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Dalitz plot distributions in presence of triangle singularities

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ABSTRACT

We discuss properties of three-particle Dalitz distributions in coupled channel systems in presence of triangle singularities. The single channel case was discussed long ago [1] where it was found that as a consequence of unitarity, effects of a triangle singularity seen in the Dalitz plot are not seen in Dalitz plot projections. In the coupled channel case we find the same is true for the sum of intensities of all interacting channels. Unlike the single channel case, however, triangle singularities do remain visible in Dalitz plot projections of individual channels.

1. Introduction

Under specific kinematic conditions [2], triangle diagrams [3] have singularities that can mimic resonance poles. For this reason partial wave peaks at energies that do not match the known hadron spectrum *e.g.* as expected from the quark model, have occasionally been attributed to such effects. Most recently, for example, triangle singularities have been discussed in the context of the XYZ quarkonium peaks [4–11], the peak in the $J^{PC} = 1^{++}[\rho\pi]$ partial wave [12], *i.e.* the $a_1(1420)$ seen in the COMPASS data on pion diffractive dissociation [13], or the pentaquark signal [14,15] reported by the LHCb collaboration [16]. Triangle singularities have a simple interpretation when the underlying amplitude is expressed as a dispersive integral. In Fig. 1 we show a triangle diagram describing decay of a quasi-stable particle D of mass M_D to three stable particles, A_α, B_α, C through coupling to a pair of particles A_β, B_β. In the following, for simplicity, we ignore all particle spins and consider a case of two coupled two-body channels, $(\alpha, \beta = 1, 2)$. The triangle diagram can be expressed through a dispersive integral in which the on-shell amplitude describing t-channel exchange of a particle of mass λ is projected onto the s-channel partial wave and unitarized. The projected amplitude (in the next section denoted by $b_{l,\alpha}(s)$) has two of its four branch points moving as a function of λ [11]. For a range of (real) λ^2, determined by the Coleman–Norton condition [2], one of these branch points, s_T is located infinitesimally below the real s-axis and above the s-channel threshold, s_β. This leads to a logarithmic branch point in the dispersive integral located on the second sheet just below the physical region (the physical region is defined as $s + i\epsilon$).

The triangle singularity is constrained by the two-body unitarity. The Coleman–Norton condition requires $\lambda \geq B + C$. Taking into account t-channel unitarity this implies that only resonances (and not stable particles) are involved. Due to the finite resonance width the singular point $s = s_T$ is shifted away from the physical region down the s-channel unitary cut and onto the second sheet.[1] The analysis is similar to that of the standard Muskhelishvili–Omnes problem [17–19] with the only difference being that in the case considered here the left hand cut is actually located in the complex s-plane and for narrow t-channel resonances may be close to the physical region, *i.e.* near the right hand cut. In other applications of triangle diagrams, however, two-body unitarity is not sufficient. For example in the analysis of the $a_1(1420)$ [12] the t-channel exchange of a stable kaon connects the $f_0(980)\pi$ and $K^*\bar{K}$, *aka* $K\bar{K}\pi$ three-particle states. In this cases it is necessary to invoke three-body unitary to constrain the triangle amplitude.

In the following we give a detailed discussion of the coupled Muskhelishvili–Omnes (MO) problem in presence of triangle singularities. In particular we determine what type of structures are to be expected in the Dalitz plot distributions. The single channel case was discussed in [1] and revisited in [20]. In particular, in [20] it was shown that inelasticities can invalidate the result derived for the single channel case [1] but the explicit formulas for the coupled channel amplitudes were not given. The reason

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[1] If the singularity was located on the physical axis it would violate the s-channel unitarity.

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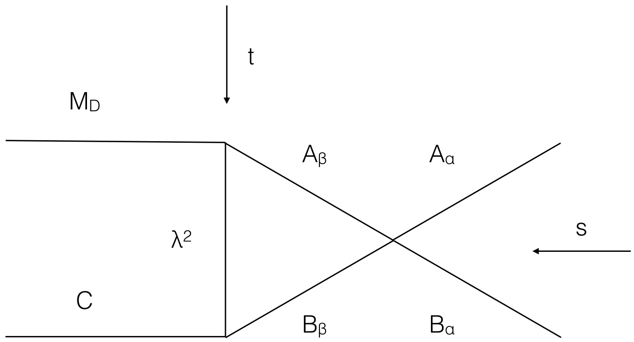

Fig. 1. Triangle diagram representing the process $D \to A_\alpha B_\alpha C$ with a t-channel exchange of a pole at $t = \lambda^2 - i\epsilon$ with couple channel interactions in the s-channel.

why generalization to coupled channels is of interest is because, for example, the XYZ phenomena tend to occur in vicinity of several open quasi-two-body channels.

2. Combining *s*, *t*, and *u*, channel isobars

We are interested in amplitudes describing a decay of a quasi-stable particle D with mass M_D to two channels, $\alpha = 1, 2$ of three distinguishable particles A_α, B_α, C. The decay amplitude $A_\alpha(s,t,u)$, depends on the three Mandelstam invariants, which we define as $s = (p_A + p_B)^2$, $t = (p_B + p_C)^2$ and $u = (p_A + p_C)^2$ and are kinematically constrained by $s + t + u = \sum_i m_i^2$. Analyticity of the S-matrix implies that, besides the decay channel, the same amplitude describes each of the three two-to-two scattering processes, *i.e.* the s-channel reaction $D + \bar{C} \to A + B$, (bar denotes an antiparticle) as well as the t and u channel scattering. Therefore, the amplitude in the physical domain of the decay process can be obtained by analytical continuation of the amplitude from, say the s-channel scattering physical region. Partial wave expansion in the s-channel,

$$A_\alpha(s,t,u) = \frac{1}{4\pi}\sum_l (2l+1) f_{l,\alpha}(s) P_l(z_s) \tag{1}$$

with z_s being cosine of scattering angle, converges in the s-channel physical region and in the decay region ($|z_s| < 1$). In the s-channel physical region, complexity of the partial waves, $f_{l,\alpha}(s)$ is determined by s-channel singularities. In the decay channel, however, in addition to the s-channel, t and u channel singularities are also physical and contribute to the complexity of the s-channel partial waves. It follows that in order to use Eq. (1) in the kinematical region of the decay process, the sum on *r.h.s.* has to be analytically continued. Therefore a finite set of s-channel partial waves cannot reproduce t or u-channel singularities, *e.g.* a resonance that appears inside the Dalitz plot. In the isobar model, in which a finite number of s-channel partial waves is considered, the omitted infinite number of waves is replaced by a finite number of t ad u waves. The amplitude has a mixed form that includes partial waves (isobars) in the three channels simultaneously,

$$A_\alpha(s,t,u) = A^{(s)}(s) + A^{(t)}(t) + A^{(u)}(u),\ A^{(x)}(x) = \frac{1}{4\pi}\sum_{l=0}^{L_{max}} (2l+1) a^{(x)}_{l,\alpha}(x) P_l(z_x),\ x = s,t,u. \tag{2}$$

We refer to the amplitudes $a^{(x)}_{l,\alpha}(x)$ as the isobaric amplitudes in the x'th channel. The isobaric amplitudes, say in the s-channel, $a^{(s)}_{l,\alpha}(s)$ contain the s-channel unitary cut and may also contain left hand cuts. To avoid double counting, however, the latter should not overlap with the cuts that originate from projections onto the s-channel partial waves of the t and u-channel isobars. In the following we ignore any remaining, distant left hand cuts of the isobaric amplitudes. In a Dalitz plot analysis, the isobaric amplitudes are typically parametrized using energy dependent Breit–Wigner formulae but this can be easily generalized [21].

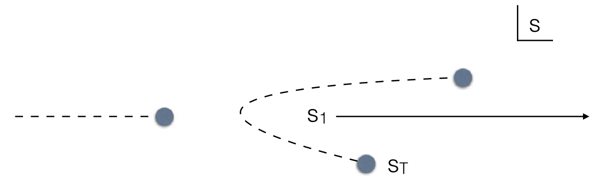

Fig. 2. Location of cuts (dashed lines) of the amplitude $b_{0,1}(s)$ in the complex s plane. The triangle singularity is due to the s_T branch point located below the real s-axis and to the right of the channel-1 threshold, s_1.

We examine implications of a triangle singularity present in the t-channel in one of the two channels, *e.g.* in $D + \bar{A}_1 \to B_1 + C$ and ignore the u-channel exchange contributions, *e.g.* set $A^{(u)} = 0$. For simplicity, we also assume that only S-wave ($l = 0$) interactions between pars A_α, B_α are strong and are given by a 2×2 set of unitary, S-wave amplitudes $t_{0,\alpha\beta}(s)$, satisfying,

$$\Delta t_{0,\alpha\beta}(s) = Im t_{0,\alpha\beta} = \sum_{\gamma=1,2} t^*_{0,\alpha,\gamma}(s)\rho_\gamma(s) t_{0,\gamma\beta}(s). \tag{3}$$

Here Δ denotes the right hand cut discontinuity, and $\rho_\alpha(s)$ is the channel phase space $\rho_\alpha(s) = \lambda(s, m^2_{A_\alpha}, m^2_{B_\alpha})/2\sqrt{s}$ with λ being the triangle function. Projecting the *r.h.s.* of Eq. (2) onto the s-channel gives the partial wave expansion of the model,

$$f_{l,\alpha}(s) = a_{l,\alpha}(s) + b_{l,\alpha}(s), \tag{4}$$

with $a_{l,\alpha}(s) = a^{(s)}_{l,\alpha}(s)$ and nonzero only for $l = 0$. For all l's,

$$b_{l,\alpha}(s) = \frac{1}{2}\int_{-1}^{1} dz_s P_l(z_s) \sum_{l'=0}^{L_{max}} (2l'+1) a^{(t)}_{l',\alpha}(t + i\epsilon) P_{l'}(z_t). \tag{5}$$

Under the integral, t and z_t, the cosine of the t-channel scattering angle, are to be considered as functions of s and z_s. The amplitude $b_{l,\alpha}(s)$ is the s-channel projection of t channel exchanges and has complex singularities in the s-plane. The location of these singularities is determined by unitarity in the t-channel. Unitarity leads to an amplitude that is analytical in the t-channel physical region *i.e.* for t infinitesimally above the real axis. Note that there is no need to make M_D complex since our amplitudes have no singularities in external masses.

Unitarity in the s-channel determines discontinuity of the $f_{l,\alpha}(s)$, partial wave across the right hand cut. With the assumption, that A_α and B_α interact strongly in the S-wave only we find,

$$\Delta f_{0,\alpha}(s) = \Delta a_{0,\alpha}(s) = \sum_{\beta=1,2} t^*_{0,\alpha\beta}\rho_\beta(s) f_{0,\beta}(s),$$
$$\Delta f_{l,\alpha}(s) = 0,\ \text{for } l > 0. \tag{6}$$

The reason why it is Δf and not $Im f$ appears on the *l.h.s.* of the unitary equation is the decay kinematics. As discussed below Eq. (1), cross channel exchanges are physical in the direct channel and lead to additional (beyond the one determined by s-channel unitarity) complexity of the s-channel partial waves. As a function of s, the projected amplitudes, $b_{l,\alpha}(s)$ have the left hand cut but do not have the right hand s-channel unitary cut. In particular, in presence of triangle singularities, when the Coleman–Norton conditions are met, [2], a portion of the left hand cut of $b_{l,\alpha}(s)$ surrounds the s-channel threshold branch point as illustrated in Fig. 2.