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Deformed phase space Kaluza–Klein cosmology and late time acceleration

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ABSTRACT

The effects of phase space deformations on Kaluza–Klein cosmology are studied. The deformation is introduced by modifying the symplectic structure of the minisuperspace variables. In the deformed model, we find an accelerating scale factor and therefore infer the existence of an effective cosmological constant from the phase space deformation parameter β.

1. Introduction

Current cosmological observations are best described by ΛCDM model [1], where the current acceleration of the universe is attributed to Λ. Unfortunately there are several theoretical problems connected to the cosmological constant, making the cosmological constant one of the central issues of modern day physics. There is a belief that the solution will come from an unconventional approach in fundamental physics (i.e. arguments are given that UV/IR mixing mechanism is needed [2]), suggesting the need of new physics. One approach to study the cosmological constant, lies in noncommutative space–time, from which several approaches to noncommutative gravity where proposed [3]. All of these formulations showed that the end result of a noncommutative theory of gravity, is a highly nonlinear theory. In order to study the effects of noncommutativity, noncommutative cosmology was presented in [4]. Although the deformations of the minisuperspace where originally studied at the quantum level, classical noncommutative formulations have been proposed [5]. The idea is based on the assumption that modifying the Poisson brackets of the classical theory gives the noncommutative equations of motion. A more general deformation of the Poisson algebra of the minisuperspace variables gives rise to deformed phase space cosmology. Phase space deformations give rise to two generally different interpretations known as the *"C-frame"* and the *"NC-frame"*, that in general are not physically equivalent [6]. For this reason we must be careful when reaching physical conclusions in phase space deformations.

These ideas have been applied in the context of the late time acceleration of the universe. In [7,8] the authors study the late time effects of minisuperspace deformations on cosmology, suggesting a relationship between late time acceleration and the deformation parameters. This was not the first time evidence was found on the possible effects of phase space deformations in the cosmological scenario. In [9] it is argued that there is a possible relation between the 4D cosmological constant and the noncommutative parameter of the compactified space in string theory. A more direct connection with the cosmological constant problem has been addressed in [10], where it is shown that by means of minisuperspace noncommutativity a small cosmological constant arises, and seems to alleviate the discrepancy between the calculated and observed vacuum energy density.

In this letter, we study deformed phase space Kaluza–Klein (KK) cosmology. We introduce the phase space deformation in the minisuperspace variables, and is achieved by modifying the symplectic structure. Finally we derive the effective cosmological constant that depends on the deformation parameters θ and β. The work is organized as follows, in section 2, we start with an empty $(4+1)$ dimensional Kaluza–Klein universe with cosmological constant and an FRW metric. In Section 3 we introduce the deformation in the phase space constructed from the minisuperspace variables and their conjugate momenta. Section 4 is devoted for conclusions and final remarks.

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2. The model

We start with an empty $(4+1)$ theory of gravity with cosmological constant Λ as shown in [11]. The action takes the form

$$I = \int \sqrt{-g}\,(R-\Lambda)\,dtd^3rd\rho. \tag{1}$$

We are interested in cosmology, so an FRW type metric is assumed

$$ds^2 = -dt^2 + \frac{a^2(t)dr^idr^i}{\left(1+\frac{\kappa r^2}{4}\right)^2} + \phi^2(t)d\rho^2, \tag{2}$$

where $\kappa = 0, \pm 1$ and $a(t)$, $\phi(t)$ are the scale factors of the universe and the compact dimension. Substituting this metric in Eq. (1), we obtain an effective lagrangian that only depends on (a,ϕ)

$$L = \frac{1}{2}\left(a\phi\dot{a}^2 + a^2\dot{a}\dot{\phi} - \kappa a\phi + \frac{1}{3}\Lambda a^3\phi\right). \tag{3}$$

Using the variables

$$x = \frac{1}{\sqrt{8}}\left(a^2 + a\phi - \frac{3\kappa}{\Lambda}\right),\ y = \frac{1}{\sqrt{8}}\left(a^2 - a\phi - \frac{3\kappa}{\Lambda}\right), \tag{4}$$

this transformation is chosen so that the Hamiltonian for the model takes the form of an isotropic oscillator-ghost-oscillator system as in [8,11],

$$H = \frac{1}{2}\left[\left(P_x^2 + \omega^2 x^2\right) - \left(P_y^2 + \omega^2 y^2\right)\right], \tag{5}$$

with $\omega^2 = -\frac{2\Lambda}{3}$. The Hamiltonian is a first-class constraint as is usual in general relativity. Since we do not have second class constraints in the model we will continue to work with the usual Poisson brackets and the commutation between the phase space variables

$$\{x_i, x_j\} = 0,\ \{P_{x_i}, P_{y_j}\} = 0,\ \{x_i, P_{x_j}\} = \delta_{ij}. \tag{6}$$

The quantum model is obtained by following the canonical formalism, from (5) we can construct the Wheeler–DeWitt (WDW) equation, and get the corresponding quantum cosmology for the model at hand. This is achieved by making the usual identifications $p_x = -i\partial/\partial x$ and $p_y = -i\partial/\partial y$,

$$\left[\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} + \omega^2(x^2 - y^2)\right]\Psi(x,y) = 0. \tag{7}$$

This equation gives the quantum description of the model and the information about the quantum behaviour would be encoded in the wave function $\Psi(x,y)$.

3. Deformed space model

As is well known, there are different approaches to include noncommutativity to physical theories. In particular, to study noncommutative cosmology, there exists a well explored path to study noncommutativity in a cosmological setting [4]. In this set up the noncommutativity is realised in the minisuperspace variables.

In canonical quantum cosmology, after canonical quantization, one formally obtains the Wheeler–DeWitt equation. This is a Klein–Gordon type equation which describes the quantum behaviour of the universe. An alternative approach to study quantum mechanical effects, is to introduce deformations to the phase space of the system. The approach is an equivalent path to quantization and is part of a complete and consistent type of quantization known as deformation quantization [12]. Our interest is in cosmology and these models are constructed in the minisuperspace, following the previous discussion we can assume the studying cosmological models in deformed phase could be interpreted as studying quantum effects to cosmological solutions [11]. In the deformed phase space approach, the deformation is introduced by the Moyal brackets $\{f,g\}_\alpha = f\star_\alpha g - g\star_\alpha f$, were the product between functions is replaced by the Moyal product $(f\star g)(x) = \exp\left[\frac{1}{2}\alpha^{ab}\partial_a^{(1)}\partial_b^{(2)}\right]f(x_1)g(x_2)|_{x_1=x_2=x}$ such that

$$\alpha = \begin{pmatrix} \theta_{ij} & \delta_{ij}+\sigma_{ij} \\ -\delta_{ij}-\sigma_{ij} & \beta_{ij} \end{pmatrix}, \tag{8}$$

where the 2×2 matrices θ_{ij} and β_{ij} are assumed to be antisymmetric and represent the noncommutativity in the coordinates and momenta respectively. The resulting α deformed algebra for the phase space variables is

$$\{x_i, x_j\}_\alpha = \theta_{ij},\ \{x_i, p_j\}_\alpha = \delta_{ij} + \sigma_{ij},\ \{p_i, p_j\}_\alpha = \beta_{ij}. \tag{9}$$

In this work we consider particular expressions for the deformations, namely $\theta_{ij} = -\theta\epsilon_{ij}$ and $\beta_{ij} = \beta\epsilon_{ij}$.

Let us consider and alternative to derive a similar algebra to Eq. (9). The resulting algebra will be the same, but the Poisson brackets are different in the two algebras. For Eq. (9) the brackets are the α deformed ones and are related to the Moyal product, for the other algebra the brackets are the usual Poisson brackets. This construction of the deformed Poisson algebra follows the approach in [7,8].

We start with the following transformation on the classical phase space variables $\{x, y, P_x, P_y\}$, that satisfy the usual Poisson algebra

$$\begin{aligned} \hat{x} &= x + \frac{\theta}{2}P_y, & \hat{y} &= y - \frac{\theta}{2}P_x, \\ \hat{P}_x &= P_x - \frac{\beta}{2}y, & \hat{P}_y &= P_y + \frac{\beta}{2}x. \end{aligned} \tag{10}$$

These new variables satisfy a deformed algebra

$$\{\hat{y},\hat{x}\} = \theta,\ \{\hat{x},\hat{P}_x\} = \{\hat{y},\hat{P}_y\} = 1+\sigma,\ \{\hat{P}_y,\hat{P}_x\} = \beta, \tag{11}$$

where $\sigma = \theta\beta/4$. Furthermore, as in [7,8], we assume that the deformed variables satisfy the same relations as their commutative counterpart.

Now that we construct the deformed theory, first we start with a Hamiltonian which is formally analogous to Eq. (5) but constructed with the variables that obey the modified algebra Eq. (11)

$$\begin{aligned} H &= \left(\frac{1}{2}\hat{P}_x^2 + \frac{\omega^2}{2}\hat{x}^2\right) - \left(\frac{1}{2}\hat{P}_y^2 + \frac{\omega^2}{2}\hat{y}^2\right) \\ &= \frac{1}{2}\left[\left(P_x^2 - P_y^2\right) - \gamma^2(xP_y + yP_x) + \widetilde{\omega}^2(x^2 - y^2)\right], \end{aligned} \tag{12}$$

where we have used the change of variables Eq. (10) and the following definitions

$$\widetilde{\omega}^2 = \frac{\omega^2 - \frac{\beta^2}{4}}{1 - \frac{\omega^2\theta^2}{4}},\quad \gamma^2 = \frac{\beta - \omega^2\theta}{1 - \frac{\omega^2\theta^2}{4}}. \tag{13}$$

The WDW equation is obtained by the usual prescription on the deformed Hamiltonian Eq. (12). The meaning of the first term in Eq. (13) is straightforward, from the definition of ω the cosmological constant is related to the oscillator frequency, then modifications to the oscillator frequency will imply modifications to the effective cosmological constant. Then $\widetilde{\omega}$ gives the effective cosmological constant $\tilde{\Lambda}_{eff}$ in the context of the WDW equation [13]. The case $\beta = 0$, is equivalent to the standard noncommutative minisuperspace model that was presented in [11] and used as a solution to the Hierarchy problem. For the physical meaning of γ