Physics Letters B 745 (2015) 90–96

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Charged isotropic non-Abelian dyonic black branes

Yves Brihaye [a,*], Ruben Manvelyan [b], Eugen Radu [c], D.H. Tchrakian [d,e]

[a] Physique-Mathématique, Universite de Mons-Hainaut, Mons, Belgium
[b] Yerevan Physics Institute, Alikhanian Br. St. 2, 0036 Yerevan, Armenia
[c] Departamento de Física da Universidade de Aveiro and CIDMA Campus de Santiago, 3810-183 Aveiro, Portugal
[d] School of Theoretical Physics – DIAS, 10 Burlington Road, Dublin 4, Ireland
[e] Department of Computer Science, National University of Ireland Maynooth, Maynooth, Ireland

ARTICLE INFO

Article history:
Received 12 March 2015
Accepted 16 April 2015
Available online 20 April 2015
Editor: M. Cvetič

ABSTRACT

We construct black holes with a Ricci-flat horizon in Einstein–Yang–Mills theory with a negative cosmological constant, which approach asymptotically an AdS_d spacetime background (with $d \geq 4$). These solutions are isotropic, i.e. all space directions in a hypersurface of constant radial and time coordinates are equivalent, and possess both electric and magnetic fields. We find that the basic properties of the non-Abelian solutions are similar to those of the dyonic isotropic branes in Einstein–Maxwell theory (which, however, exist in even spacetime dimensions only). These black branes possess a nonzero magnetic field strength on the flat boundary metric, which leads to a divergent mass of these solutions, as defined in the usual way. However, a different picture is found for odd spacetime dimensions, where a non-Abelian Chern–Simons term can be incorporated in the action. This allows for black brane solutions with a magnetic field which vanishes asymptotically.

1. Introduction and motivation

In recent years there has been some interest in studying the AdS/CFT correspondence [1,2], in the presence of a background magnetic field. On the bulk side, this corresponds to solving the Einstein-gauge field system of equations, with suitable boundary conditions such that the AdS background is approached asymptotically, while the magnetic field does not trivialize. Several new classes of such solutions have been found in this way, most of them for the case of main interest of asymptotically AdS_5 configurations with Abelian fields. For example, the results in [3,4] revealed the existence of a variety of unexpected features of these solutions; here we mention only that their study is relevant for the issue of the third law of thermodynamics in the AdS/CFT context.

The investigation of the non-Abelian (nA) generalizations of these solutions is only in its beginning stages. Considering such configurations is a legitimate task, since the gauged supersymmetric models generically contain Yang–Mills fields (although usually only Abelian truncations are considered). To date, the only case investigated systematically corresponds to that in four ($d = 4$) spacetime dimensions (see [5] for a review of these solutions). The four-dimensional nA asymptotically-AdS (AAdS) solutions exhibit many new features which are absent for $\Lambda \geq 0$. For example, stable[1] solitons and black holes, possessing a global magnetic charge, are known to exist in a globally AdS_4 background even in the absence of a Higgs field [6,7]. However, the results in [8,9] show that these Einstein–Yang–Mills (EYM) black holes solutions have also generalizations with a nonspherical event horizon topology, in particular with a Ricci-flat horizon and a magnetic field which does not vanish asymptotically. They share many of the features of the spherical configurations in [6,7], in particular the existence of solutions stable against linear fluctuations. The only $d > 4$ nA AAdS solutions black holes studied more systematically so far are those possessing spherical event horizon topology [11–14], though some solutions with Ricci-flat horizon have been studied in [15,16].

In an unexpected development, the study of the $d = 4, 5$ EYM black brane solutions has led to the discovery of holographic superconductors and holographic superfluids, describing condensed phases of strongly coupled, planar, gauge theories [10]. Studying such solutions involves the construction of AAdS electrically charged black branes, which, below a critical temperature become unstable to forming YM hair. However, the magnetic field of these

* Corresponding author.
E-mail address: yves.brihaye@umons.ac.be (Y. Brihaye).

[1] The stability is against linear perturbations, and is not topological.

http://dx.doi.org/10.1016/j.physletb.2015.04.029

configurations vanishes on the boundary, leading to a vanishing background magnetic field for the dual theory.

The main purpose of this work is to present an investigation of $d \geq 4$ AAdS isotropic black branes supporting both electric and magnetic nA fields. In contrast to previous studies in the literature, the magnetic fields of these solutions do not vanish on the boundary, which leads to a variety of interesting features. For example, we find that the mass of these asymptotically AdS solutions, as defined in the usual way, always diverges, while the solutions do not posses a regular extremal limit. In odd-dimensional spacetimes, when a Chern–Simons term is added to the total action, it is found that a special class of solutions exhibit a nontrivial magnetic field in the bulk while vanishing asymptotically.

2. The Einstein–Yang–Mills system

We consider the EYM theory in a d-dimensional spacetime, with a cosmological constant $\Lambda = -(d-2)(d-1)/(2L^2)$. The action is

$$I = \int_{\mathcal{M}} d^d x\sqrt{-g}\left(\frac{1}{16\pi G}(R-2\Lambda) - \frac{1}{4}{*F}\wedge F\right) + S_{bndy}. \qquad (1)$$

The boundary terms S_{bndy} include the Gibbons–Hawking term [17] as well as the counterterms required for the on-shell action to be finite [18]. The Einstein and Yang–Mills equations derived from the above action are

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G\, T_{\mu\nu}, \qquad D_\mu F^{\mu\nu} = 0, \qquad (2)$$

where D_μ is the gauge derivative and the Yang–Mills stress-energy tensor

$$T_{\mu\nu} = \frac{1}{2}\left(F^{IJ}_{\mu\rho}F^{IJ}_{\nu\sigma}g^{\rho\sigma} - \frac{1}{4}g_{\mu\nu}F^{IJ}_{\rho\sigma}F^{IJ\rho\sigma}\right). \qquad (3)$$

We are interested in static Ricci-flat solutions which approach asymptotically a (planar) AdS$_d$ background. Also, to simplify the picture, we shall restrict our study to the following case: denoting the radial and time coordinate by r and t respectively and considering the hypersurfaces parametrized by x^i ($i = 1, \ldots, d-2$ and r, t fixed), we assume that all space directions in these hypersurfaces are equivalent. Thus the field strength and the metric are taken to be invariant under space translations and rotations in the planes (x^i, x^j); they are also time independent. Without any loss of generality, a line element with this property can be written in the form

$$ds^2 = g_{rr}(r)dr^2 + g_{\Sigma\Sigma}(r)d\Sigma^2_{d-2} + g_{tt}(r)dt^2, \qquad (4)$$

with $d\Sigma^2_{d-2} = (dx^1)^2 + \ldots + (dx^{d-2})^2$ the metric on the $(d-2)$-flat space.

The above symmetry requirements imply some restrictions on the choice of the gauge group. Restricting to $SO(n)$ YM fields, one finds that a YM ansatz leading to an isotropic energy–momentum tensor for *both* even and odd values of d is possible for $n \geq d+1$ only.[2]

In this work we shall consider an $SO(d+1)$ gauge group, with $d(d-1)/2$ $SO(d+1)$ nA gauge fields represented by the 1-form potential A^{IJ} antisymmetric in I and J (with $I, J = 1, \ldots, d+1$) and $F^{IJ} = dA^{IJ} + \frac{1}{\hat{g}}A^{IK}\wedge A^{KJ}$, with $\hat{g}$ the Yang–Mills coupling. Also, to simplify the relations, it is convenient to define

$$\alpha^2 = \frac{4\pi G}{\hat{g}^2}. \qquad (5)$$

3. Embedded Abelian solutions

Before proceeding to the non-Abelian case, it is instructive to consider the dyonic black branes in Einstein–Maxwell theory, (i.e. the gauge fields taking their values in the $U(1)$ subgroup of $SO(d+1)$). A gauge field ansatz compatible with the symmetries of the line-element (4) can be constructed for an even number of spacetime dimensions only, $d = 2n+2$ and reads[3]

$$A_1^{IJ} = \frac{w_0^2}{\hat{g}}x^2\delta^I_{[d}\delta^J_{d+1]},\ A_2^{IJ} = -\frac{w_0^2}{\hat{g}}x^1\delta^I_{[d}\delta^J_{d+1]},$$
$$\ldots,$$
$$A_{2n-1}^{IJ} = \frac{w_0^2}{\hat{g}}x^{2n}\delta^I_{[d}\delta^J_{d+1]},\ A_{2n}^{IJ} = -\frac{w_0^2}{\hat{g}}x^{2n-1}\delta^I_{[d}\delta^J_{d+1]},$$
$$A_r^{IJ} = 0,\ A_t^{IJ} = \frac{V(r)}{\hat{g}}\delta^I_{[d}\delta^J_{d+1]}, \qquad (6)$$

with w_0 an arbitrary parameter which fixes the magnetic field in a two plane, $F^{IJ}_{21} = \ldots = F^{IJ}_{2n2n-1} = \frac{2w_0^2}{\hat{g}}\delta^I_{[d}\delta^J_{d+1]}$. Choosing a metric gauge with $g_{\Sigma\Sigma} = r^2$, one finds[4] a black brane solution with $1/g_{rr} = -g_{tt} = N(r)$, where

$$N(r) = \frac{r^2}{L^2} - \frac{M_0}{r^{d-3}} + \frac{2}{(d-3)(d-2)}\frac{\alpha^2 Q^2}{r^{2(d-3)}} - \frac{4}{(d-5)}\frac{\alpha^2 w_0^4}{r^2}, \qquad (7)$$

and

$$V(r) = V_0 - \frac{Q}{(d-3)r^{d-3}}, \qquad (8)$$

with V_0 a constant which is fixed by requiring that the electric potential vanish at the horizon. Apart from w_0, this solutions posseses two more parameters: M_0 and Q, which fixes the mass and the electric charge densities, respectively.

This black brane possesses an horizon at $r = r_H > 0$, where $N(r_H) = 0$ (and $N'(r_H) \geq 0$). The Hawking temperature T_H, the event horizon area density A_H, the chemical potential Φ and the electric charge density Q_e of this solution are

$$T_H = \frac{1}{4\pi}\left((d-1)\frac{r_H}{L^2} - \frac{2\alpha^2}{r_H}\left(\frac{2w_0^4}{r_H^2} + \frac{1}{(d-2)}\frac{Q^2}{r_H^{2(d-3)}}\right)\right),$$
$$A_H = r_H^{d-2},$$
$$\Phi = \frac{1}{d-3}\frac{Q}{r_H^{d-3}},\quad Q_e = \frac{\alpha^2}{4\pi}Q. \qquad (9)$$

One can easily verify that the total mass of the solutions, as defined according to the counterterm prescription in [18], diverges for any (even) $d > 4$ due to the slow decay of the magnetic fields, despite the fact that the spacetime is still AAdS. A finite mass density results when a boundary term

[2] Note that, for even values of d, one can consider instead a gauge group $SO(d-1)$, which leads to isotropic EYM branes. A study of this case has been proposed in [15] (Ansatz I there). However, the properties of those solutions are rather different to the case of interest here.

[3] The ansatz (6), (4) can be extended to the case of odd d by adding a number of codimensions y^μ, with $A^{IJ}_\mu = 0$; however, this leads to anisotropic configurations.

[4] A version of this solution has been considered in a more general context in [24]. Also, its purely magnetic limit, $Q = 0$, has been discussed in [3].