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The bag and the string: Are they opposed?

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ABSTRACT

The Cornell potential is under certain conditions converted to an effective potential which is suggestive of the bag model once all spin degrees of freedom of a quark driven by this static field have been integrated out. We argue for the view that such conditions arise from a quark Q moving in a relativistic mean field generated by two quarks Q' and Q'', which together with Q form a nucleon $QQ'Q''$, and the nucleon $QQ'Q''$ is a constituent of some nucleus. This view opens up a new avenue of attack on the problem of saturation of the nuclear force.

1. Introduction

Among many phenomenological models of quarks confined to hadrons, two have enjoyed popularity for years: the nonrelativistic potential model with the Cornell potential [1,2], and the bag model [3–6]. The pictorial renditions of these models are different. They refer respectively to an elastic string with quarks fixed at its ends, and a spherical cavity in which free valence quarks are permanently held. The bag states of high angular momentum are likely to deform into rotating tubes with quarks at the ends and a flux of color fields connecting them. Such a structure resembles a "string" with a constant energy per unit length [7]. However, the converse, that is, whether a spherical bag could arise from a relativistic string, is not evident. A central idea of this paper is that the string is under certain conditions converted to the bag.

The Cornell potential was proposed in an effort to determine the quarkonium levels through the use of the Schrödinger equation. The basic assumption is that the quarkonium properties are adequately described by the degrees of freedom of a heavy quark Q and its own antiquark $\bar{Q}$ whose motions in quarkonia are nonrelativistic.[1] The main concern there is with the low-lying spectrum of the Hamiltonian

$$H = 2m + \frac{\mathbf{p}^2}{2m} + V_{\rm Cornell}(r)\,, \tag{1}$$

where m is the quark mass (we use units in which $\hbar = 1$, $c = 1$), and $V_{\rm Cornell}$ is the static quark–antiquark potential

$$V_{\rm Cornell}(r) = -\frac{\alpha_s}{r} + \sigma r\,. \tag{2}$$

The first term in (2) arises from the single gluon exchange between Q and $\bar{Q}$. This term is responsible for short-distance effects, and is known as the Coulomb part of the Cornell potential. The second term is responsible for the long-distance confinement effects. This linearly rising term is associated with a string-like configuration of the gluon field between Q and $\bar{Q}$ [8–11]. The parameter σ, the so-called string tension, is common to both charmonium and bottomonium states. It can be obtained from fits to lattice calculations of Wilson loops exhibiting the heavy quark potential (2) at the leading order in $1/m$ expansion [12]. The standard assumption is that the flavor dependence of the level structure stems from m and α_s. The parameters that follow from fitting

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[1] The speed of charm quarks is estimated to be about 0.3 times the speed of light for charmonia, and that of bottom quarks is about 0.1 times the speed of light for bottomonia.

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the masses of the 11 well established $\bar{c}c$ states are $\alpha_s = 0.7281$, $\sigma = 0.1425$ GeV^2 [13].

It transpires that this model allows a "relativized" extension to mesons composed of one heavy and one light quark [14], and that all mesons – from the π to the Υ – can be described in this unified framework [15]. The Hamiltonian consists of a relativistic kinetic term,

$$H = \sqrt{\mathbf{p}_{\bar{Q}}^2 + m_{\bar{Q}}^2} + \sqrt{\mathbf{p}_Q^2 + m_Q^2}\,, \tag{3}$$

and a generalized quark–antiquark potential which, to first order in v_Q^2, reduces to that given by (2) with α_s replaced by a running coupling constant

$$\alpha_s(r) = -\frac{8\pi}{33 - 2n_f}\,\frac{1}{\ln(\Lambda r)}\,, \quad \Lambda r \ll 1\,, \tag{4}$$

where n_f is the number of quark flavors which are active in pair production, and Λ the quantum chromodynamics (QCD) scale. Reasonably accurate results for the spectrum and matrix elements of meson systems of all u, d, s, c, b quark flavors have been found by taking $\Lambda = 398$ MeV, $m_c = 1628$ MeV, $m_b = 4884$ MeV, and $\sigma = 0.18$ GeV^2 [13].

It would be erroneous to think of the potential (2) as a non-Abelian static solution to the classical Yang–Mills equations. If this was the case, the linearly rising term of the Yang–Mills vector potential would lead to a problem with Gauss' law because integrating the field strength $F^a_{\mu\nu}$ over two-dimensional spheres of increasing radii would result in successively higher magnitudes of the color charge of the source Q^a. Meanwhile an exact non-Abelian solution A^a_μ to the classical SU(3) Yang–Mills equations with the source composed of two arbitrarily moving colored point charges has long been known [16–19]. This A^a_μ comprises two parts related to their respective quarks, each involving generalized Liénard–Wiechert terms and linearly rising terms. For this solution, Gauss' law holds because the flux of the Liénard–Wiechert term of $F^a_{\mu\nu}$ through any two-dimensional surface enclosing the source equals $4\pi Q^a$, the remaining terms cancel each other. Although the solution A^a_μ contains two linearly rising terms, both give rise to no force. The general reason for this surprising result is conformal invariance of the Yang–Mills equations. The linearly rising term depends on a dimensional parameter whereby scale invariance is violated. While this violation is allowable for the gauge quantities A^a_μ and $F^a_{\mu\nu}$, it cannot be tolerated for observables. In actual fact, the stress–energy tensor $T_{\mu\nu}$ and the four-force f^μ are free of scale-violating terms. A dimensional parameter, which measures a gap in the energy spectrum and violates scale symmetry, only occurs on the quantum level due to quantum anomalies and dimensional transmutations.

The confining potential (2) may arise at a certain stage of derivation of an effective theory to low-energy QCD when irrelevant field variables have been integrated out from the field equations or the QCD path integral.[2] However, a systematic implementation of this project in QCD is still a good distance in the future. By now, the nature of the confining potential is open to speculation; we even cannot say whether it is a scalar U_S, or pseudoscalar U_{PS}, or the time component of a four-vector potential U_V, or their combination. This sends us in search of phenomenological hints. One possibility is to invoke the spin symmetry condition $U_S = U_V$ or the pseudospin symmetry condition $U_S = -U_V$ [20], accompanied by the assumption that $2U_V$ is identical to $V_{\rm Cornell}$. For simplicity, we ignore U_{PS} because the qualitative conclusions of our consideration are unaffected by the availability of U_{PS}. We thus regard $V_{\rm Cornell}$ as an effective potential. More precisely, $V_{\rm Cornell}$ is a semi-finished product of the effective theory; the net effective potential could be found upon completion of integrating out spin degrees of freedom of a quark which moves in the potential $V_{\rm Cornell}$.

The bag model was formulated in such a way as to combine two basic features of QCD: asymptotic freedom and confinement. A ground-state hadron is imagined as a spherical cavity, or a bag, to which quark and gluon fields are confined. The bag is characterized by a constant B, positive energy per unit volume. The characteristic linear dimension of a hadron is thus scaled by $(1/B)^{1/4}$. The energy of a free quark of mass m confined to a sphere of radius R is given by $\varepsilon = \left(m^2 + p^2/R^2\right)^{1/2}$, where p is the quark momentum in units of $1/R$. This system is unstable in the sense that increasing R decreases the energy monotonically until $R = \infty$. It is this instability which leads to introducing the quantity B, a "pressure" that stabilizes the system. The total energy of N quarks in the bag becomes

$$E(R) = \sum_{i=1}^{N}\left(m^2 + \frac{p_i^2}{R^2}\right)^{\frac{1}{2}} + B\,\frac{4\pi R^3}{3}\,, \tag{5}$$

and equilibrium is attained when $E(R)$ is minimized, $\partial E/\partial R = 0$. If we restrict our consideration to massless quarks, we obtain the size of a stable hadron

$$R = \frac{4}{3}\,\frac{p_{\min} N}{E}\,, \tag{6}$$

where $p_{\min} = 2.04$ is the minimal value of the momentum corresponding to $mR = 0$ [21]. On putting $N = 3$ and $E = M_{\rm nucleon}$, we find that the size required to generate a nucleon mass from the kinetic energy of massless quarks and the confining pressure is $R = 1.6$ fm. Note that this size is typical of light nuclei.

A hadron can execute rotations which makes its form stretched. This points to the existence of stringlike solutions in the bag model. On the assumption that stringlike configurations of the bag maximize angular momentum for fixed mass the bag string tension is shown to be $\sigma = (32\pi\alpha_s B/3)^{1/2}$ [7].

The *ad hoc* introduction of the pressure B may seem rather awkward. However, the mechanism of confinement in the bag model is amenable to more fundamental treatments. The interested reader may consult Ref. [6] where successes and drawbacks of the MIT bag model [3], the SLAC bag model [4], and the soliton bag model [5] are considered at length. Of special interest to our discussion are solutions of the classical Yang–Mills equations that were argued to have a direct bearing on the bag [22,23]. A solution of this kind is singular on a sphere of radius r_0, and hence is interpreted as a Yang–Mills black hole with the color "event horizon" at $r = r_0$.[3]

[2] To illustrate, we solve the classical Yang–Mills equations in the Yang–Mills–Wong theory [19] by expressing the retarded gauge field $A^a_\mu(x)$ in terms of variables describing the world lines of N color particles, $z^\mu_I(s_I)$, $I = 1, \ldots, N$. This operation corresponds to integrating out the field degrees of freedom $A^a_\mu(x)$ from the particle dynamics. Likewise, integrating out gluon and ghost variables from the QCD path integral will supposedly result in the starting point of our consideration, Eq. (7), where $\mathbf{A}(\mathbf{r})$, $A_0(\mathbf{r})$, and $U_S(\mathbf{r})$ are classical background fields which form a nonperturbative relief of the gluon vacuum.

[3] This brings up the question: How did the scale invariance violating quantity r_0 occur in the Yang–Mills theory? The classical Yang–Mills equations are invariant under conformal transformations only in spacetime dimension $D = 4$. The configuration studied by Lunev [22] is a solution of the Euclidean three-dimensional Yang–Mills theory in which the Yang–Mills coupling constant g has dimension (length)$^{-1/2}$. With this in mind it is difficult if not impossible to regard this configuration as a static solution of the four-dimensional Yang–Mills theory. The configuration found by Singleton in [23] is a solution to the equations which govern the Yang–Mills fields interacting with a massless scalar field in four-dimensional space-