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Non-minimal two-loop inflation

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ABSTRACT

We investigate the chaotic inflationary model using the two-loop effective potential of a self-interacting scalar field theory in curved spacetime. We use the potential which contains a non-minimal scalar curvature coupling and a quartic scalar self-interaction and which was found in Ref. [1]. We analyze the Lyapunov stability of de Sitter solution and show the stability bound. Calculating the inflationary parameters, we systematically explore the spectral index n_s and the tensor-to-scalar ratio r, with varying the four parameters, the scalar-curvature coupling ξ_0, the scalar quartic coupling λ_0, the renormalization scale μ and the e-folding number N. It is found that the two-loop correction on n_s is much larger than the leading-log correction, which has previously been studied in Ref. [2]. We show that the model is consistent with the observation by Planck with WMAP [3,4] and a recent joint analysis of BICEP2 [5].

1. Introduction

The measurements of the cosmic microwave background (CMB) fluctuations become increasingly important from the perspective of not only cosmology but also elementary particle physics. Useful indicators of the CMB fluctuations are given by the scalar (or density) fluctuations, δ, the spectral index, n_s, and the tensor-to-scalar ratio, r. In the inflation scenario a non-vanishing potential energy density of a scalar field induces exponential expansion on the universe. The origin of the CMB fluctuations is found in the quantum fluctuations of the scalar field. It is expected that these inflationary parameters restrict the models of particle physics. Although we have observed only one elementary scalar field, i.e. Higgs, it is quite natural to assume that other scalar fields exist and play a decisive role for the energy density and its fluctuation at early universe.

In this paper we consider that the inflaton field is a real scalar field with a quartic self-interaction and a non-minimal scalar-curvature interaction at high-energy scale and study a possible model consistent with the CMB fluctuations at the two loop level. Inflation is thought to occur near the Planck scale. In such high energy scale the quantum correction may have some remarkable effect on the inflationary parameters. It is known that the spectral index and the tensor-to-scalar ratio are independent of the scalar quartic coupling, λ_0, at the tree level. It is also known that there is an attractor on the (n_s, r) plane. The inflationary parameters, n_s and r, converge to their universal model-independent values at the large scalar-curvature coupling limit [6,7]. Note that RG behavior of scalar curvature coupling ξ is defined by the behavior of the corresponding quantum field theory at high energy (see: [8,9]) so that it maybe tend to large or small asymptotic value at high-energy limit.

The inflationary parameters have been investigated up to the leading log level with respect to the scalar quartic coupling in Ref. [2]. The quantum corrections introduce the quartic coupling dependence for n_s and r, but do not alter the attractor behavior. The standard model (SM) Higgs inflation has been investigated up to the next to leading log level in Refs. [10,11]. In SM the scalar quartic coupling is extremely suppressed near the Planck scale. It has been pointed out that a large non-minimal scalar curvature coupling is necessary to reproduce the observed Higgs mass, n_s and r. The remark is in order. Quantum field theory in curved spacetime induces log terms in the scalar four-point as well as in

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the non-minimal scalar-curvature sector, for the corresponding effective potential [12,13]. The account of such quantum-corrected terms in the potential, especially RG improved effective potential, is done for the study of inflation in [14–26]. It is also interesting to note that the reconstruction of the inflationary scalar potential as is done in [27] maybe applied to such non-minimal inflationary scalar potential.

The paper is organized as follows. Following Ref. [1] we introduce the effective Lagrangian up to the two-loop level in Section 2. The Lagrangian in the Einstein frame is formulated by the Weyl transformation. We show the concrete expressions for the scalar fluctuations, the spectral index and the tensor-to-scalar ratio at the two loop level in Section 3. The Lyapunov stability of de Sitter solutions is studied in Section 4 in close analogy with the corresponding study for log-corrected higher-derivative quantum gravity [28]. In Section 5, we numerically evaluate the inflationary parameters and show possible parameters consistent with the observed data. Finally we give some concluding remarks.

2. Two-loop effective Lagrangian in Einstein frame

Here we consider a massless scalar field with a non-minimal scalar-curvature coupling, ξ_0, and adopt a simple chaotic inflation scenario near the Planck scale. We start from a Lagrangian density,

$$\mathcal{L}^{(J)} = \sqrt{-g}\left(\frac{1}{2}R + \frac{1}{2}\xi_0 R\phi^2 - \frac{1}{2}g^{\mu\nu}(\partial_\mu\phi)(\partial_\nu\phi) - V^{(J)}\right), \tag{2.1}$$

$$V^{(J)} = \frac{\lambda_0}{24}\phi^4, \tag{2.2}$$

where the superscript (J) denotes the Jordan frame, g is the determinant of the metric tensor, $g_{\mu\nu}$, ξ_0, and λ_0 represents the scalar-curvature and scalar quartic couplings, respectively. The Jordan frame is characterized by the existence of the ξ-term. Here the reduced Planck mass is set as $M_p = (8\pi G)^{-1/2} = 1$.

In Ref. [1] the closed expression for the two-loop effective potential is given under the linear curvature approximation,

$$\begin{aligned} V = {} & \frac{\lambda_0}{24}\phi^4 - \frac{1}{2}\xi_0 R\phi^2 + \frac{\lambda_0^2\phi^4}{(16\pi)^2}\ln\frac{\phi^2}{\mu^2} \\ & - \frac{\lambda_0(\xi_0 - 1/6)}{(8\pi)^2}R\phi^2\ln\frac{\phi^2}{\mu^2} - \frac{\lambda_0^3\phi^4}{8(4\pi)^4}\ln\frac{\phi^2}{\mu^2} \\ & + \frac{3\lambda_0^3\phi^4}{32(4\pi)^4}\left(\ln\frac{\phi^2}{\mu^2}\right)^2 \\ & - \frac{\lambda_0^2}{4(4\pi)^4}\left[\left(\xi_0 - \frac{1}{6}\right) + \frac{1}{36}\right]R\phi^2\ln\frac{\phi^2}{\mu^2} \\ & - \frac{\lambda_0^2(\xi_0 - 1/6)}{4(4\pi)^4}R\phi^2\left(\ln\frac{\phi^2}{\mu^2}\right)^2, \end{aligned} \tag{2.3}$$

where μ represents the renormalization scale. It should be noted that some terms in the effective potential vanish if we set the scalar-curvature coupling as $\xi_0 = 1/6$, called the conformal one. From Eqs. (2.1)–(2.3) we define effective scalar-dependent couplings, ξ and λ, as

$$\begin{aligned} \frac{1}{2}\xi \equiv {} & \frac{1}{2}\xi_0 + \frac{\lambda_0(\xi_0 - 1/6)}{(8\pi)^2}\ln\frac{\phi^2}{\mu^2} \\ & + \frac{\lambda_0^2}{4(4\pi)^4}\left[\left(\xi_0 - \frac{1}{6}\right) + \frac{1}{36}\right]\ln\frac{\phi^2}{\mu^2} \\ & + \frac{\lambda_0^2(\xi_0 - 1/6)}{4(4\pi)^4}\left(\ln\frac{\phi^2}{\mu^2}\right)^2, \end{aligned} \tag{2.4}$$

$$\begin{aligned} \frac{\lambda}{24} \equiv {} & \frac{\lambda_0}{24} + \frac{\lambda_0^2}{(16\pi)^2}\ln\frac{\phi^2}{\mu^2} - \frac{\lambda_0^3}{8(4\pi)^4}\ln\frac{\phi^2}{\mu^2} \\ & + \frac{3\lambda_0^3}{32(4\pi)^4}\left(\ln\frac{\phi^2}{\mu^2}\right)^2. \end{aligned} \tag{2.5}$$

Thus the two-loop effective Lagrangian is given by

$$\mathcal{L}_{eff}^{(J)} = \sqrt{-g}\left(\frac{1}{2}R + \frac{1}{2}\xi R\phi^2 - \frac{1}{2}g^{\mu\nu}(\partial_\mu\phi)(\partial_\nu\phi) - V_{eff}^{(J)}\right), \tag{2.6}$$

$$V_{eff}^{(J)} = \frac{\lambda}{24}\phi^4. \tag{2.7}$$

The Lagrangian, $\mathcal{L}_{eff}^{(J)}$, has a similar form with (2.1) thanks to the definitions (2.4) and (2.5). This potential has the renormalization scale dependence which stems from the radiative corrections for ϕ^4 theory in curved spacetime.

For calculations of the inflationary parameters, it is more convenient to change the frame into the Einstein frame where the ξ-term disappears. In order to change the frame, we consider the Weyl transformation,

$$\tilde{g}^{\mu\nu} = \Omega^{-2}(x)g^{\mu\nu}, \tag{2.8}$$

where $\tilde{g}^{\mu\nu}$ is the metric tensor in the transformed frame. The Weyl factor, Ω, is an analytic function with respect to the space–time coordinate. After this Weyl transformation the two-loop effective Lagrangian is rewritten as

$$\begin{aligned} \mathcal{L}_{eff}^{(J)} \to {} & \Omega^{-2}\sqrt{-\tilde{g}}\Big[\frac{1}{2}(1+\xi\phi^2)\tilde{R} - \frac{1}{2}\tilde{g}^{\mu\nu}(\partial_\mu\phi)(\partial_\nu\phi) - \Omega^{-2}V_{eff}^{(J)} \\ & + 3\left[\tilde{\Box}\ln\Omega - \tilde{g}^{\mu\nu}(\partial_\mu\ln\Omega)(\partial_\nu\ln\Omega)\right]\left(1+\xi\phi^2\right)\Big], \end{aligned} \tag{2.9}$$

where $\tilde{g}$, $\tilde{R}$ and $\tilde{\Box}$ are the determinant of the metric tensor, the Ricci scalar and the d'Alembert operator in the transformed frame, respectively. We can transform the Jordan frame into the Einstein frame by choosing the Weyl factor in order that the ξ-term is eliminated. The suitable choice is

$$\Omega^2 = 1 + \xi\phi^2. \tag{2.10}$$

Then we redefine the scalar field to obtain the canonical kinetic term for the scalar field. The redefined scalar field is given by the relation,

$$\frac{\partial\varphi}{\partial\phi} = \frac{\sqrt{\Omega^2 + \frac{3}{2}\left(\frac{\partial\Omega^2}{\partial\phi}\right)^2}}{\Omega^2}, \tag{2.11}$$

where φ is the redefined canonical scalar field. With these techniques we finally obtain the Lagrangian in the Einstein frame,

$$\mathcal{L}_{eff}^{(E)} = \sqrt{-\tilde{g}}\left[\frac{1}{2}\tilde{R} - \frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - V_{eff}^{(E)}\right], \tag{2.12}$$

$$V_{eff}^{(E)} = \Omega^{-4}V_{eff}^{(J)}, \tag{2.13}$$

where the superscript (E) represents the Einstein frame. The effective potential (2.13) has the ξ- and μ-dependences in addition to the λ-dependence. The first one comes from the Weyl transformation and the second one from the quantum corrections.