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Resumming double logarithms in the QCD evolution of color dipoles

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ABSTRACT

The higher-order perturbative corrections, beyond leading logarithmic accuracy, to the BFKL evolution in QCD at high energy are well known to suffer from a severe lack-of-convergence problem, due to radiative corrections enhanced by double collinear logarithms. Via an explicit calculation of Feynman graphs in light cone (time-ordered) perturbation theory, we show that the corrections enhanced by double logarithms (either energy-collinear, or double collinear) are associated with soft gluon emissions which are strictly ordered in lifetime. These corrections can be resummed to all orders by solving an evolution equation which is non-local in rapidity. This equation can be equivalently rewritten in *local* form, but with modified kernel and initial conditions, which resum double collinear logs to all orders. We extend this resummation to the next-to-leading order BFKL and BK equations. The first numerical studies of the collinearly-improved BK equation demonstrate the essential role of the resummation in both stabilizing and slowing down the evolution.

1. Introduction

It is by now well established that the Balitsky–JIMWLK hierarchy[1] [1–7] and its mean field approximation known as the Balitsky–Kovchegov (BK) equation [8] govern the high-energy evolution of scattering amplitudes in presence of non-linear effects (multiple scattering and gluon saturation) responsible for unitarization. Some of the most remarkable recent developments in that context refer to the first calculations of the next-to-leading order (NLO) corrections [9–11] to the B–JIMWLK and BK equations. These new developments parallel and extend previous efforts, towards the end of nineties, which established the NLO version [12–17] of the Balitsky–Fadin–Kuraev–Lipatov (BFKL) equation [18–20] — the linearized version of the BK equation which applies so long as the scattering is weak. Although the BFKL and B–JIMWLK equations are based on a common evolution mechanism, they differ in the way how they treat the scattering problem: the BFKL equation deals only with single scattering, as appropriate for a dilute target, whereas the B–JIMWLK hierarchy includes the interplay between evolution and multiple scatterings. The former is usually written in transverse *momentum* space, as an equation for the unintegrated gluon distribution, while the latter is formulated in terms of transverse *coordinates* (better suited for implementing the eikonal approximation) and keeps trace of the multiple scattering of the individual partons in the projectile — each of them represented by a Wilson line. Such differences explain the difficulty to adapt to the NLO B–JIMWLK evolution the 'collinear resummations' originally developed in the context of NLO BFKL [21–25], which aim at improving the convergence of the perturbative expansion for the BFKL kernel.

The collinear resummations refer to perturbative corrections, starting at NLO, which are enhanced by large, single or double, *transverse* logarithms. Without a proper resummation, which, strictly speaking, goes beyond the order-by-order expansion of the BFKL kernel, these large logarithms deprive the NLO BFKL formalism of its predictive power.

There is no reason to expect this lack-of-convergence problem to be attenuated by the non-linear terms in the B–JIMWLK equations: indeed, the 'collinear' corrections arise from regions in phase-space where the scattering is weak and the non-linear effects are negligible. This was anticipated in a semi-analytic study [26] and later on confirmed by the numerical observation that

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[1] The acronym JIMWLK stands for Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov and Kovner.

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adding a unitarity constraint (in the form of a 'saturation boundary') to the NLO BFKL equation does not help improving the stability of the solution [27]. Very recently, while our work was being completed, this has been corroborated by a numerical study [28] of the NLO BK equation [9]: the numerical solution turns out to be unstable (the scattering amplitude decreases with increasing energy and can even turn negative) for the physically interesting initial conditions. As also shown in Ref. [28], this instability can be traced back to a large NLO correction to the BFKL kernel enhanced by a double transverse logarithm. This kind of correction, which can be associated with the choice of the reference scale in the energy logarithm, is well understood at BFKL level, where it is successfully resummed to all orders by the schemes proposed in Refs. [22–25]. It is our main objective in this paper to propose a similar resummation at the level of the BK equation.

More precisely, our goals are twofold: first, we would like to unambiguously identify the origin of the double-collinear logarithms in Feynman graphs to all orders and devise a method for their resummation; second, we would like to reformulate this resummation as a change in the kernel of the BK equation, which is energy independent. Unlike the corresponding method in the context of NLO BFKL, where the resummation is generally implemented in double Mellin space[2] [22,23], our resummation will be directly implemented in transverse coordinate space, in order to be consistent with the non-linear structure of the BK equation.

Concerning the first objective above, our main finding is that the double-collinear logs arise due to a reduction in the longitudinal phase-space for the high-energy evolution, as introduced by the condition that successive gluon emissions be strictly ordered in lifetime. The interplay between this 'kinematical constraint' and the double transverse logarithms has already been recognized in the literature [22,29–31] (see [32] for a recent discussion and more references), but we are not aware of any systematic derivation of this prescription from Feynman graphs. To emphasize that this is indeed non-trivial, we notice that double collinear logs are also generated by diagrams with *anti*-time ordering, but they mutually cancel when all such graphs are summed together (see the discussion in Section 3 below). This observation helps understanding the peculiar way how the double transverse logs arise in the context of the NLO BK calculation in [9]. The main outcome of this diagrammatic analysis is Eq. (17), which governs the evolution in the double-logarithmic approximation (DLA): it resums to all orders the perturbative corrections in which each power of the coupling is accompanied by a double logarithm (either energy-collinear, or double collinear).

Eq. (17) however is *non-local* in 'rapidity' (the logarithm of the longitudinal momentum, which is our evolution variable), so it does not fully match our goals for a collinearly-improved evolution equation.[3] To cope with that, in Section 4 we demonstrate that the non-local Eq. (17) can be reformulated in a *local* form, modulo an analytic continuation and a reshuffling of the perturbative expansion. The new, local, Eq. (30) involves an 'improved' kernel and (for consistency) a modified initial condition, which both resum double-collinear logs to all orders.

It is then straightforward to extend this resummation to the BFKL and BK equations and thus obtain the collinearly-improved BK equation (32), which is our main result in this paper. It is furthermore possible to promote this result to full NLL accuracy, by adding the remaining NLO BK corrections from Ref. [9]. Notice however that the NLO terms include single transverse logarithms, which may require additional resummations, as was already the case in the context of NLO BFKL [22–24].

Finally, in Section 5 we present the first numerical studies of the resummed BK equation (32). These studies clearly demonstrate the role of the resummation in both stabilizing and significantly slowing down the evolution: the saturation exponent extracted from the numerical solution is smaller by, roughly, a factor of two than in the absence of the resummation.

2. The double-logarithmic limit of the BFKL equation

In order to fix the notations and for comparison with the more refined results that we shall later obtain, it is instructive to recall the derivation of the 'naive DLA', by which we mean the version of this approximation which neglects the time-ordering of successive gluon emissions, from the leading-order (LO) BFKL equation [18–20]. The LO BFKL equation resums the perturbative corrections in which each power of the QCD coupling $\bar{\alpha}_s \equiv \alpha_s N_c/\pi$, assumed to be fixed and small, is accompanied by the energy logarithm $Y \equiv \ln(s/Q_0^2)$ (the 'rapidity'), with s the center-of-mass energy squared and Q_0 the characteristic transverse scale of the target.[4] In this leading-log approximation (LLA), valid when $\bar{\alpha}_s Y \gtrsim 1$, it is consistent to treat the scattering and the evolution in the eikonal approximation. The LO BFKL equation can then be written as the linearized version of the BK equation [1,8], i.e. as an equation for the high-energy evolution of the scattering amplitude $T_{\boldsymbol{xy}}(Y)$ of a quark–antiquark dipole, with a quark leg at transverse coordinate $\boldsymbol{x}$ and an antiquark leg at transverse coordinate $\boldsymbol{y}$, which undergoes weak scattering off a generic target (a nucleus, or a 'shockwave'):

$$\frac{\partial T_{\boldsymbol{xy}}(Y)}{\partial Y} = \frac{\bar{\alpha}_s}{2\pi} \int \mathrm{d}^2\boldsymbol{z}\, \mathcal{M}_{\boldsymbol{xyz}} \big[T_{\boldsymbol{xz}}(Y) + T_{\boldsymbol{zy}}(Y) - T_{\boldsymbol{xy}}(Y) \big]. \quad (1)$$

This equation involves the 'dipole' version of the BFKL kernel,

$$\mathcal{M}_{\boldsymbol{xyz}} \equiv \frac{(\boldsymbol{x}-\boldsymbol{y})^2}{(\boldsymbol{x}-\boldsymbol{z})^2(\boldsymbol{z}-\boldsymbol{y})^2}, \quad (2)$$

which describes the emission of a soft gluon with transverse coordinate $\boldsymbol{z}$ by either the quark or the antiquark leg of the dipole, followed by its reabsorption (see Fig. 1). In the limit of a large number of colors $N_c \to \infty$, the positive quantity $(\bar{\alpha}_s/2\pi)\mathcal{M}_{\boldsymbol{xyz}}\mathrm{d}^2\boldsymbol{z}$ can be interpreted [34] as the differential probability for the splitting of the original color dipole $(\boldsymbol{x}, \boldsymbol{y})$ into a pair of dipoles $(\boldsymbol{x}, \boldsymbol{z})$ and $(\boldsymbol{z}, \boldsymbol{y})$. The first two terms within the square brackets, $T_{\boldsymbol{xz}}$ and $T_{\boldsymbol{zy}}$, are the 'real' terms describing the scattering of the daughter dipoles, whereas the last one, $-T_{\boldsymbol{xy}}$, is the 'virtual' term expressing the reduction in the probability for the parent dipole to survive at the time of scattering.

Eq. (1) is valid so long as the scattering is weak, $T \ll 1$, for all the dipoles. For a dense target, such as a large nucleus, this is indeed the case provided all dipoles look small on the scale set by the target saturation momentum[5] Q_0: $(\boldsymbol{x}-\boldsymbol{y})^2 Q_0^2 \ll 1$, etc. In this regime, the integration over $\boldsymbol{z}$ in the r.h.s. of Eq. (1) becomes

[2] Note however some similarity between our strategy and that proposed in [25], where the ω-shift in Mellin space [22,23] has been approximately reformulated as an improvement of the BFKL kernel in transverse momentum space.

[3] Collinearly-improved versions of the BK equation which are non-local in rapidity have been proposed too in the literature [32,33], but they suffer from some shortcomings, concerning either the systematics of the resummation (for the approach in [33]), or its feasibility in practice (for [32]).

[4] This scale Q_0 is assumed to be hard enough for perturbation theory to apply: $Q_0^2 \gg \Lambda_{\rm QCD}^2$. E.g., if the target is a small dipole, then $1/Q_0$ is the size of that dipole. If the target is a large nucleus described by the McLerran–Venugopalan (MV) model [35], then Q_0 is the target saturation momentum in a frame where the projectile carries most of the total energy.

[5] In the full BK equation, which includes unitarity corrections, this condition is eventually replaced by $r^2 Q_s^2(Y) \ll 1$, where $Q_s(Y)$ is the saturation momentum of the target, which obeys $Q_s(0) = Q_0$ and increases with Y.