takes the form [12, 20]:

$$\kappa \langle \hat{T}^\mu{}_\nu \rangle_{\rm ren} = \frac{l_P}{r^3} F_{BH}(M) \mathrm{diag}(1, 1, -2), \tag{4}$$

in $\{t, r, \theta\}$ coordinates, where $l_P = \hbar G$ is the Planck length and $F_{BH}(M)$ is a function that we give in Eq. (8) below. The two-point function in the BTZ black hole case can be calculated via the

[1] This result was extended to non-conformal coupling for the massless black hole in [13] and to the four-dimensional planar massless black hole metric, in the conformal case, in [14].