Physics Letters B 757 (2016) 312–316

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Quartet excitations and cluster spectra in light nuclei

J. Cseh *, G. Riczu

Institute for Nuclear Research, Hungarian Academy of Sciences, Debrecen, Pf. 51, 4001, Hungary

ARTICLE INFO

Article history:
Received 26 August 2015
Received in revised form 18 February 2016
Accepted 29 March 2016
Available online 4 April 2016
Editor: J.-P. Blaizot

Keywords:
Quarteting and clustering
Excitation spectrum
Multichannel dynamical symmetry

ABSTRACT

The relation of quarteting and clustering in atomic nuclei is discussed based on symmetry-considerations. This connection enables us to predict a complete high-energy cluster spectrum from the description of the low-energy quartet part. As an example the ^{28}Si nucleus is considered, including its well-established ground-state region, the recently proposed superdeformed band, and the high-lying molecular resonances.

Most of the atomic nuclei are typical mesoscopic systems, which allow neither ab initio, nor statistical description. Therefore, models play the crucial role in the understanding the nuclear structure. The fundamental structure models are based on different physical pictures, e.g. shell, cluster or liquid drop, therefore, their interrelation is not trivial. Symmetry-considerations are very helpful in finding their connection, as well as in describing complex spectra. In this letter we show how the nucleon-quarteting, which is a shell model phenomenon, is related to the clusterization, i.e. to the appearance of a molecule-like configuration. We do so by applying a semimicroscopic algebraic description for both phenomena, which reveals a special symmetry, called multichannel dynamical symmetry. This symmetry allows us to obtain a high-lying cluster spectrum from the quartet model fitted to the low-energy part. We do not know any other method of this ability.

The investigation of quarteting and clustering has a long history, and a large variety of models have been invented for their description. When the cluster is an alpha-particle, which is the most typical and best studied case, the two structures are obviously related to each other: in both cases the basic building block is composed of two protons and two neutrons. In the phenomenological approaches, which do not respect the Pauli-exclusion principle, the wavefunction of the shell-like and molecule-like configurations (or those of two different cluster configurations) are orthogonal to each other. In fact, however, the antisymmetrization modifies the simple geometric picture, and as a result, the overlap can be finite, up to 100 percent. One needs microscopically constructed model spaces for the study of this connection. (Whether the interactions are also microscopic or not, i.e. if the description is fully microscopic, or semimicroscopic is less relevant in this respect.)

In what follows we apply semimicroscopic algebraic models for the description of both quarteting and clustering. This approach takes into account the exclusion principle, furthermore, due to its fully algebraic nature it has rather transparent symmetry properties. (We call a model fully algebraic when not only the basis states, but the physical operators as well are characterized by group representations.)

The semimicroscopic algebraic quartet model (SAQM) [1] is a symmetry-governed truncation of the no-core shell model [2], that describes the quartet excitations in a nucleus. A quartet is formed by two protons and two neutrons, which interact with each other very strongly, as a consequence of the short-range attractive forces between the nucleons inside a nucleus [3]. The interaction between the different quartets is weaker. In this approach the L–S coupling is applied, the model space has a spin–isospin sector, characterized by Wigner's $U^{ST}(4)$ group [4], and a space part described by Elliott's U(3) [5]. Four nucleons form a quartet [6] when their spin–isospin symmetry is $\{1,1,1,1\}$, and their permutational symmetry is $\{4\}$. This definition allows two protons and two neutrons to form a quartet even if they sit in different shells. As a consequence the quartet model space incorporates $0,1,2,3,4,\ldots$ major shell excitations (in the language of the shell model), contrary to the original interpretation of [3], when the four nucleons had to occupy the same single-particle orbital, therefore, only $0,4,8,\ldots$ major shell excitations could be described.

The model is fully algebraic, therefore, group theoretical methods can be applied in calculating the matrix elements. The opera-

* Corresponding author.
E-mail address: cseh@atomki.mta.hu (J. Cseh).

http://dx.doi.org/10.1016/j.physletb.2016.03.080

tors contain parameters to fit to the experimental data, that is why the model is called semimicroscopic: phenomenologic operators are combined with microscopic model space. Due to the quartet symmetry only a single $\{1,1,1,1\}$ $U^{ST}(4)$ sector plays a role in the calculation of the physical quantities, thus the U(3) space-group and its subgroups are sufficient for characterizing the situation:

$$U(3) \supset SU(3) \supset SO(3) \supset SO(2)$$
$$|[n_1,n_2,n_3],(\lambda,\mu),\ K\ ,\ \ L\ \ ,\ \ M\rangle. \tag{1}$$

In Eq. (1) we have indicated also the representation labels of the groups which serve as quantum numbers of the basis states. Here $n = n_1 + n_2 + n_3$ is the number of the oscillator quanta, and $\lambda = n_1 - n_2$, $\mu = n_2 - n_3$. The angular momentum content of a (λ,μ) representation is as follows [5]: $L = K, K+1, \ldots, K + max(\lambda,\mu)$, $K = min(\lambda,\mu), min(\lambda,\mu) - 2, \ldots, 1$ or 0, with the exception of $K_L = 0$, for which $L = max(\lambda,\mu), max(\lambda,\mu) - 2, \ldots, 1$ or 0. In the limiting case of the dynamical symmetry, when the Hamiltonian is expressed in terms of the invariant operators of this group-chain, an analytical solution is available for the energy-eigenvalue problem (an example is shown below).

The SAQM can be considered as an effective model in the sense of [7]: the bands of different quadrupole shapes are described by their lowest-grade U(3) irreducible representations (irreps) without taking into account the giant-resonance excitations, built upon them, and the model parameters are renormalised for the subspace of the lowest U(3) irreps.

The semimicroscopic algebraic cluster model (SACM) [8], just like the other cluster models, classifies the relevant degrees of freedom of the nucleus into two categories: they belong either to the internal structure of the clusters, or to their relative motion. In other words: the description is based on a molecule-like picture. The internal structure of the clusters is handled in terms of Elliott's shell model [5] with $U^{ST}(4) \otimes U(3)$ group structure (as discussed beforehand). The relative motion is taken care of by the vibron model [9], which is an algebraic model of the dipole motion, and it has a U(3) basis, too. For a two-cluster-configuration this model has a group-structure of $U^{ST}_{C_1}(4) \otimes U_{C_1}(3) \otimes U^{ST}_{C_2}(4) \otimes U_{C_2}(3) \otimes U_R(4)$.

The model space is constructed also in this case in a microscopic way, i.e. the Pauli-forbidden states are excluded. It requires the truncation of the basis of the vibron model, as given by the Wildermuth-condition (see below for some specific examples). This condition determines the lowest-allowed quantum number of the relative motion, i.e. the allowed major shells of the (united) nucleus. Furthermore, one needs to distinguish between the Pauli-allowed and forbidden states within a major shell, too. Different methods can be applied to this purpose; e.g. by making an intersection with the U(3) shell model basis of the nucleus, which is constructed to be free from the forbidden states. The SACM is fully algebraic, and semimicroscopic in the sense discussed above.

When we are interested only in spin–isospin zero states of the nucleus (a typical problem in cluster studies, and being our case here, too), then only the space symmetries are relevant (apart from the construction of the model space). Considering, for the sake of simplicity, a binary cluster configuration the corresponding group-chain is:

$$U_{C_1}(3) \otimes U_{C_2}(3) \otimes\ U_R(4) \supset U_C(3) \otimes U_R(3) \supset$$
$$U(3) \supset SU(3) \supset SO(3) \supset SO(2). \tag{2}$$

The basis defined by this chain is especially useful for treating the exclusion principle, since the U(3) generators commute with those of the permutation group, therefore, all the basis states of an irrep are either Pauli-allowed, or forbidden [10]. In particular, this U(3) basis allows us to pick up the allowed cluster states from the U(3) shell model basis (1).

A Hamiltonian corresponding to the dynamical symmetry of group-chain (2) reads as:

$$\hat{H} = \hat{H}_{C_1} + \hat{H}_{C_2} + \hat{H}_{U_R(4)} + \hat{H}_{U_C(3)} + \hat{H}_{U_R(3)} + \hat{H}_{U(3)} + \hat{H}_{SU(3)} + \hat{H}_{SO(3)}. \tag{3}$$

We note here, that the first part

$$\hat{H}_{CM} = \hat{H}_{C_1} + \hat{H}_{C_2} + \hat{H}_{U_R(4)} + \hat{H}_{U_C(3)} + \hat{H}_{U_R(3)} \tag{4}$$

is an operator that corresponds to the pure cluster picture, while the second part

$$\hat{H}_{SM} = \hat{H}_{U(3)} + \hat{H}_{SU(3)} + \hat{H}_{SO(3)} \tag{5}$$

is a shell model Hamiltonian (of the united nucleus).

The multichannel dynamical symmetry (MUSY) [11,12] connects different cluster configurations (including the shell model limit) in a nucleus. Here the word channel refers to the reaction channel, that defines the cluster configuration.

The simplest case is a two-channel symmetry connecting two different clusterizations. It holds, when both cluster configurations can be described by an U(3) dynamical symmetry and in addition a further symmetry connects them to each other. This latter symmetry is that of the Talmi–Moshinsky transformation. It acts in the pseudo space of the particle indices, or geometrically it corresponds to the transformations between the different sets of Jacobi-coordinates associated to the cluster configurations [13,12]. The $\hat{H}_{SM}$ Hamiltonian of Eq. (5) is symmetric with respect to these transformations, therefore, it is invariant under the changes from one clusterization to the other. The cluster part of the Hamiltonian $\hat{H}_{CM}$ is affected by the transformation from one configuration to the other, of course. Nevertheless, it may remain invariant, which is the case for simple operators, like the harmonic oscillator Hamiltonian, or the quadrupole operator [12]. Due to this symmetry of the quadrupole operator, the $E2$ transitions of different clusterizations also coincide, when the MUSY holds, just like the energy eigenvalues of the symmetric Hamiltonians [12].

The MUSY is a composite symmetry of a composite system. Its logical structure is somewhat similar to that of the dynamical supersymmetry (SUSY) of nuclear spectroscopy. In the SUSY case the system has two components, a bosonic and a fermionic one, each of them showing a dynamical symmetry, and a further symmetry connects them to each other. The connecting symmetry is that of the supertransformations which change bosons into fermions or vice versa. In the MUSY case the system has two (or more) different clusterizations, each of them having dynamical symmetries which are connected to each other by the symmetry of the (Talmi–Moshinsky) transformations that change from one configuration to the other.

When the multichannel dynamical symmetry holds then the spectra of different clusterizations are related to each other by very strong constraints. The MUSY provides us with a unified multiplet structure of different cluster configurations, furthermore the corresponding energies and E2 transitions coincide exactly. Of course, it cannot be decided a priori whether the MUSY holds or not, rather one can suppose the symmetry and compare its consequences with the experimental data. In what follows we derive the spectra of two clusterizations from the quartet spectrum of the ^{28}Si nucleus.

The ^{28}Si nucleus provides us with many reasons to be chosen as an illustrative example. i) It has a well-established band-structure in the low-energy region, and to several bands SU(3) quantum numbers could be associated as a joint conclusion of experimental and theoretical investigations [14]. ii) More recently a new