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High-energy limit of quantum electrodynamics beyond Sudakov approximation

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ABSTRACT

We study the high-energy behavior of the scattering amplitudes in quantum electrodynamics beyond the leading order of the small electron mass expansion in the leading logarithmic approximation. In contrast to the Sudakov logarithms, the mass-suppressed double-logarithmic radiative corrections are induced by a soft electron pair exchange and result in enhancement of the power-suppressed contribution, which dominates the amplitudes at extremely high energies. Possible applications of our result to the analysis of the high-energy processes in quantum chromodynamics is also discussed.

In a renowned paper [1] V.V. Sudakov derived the leading asymptotic behavior of an electron scattering amplitude in quantum electrodynamics (QED) at high energy. It is determined by the "Sudakov" radiative corrections, which include the second power of the large logarithm of the electron mass m_e divided by a characteristic momentum transfer of the process per each power of the fine structure constant α. Sudakov double logarithms exponentiate and result in a strong universal suppression of any electron scattering amplitude with a fixed number of emitted photons in the limit when all the kinematic invariants of the process are large. This result plays a fundamental role in particle physics. Within different approaches it has been extended to the nonabelian gauge theories and to the subleading logarithms [2–7], which is crucial for a wide class of applications from deep inelastic scattering to Drell–Yan processes and the Higgs boson production. At the same time no significant progress has been achieved in the study of the logarithmically enhanced corrections to the subleading contributions suppressed by a power of electron mass at high energies. However, the power-suppressed contributions are of great interest. They can become asymptotically dominant at very high energies due to Sudakov suppression of the leading terms. At the intermediate energies the power corrections in many cases are phenomenologically important [8–11]. Moreover, in contrast to the Euclidean operator product expansion [12] or nonrelativistic threshold dynamics [13] very little is known about the general all-order structure of the large logarithms beyond the leading-power approximation in the high-energy limit, which is a real challenge for the effective field theory approach. This problem is now actively discussed in various contexts (see *e.g.* [14–19]). In this Letter we make the first step toward the solution of the problem and generalize the result of Ref. [1] to the leading power-suppressed contribution. We present a detailed analysis of the electron scattering in the external field and later discuss the extension of the result to more complex processes.

The amplitude $\mathcal{F}$ of the electron scattering in an external field can be parameterized in the standard way by the Dirac and Pauli form factors

$$\mathcal{F} = \bar{\psi}(p_1)\left(\gamma_\mu F_1 + \frac{i\sigma_{\mu\nu}q^\nu}{2m_e}F_2\right)\psi(p_2). \tag{1}$$

The Pauli form factor F_2 does not contribute in the approximation discussed in this Letter and we mainly focus on the high-energy behavior of the Dirac form factor F_1. We consider the limit of the on-shell electron $p_1^2 = p_2^2 = m_e^2$ and the large Euclidean momentum transfer $Q^2 = -(p_2 - p_1)^2$ when the ratio $\rho \equiv m_e^2/Q^2$ is positive and small. The Dirac form factor can then be expanded in an asymptotic series in ρ

$$F_1 = S_\lambda \sum_{n=0}^{\infty} \rho^n F_1^{(n)}, \tag{2}$$

where $F_1^{(n)}$ are given by the power series in α with the coefficients depending on ρ only logarithmically. The factor $S_\lambda = \exp\left[-\frac{\alpha}{2\pi}B(\rho)\ln\left(\lambda^2/m_e^2\right)\right]$ with $B(\rho) = \ln\rho + \mathcal{O}(1)$ accounts for the universal singular dependence of the amplitude on the auxiliary photon mass λ introduced to regulate the infrared diver-

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gences [20]. In the double-logarithmic approximation the leading term is given by the Sudakov exponent $F_1^{(0)} = e^{-x}$, with $x = \frac{\alpha}{4\pi}\ln^2\rho$ [21]. Let us outline our approach for the analysis of the power-suppressed double logarithmic contributions. We use the expansion by regions method [22–24] to get a systematic expansion of the Feynman integrals in ρ. In this method the coefficients $F_1^{(n)}$ are given by the sum over contributions of different virtual momentum regions. Each contribution is represented by a Feynman integral which in general is divergent. These spurious divergences result from the process of scale separation and have to be dimensionally regulated. The singular terms cancel out in the sum of all regions but can be used to determine the logarithmic contributions to $F_1^{(n)}$. The double logarithmic contributions are determined by the leading singular behavior of the integrals and can be found by the method developed in Refs. [1,21,25]. Though the method is blind to the power corrections, it can be applied in this case since the expansion by regions provides the integrals which are *homogeneous* in the expansion parameter. Sudakov logarithms are produced by the soft virtual photons, which are collinear to either p_1 or p_2. We have found that such a configuration of virtual momenta does not produce double logarithms in the first order in ρ. This observation agrees with the analysis [26] of the cusp anomalous dimension, which determines the double-logarithmic corrections to the light-like Wilson line with a cusp. For the large cusp angle corresponding to the limit $\rho \to 0$ from the result of Ref. [26] one gets

$$\Gamma_{cusp} = -\frac{\alpha}{\pi}\ln\rho\left(1 + \mathcal{O}(\rho^2)\right), \tag{3}$$

with vanishing first-order term in ρ. Nevertheless, the $\mathcal{O}(\rho)$ double-logarithmic contribution does exist but originates from completely different virtual momentum configuration described below. Let us consider an electron propagator $S = \frac{\not p_i - \not l + m_e}{(p_i - l)^2 - m_e^2}$, where l is the momentum of a virtual photon with the propagator $D = \frac{-g_{\mu\nu}}{l^2 - \lambda^2}$. In the soft-photon limit $l \to 0$ the electron propagator becomes eikonal $S \approx -\frac{\not p_i + m_e}{2p_i l}$ and develops a collinear singularity when l is parallel to p_i. Alternatively, we may consider the soft-electron limit $l' \to 0$, where $l' = p_i - l$. Then the electron propagator becomes scalar $S \approx \frac{m_e}{l'^2 - m_e^2}$ while the photon propagator becomes eikonal $D \approx \frac{g_{\mu\nu}}{2p_i l' - m_e^2 + \lambda^2}$. Thus the roles of the electron and photon propagators are exchanged. The existence of non-Sudakov double-logarithmic contributions due to soft electron exchange has actually been known for a long time [25,27]. However in our case this virtual momentum configuration does not produce a double-logarithmic contribution in one loop because the momentum shift distorts the eikonal structure of the second electron propagator and removes the soft singularity at small l' necessary to get the second power of the large logarithm. This may be avoided only in the two-loop diagram of nonplanar topology, Fig. 1(a). After shifting the photon virtual momenta by p_1 and p_2 the diagram can be twisted into the shape of Fig. 1(b), (c) with soft electron pair exchange between the eikonal lines. The corresponding contribution has an explicit suppression factor m_e^2 from two soft electron propagators. Hence the integration over the virtual momenta can be performed in the leading order of the small electron mass expansion. Note that in the case under consideration the electron mass regulates both soft and collinear divergences and we can put $\lambda = 0$. The calculation is conveniently performed by using the light-cone coordinates where $p_1 \approx p_{1-}$ and $p_2 \approx p_{2+}$. In this representation only the interaction of the transverse photons to soft electrons is not mass-suppressed and we can use $\frac{g_{kl}^{\perp}}{2p_i l}$ for the eikonal photon propagators. To get the double-logarithmic part of the correction

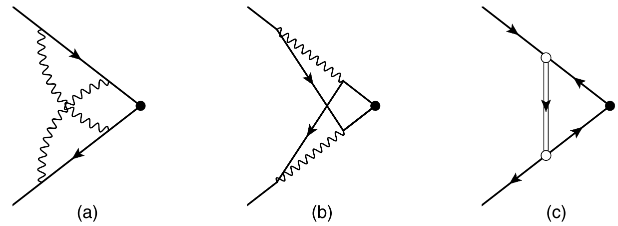

Fig. 1. Different representations of the two-loop Feynman diagram giving the leading power-suppressed double-logarithmic contribution. In figure (c) the double line arrow represents the soft electron pair propagator and the empty blobs represent the nonlocal interaction of the soft electron pair to the eikonal electrons and positrons.

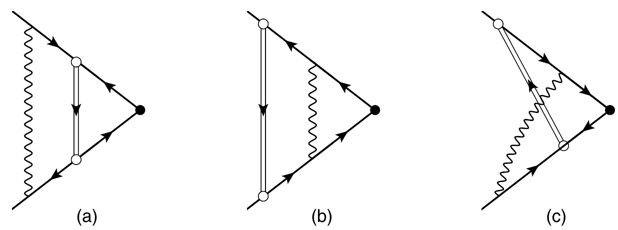

Fig. 2. Feynman diagrams contributing to the double-logarithmic correction factors $\phi^{a,b,c}$, Eq. (7).

we use Sudakov parametrization of a virtual photon momentum $l = up_1 + vp_2 + l_\perp$. After integrating over the transverse components $l_\perp$ we get the following representation of the two-loop power-suppressed form factor

$$F_1^{(1)}\Big|_{2\text{-}loop} = -4x^2\int K(\eta_1,\eta_2,\xi_1,\xi_2)\,d\eta_1 d\eta_2 d\xi_1 d\xi_2, \tag{4}$$

where $\eta = \ln v/\ln\rho$, $\xi = \ln u/\ln\rho$, the integration goes over the four-dimensional cube $0 < \eta_i, \xi_i < 1$, and the kernel

$$K(\eta_1,\eta_2,\xi_1,\xi_2) = \theta(1-\eta_1-\xi_1)\theta(1-\eta_2-\xi_2) \times\theta(\eta_2-\eta_1)\theta(\xi_1-\xi_2) \tag{5}$$

selects the kinematically allowed region of double-logarithmic integration. This gives $F_1^{(1)} = -\frac{x^2}{3} + \mathcal{O}(x^3)$, in agreement with [8,28]. The higher-order double-logarithmic corrections are generated in a usual way through the exchange of soft photons with the propagator $\frac{-g_{\mu\nu}}{l^2-\lambda^2}$. A key observation here is that an exchange of a soft photon between an eikonal and a soft electron line does not produce double logarithms. The reason for this is that such a loop is always separated from the second eikonal line by a scalar electron propagator, which does not communicate any information on the second external momentum. Hence the loop integral cannot depend on the scalar product p_1p_2, which is the only large scale in the problem. Thus it is sufficient to consider only the diagrams of the topologies given in Fig. 2. By using the factorization properties of the soft photon contribution [1] after separating the singular factor S_λ we find the following representation of the all-order double-logarithmic result

$$F_1^{(1)} = -4x^2\int \phi^b(\eta_1,\xi_2)\phi^c(\eta_1,\xi_1)\phi^c(\xi_2,\eta_2) \times\Big[\phi^a(\eta_2,\xi_1)K_1(\eta_1,\eta_2,\xi_1,\xi_2) + K_2(\eta_1,\eta_2,\xi_1,\xi_2)\Big]d\eta_1 d\eta_2 d\xi_1 d\xi_2, \tag{6}$$

where the Sudakov correction factors corresponding to Fig. 2(a)–(c) are