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Time of flight of ultra-relativistic particles in a realistic Universe: A viable tool for fundamental physics?

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ABSTRACT

Including the metric fluctuations of a realistic cosmological geometry we reconsider an earlier suggestion that measuring the relative time-of-flight of ultra-relativistic particles can provide interesting constraints on fundamental cosmological and/or particle parameters. Using convenient properties of the geodetic light-cone coordinates we first compute, to leading order in the Lorentz factor and for a generic (inhomogeneous, anisotropic) space–time, the relative arrival times of two ultra-relativistic particles as a function of their masses and energies as well as of the details of the large-scale geometry. Remarkably, the result can be written as an integral over the unperturbed line-of-sight of a simple function of the local, inhomogeneous redshift. We then evaluate the irreducible scatter of the expected data-points due to first-order metric perturbations, and discuss, for an ideal source of ultra-relativistic particles, the resulting attainable precision on the determination of different physical parameters.

It is well known that times of flight of ultra-relativistic (UR) particles received from a distant astrophysical source depend on the particle mass m, on the particle energy E measured by the observer, and on the details of the space–time geometry in which the particle trajectory is embedded.

The first pioneer study on this subject [1] has shown, in particular, that the observation of the relative arrival times of neutrinos of different energies emitted in Supernovae explosions can provide significant information on neutrino masses. In a later, complementary paper [2] it has been pointed out that measuring the relative arrival times of neutrinos and photons (or of different neutrino species), and knowing neutrino masses, energies, and the redshift of the source, one can in principle obtain numerical estimates of cosmological parameters (such as the present values of the Hubble and deceleration parameters).

The results presented in [1,2] are both based on the homogeneous and isotropic cosmology described by the standard Friedman–Lemaître–Robertson–Walker (FLRW) metric. In this case the flight-time difference between two UR particles, emitted by the same source at time τ_s, can be expressed, to lowest order in the inverse Lorentz factor $\gamma^{-1} = m/E$, as [2]:

$$\Delta\tau = \tau_1 - \tau_2 = \left(\frac{m_1^2}{2E_1^2} - \frac{m_2^2}{2E_2^2}\right) \int_{\tau_s}^{\tau_o} \frac{d\tau}{1+z(\tau)}. \quad (1)$$

Here τ is the proper time of the observer (with τ_o the arrival time at the observer of massless particles emitted by a source at time τ_s), $m_{1,2}$ and $E_{1,2} \gg m_{1,2}$ are energies and masses of the two particles as measured by the observer, and z is the cosmological redshift $1+z = a_o/a$, where a is the scale factor of the FLRW geometry.

The Universe, however, is full of structure at different length scales. An interesting question is how Eq. (1) is affected by inhomogeneities when these are not assumed to be negligible. A priori one might expect that inhomogeneities could alter (1) by terms proportional to a lower power of m/E. More generally, such effects should be taken into account if one wants to connect precisely the data to cosmological and/or particle physics parameters. In this

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Letter we exploit the remarkable properties of the so-called geodetic light-cone (GLC) coordinates [3] to answer the above questions. The basic simplification is that null geodesics are extremely simple to describe in GLC coordinates. UR (or nearly null) geodesics turn out to be sufficiently simple for the problem to be tractable.

We start by recalling the definition of GLC coordinates [3] and some already well known properties of them (see also [4] for a recent discussion). They consist of a timelike coordinate τ, a null coordinate w, and two angular coordinates $\tilde{\theta}^a$ ($a=1,2$). The parameter τ can be identified with the proper time in the synchronous gauge and thus provides the four-velocity of a static geodesic observer in the form $u_\mu = -\partial_\mu \tau$. The GLC line-element depends on six arbitrary functions ($\Upsilon, U^a, \gamma_{ab}=\gamma_{ba}, a,b=1,2$), and takes the form:

$$ds^2 = \Upsilon^2 dw^2 - 2\Upsilon dw d\tau + \gamma_{ab}(d\tilde{\theta}^a - U^a dw)(d\tilde{\theta}^b - U^b dw), \quad (2)$$

where γ_{ab} and its inverse γ^{ab} lower and raise the two-dimensional indices.

In the GLC coordinates the (interior of the) past light-cone of a given observer is defined by $w = (<)\ w_o =$ constant. Furthermore, null geodesics stay at fixed values of the angular coordinates $\tilde{\theta}^a = \tilde{\theta}^a_o =$ constant, with $\tilde{\theta}^a_o$ specifying the source direction at the observer position. Finally, the redshift z of a signal propagating along a light-cone, emitted at time τ by a comoving source and received at time τ_o by a comoving observer, is given by a simple generalization of the standard FLRW expression:

$$1+z = \Upsilon(\tau_o, w_o, \tilde{\theta}^a_o)/\Upsilon(\tau, w_o, \tilde{\theta}^a_o). \quad (3)$$

The above properties of the GLC coordinates have already found several interesting applications [3–15]. In the present context we are interested in describing a family of almost null geodesics that start from a source lying on a past light-cone $w = w_o$ at a given z. The geodesics, however, reach the observer at later values of w, $w = w_i$. The latter will depend on the Lorentz factor γ_i of the ith particle which thus travels between the two light-cones $w = w_o$ and $w = w_i$.

We then write down the standard geodesic equation and mass-shell constraint for a point particle of mass m, propagating in the metric (2). The latter condition reads

$$2(\Upsilon\dot{\tau})\dot{w} - \gamma_{ab}\dot{\tilde{\theta}}^a\dot{\tilde{\theta}}^b + 2U_a\dot{\tilde{\theta}}^a\dot{w} - (\Upsilon^2+U^2)\dot{w}^2 = 1\,, \quad (4)$$

where a dot denotes differentiation with respect to the particle's proper time $ds = \sqrt{-dx^\mu dx^\nu g_{\mu\nu}}$. In order to make the extrapolation to the massless limit smooth, let us rescale proper time by the Lorentz factor at the observer, γ_o. In that case the r.h.s. of Eq. (4) becomes $m^2/E^2 \ll 1$, with E the energy measured by the observer. Our claim now is that there is a perturbative hierarchy among the quantities $\dot{\tau}, \dot{w}, \dot{\tilde{\theta}}^a$, with:

$$\dot{\tau} \sim \gamma^0, \quad \dot{\tilde{\theta}}^a \sim \gamma^{-1}, \quad \dot{w} \sim \gamma^{-2}\,. \quad (5)$$

We will check below that such an assumption is self consistent. Assuming it, we can rewrite (4) in the form:

$$2(\Upsilon\dot{\tau})\dot{w} - \gamma_{ab}\dot{\tilde{\theta}}^a\dot{\tilde{\theta}}^b + 2U_a\dot{\tilde{\theta}}^a\dot{w} + \dots = \frac{m^2}{E^2}\,, \quad (6)$$

where the dots represent next-to-next-to-leading contributions. Analogously, the geodesic equations read:

$$(\Upsilon\dot{\tau})^{\cdot} = \left(U^a\gamma_{ab,\tau} - U_{b,\tau}\right)\dot{\tau}\dot{\tilde{\theta}}^b + \dots\,, \quad (7)$$

$$\ddot{w} = -\frac{1}{2\Upsilon}\gamma_{ab,\tau}\dot{\tilde{\theta}}^a\dot{\tilde{\theta}}^b - \frac{1}{\Upsilon}\left(\Upsilon_{,a} - U_{a,\tau}\right)\dot{w}\dot{\tilde{\theta}}^a + \dots\,, \quad (8)$$

$$\ddot{\tilde{\theta}}^a = -\gamma^{ab}\gamma_{bc,\tau}\dot{\tau}\dot{\tilde{\theta}}^c - \gamma^{ab}\left(\Upsilon_{,b} - U_{b,\tau}\right)\dot{\tau}\dot{w} - \left(\gamma^{ab}\Gamma_{cd\ b} + \frac{1}{2\Upsilon}U^a\gamma_{cd,\tau}\right)\dot{\tilde{\theta}}^c\dot{\tilde{\theta}}^d + \dots\,, \quad (9)$$

where $\Gamma_{cd\ b} = \frac{1}{2}(\gamma_{cb,d} + \gamma_{db,c} - \gamma_{cd,b})$. It is a straightforward (though tedious) exercise to verify that, to next to leading order included, the constraint (4) is preserved by the evolution equations (7), (8) and (9).

At the same level of approximation, we find immediately from (6) that

$$2\dot{w} = \frac{\frac{m^2}{E^2} + \gamma^{ab}J_aJ_b}{\Upsilon\dot{\tau} + U_a\dot{\tilde{\theta}}^a}\,, \quad (10)$$

where $J_a \equiv \gamma_{ab}\dot{\tilde{\theta}}^b$. This equation is clearly consistent with (5) since the numerator is of order γ^{-2} while the denominator is of $O(1)$ with a relative correction $O(\gamma^{-1})$. Another straightforward calculation shows that (10) gives the correct result for $\ddot{w}$ once Eqs. (7)–(9) are used. A useful input for this check is the smallness ($O(\gamma^{-2})$) of the first derivative of J_a

$$\dot{J}_a = \frac{1}{2}\left(\gamma_{bc,a} - \frac{1}{\Upsilon}U_a\gamma_{bc,\tau}\right)\dot{\tilde{\theta}}^b\dot{\tilde{\theta}}^c - \left(\Upsilon_{,a} - U_{a,\tau}\right)\dot{\tau}\dot{w}\,. \quad (11)$$

The quantity we need to compute is $dw/d\tau = \dot{w}/\dot{\tau}$. From (10) we obtain, to leading order in m/E,

$$\frac{dw}{d\tau} = \frac{\Upsilon}{2(\Upsilon\dot{\tau})^2}\left(\frac{m^2}{E^2} + \gamma^{ab}J_aJ_b\right). \quad (12)$$

We now note that the time dependence of both $\Upsilon\dot{\tau}$ and J_a appears only at higher order in m/E thanks to Eqs. (7) and (11), respectively. Evaluating $\Upsilon\dot{\tau}$ at the observer gives simply $\Upsilon_o \equiv \Upsilon(\tau_o, w_o, \tilde{\theta}_o)$ (because of the rescaling we adopted on the proper time). Integrating now (12) from the source to the observer (along the geodesic) gives:

$$w_i - w_o = \frac{1}{2}\int_{\tau_s}^{\tau_o} d\tau \frac{\Upsilon}{\Upsilon_o^2}\left(\frac{m_i^2}{E_i^2} + \gamma^{ab}J_aJ_b\right), \quad (13)$$

where, to this order in γ^{-1}, τ_i has been taken equal to τ_o. There are two further simplifications that we can apply to our final result (13). The first is that J_a is zero at the observer (and then also all along the geodesic, because of its approximate constancy) for a geodesic arriving exactly at the observer. The same is true for the quantity $\gamma^{ab}J_aJ_b$ appearing in (13), since it can also be written as $\gamma_{ab}\dot{\tilde{\theta}}^a\dot{\tilde{\theta}}^b$. The second observation is that the integral in Eq. (13) can be taken along the unperturbed null geodesic (with constant $\tilde{\theta}^a$ and w), since deviations from it are subleading.

Let us finally, compare two such geodesics starting from the same source at the same time τ_s. Their relative time delay can be easily obtained by subtracting two equations like (13) to yield:

$$w_1 - w_2 = \left(\frac{m_1^2}{2E_1^2} - \frac{m_2^2}{2E_2^2}\right)\int_{\tau_s}^{\tau_o} d\tau \frac{\Upsilon}{\Upsilon_o^2}(\tau, w_o, \tilde{\theta}^a_o)\,,$$

$$\tau_1 - \tau_2 = \left(\frac{m_1^2}{2E_1^2} - \frac{m_2^2}{2E_2^2}\right)\int_{\tau_s}^{\tau_o} \frac{d\tau}{1+z(\tau, w_o, \tilde{\theta}^a_o)}\,, \quad (14)$$

where we used Eq. (3) and $\Delta\tau \equiv \tau_1 - \tau_2 = \Upsilon_o(w_1 - w_2)$ (see also [3]).

This is our main result showing that, to leading order in $\gamma_{1,2}^{-1}$, the arrival-time difference is very similar to the FLRW expression in Eq. (1), with the only difference that the redshift along the (massless) line-of-sight, being the exact redshift associated with a generic (inhomogeneous and anisotropic) geometry, is no longer just a function of time. The obtained geometric corrections, to leading order again, are the same for the two particles and thus