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Counting ghosts in the "ghost-free" non-local gravity

Ilya L. Shapiro [a,b,c,*]

[a] Département de Physique Théorique and Center for Astroparticle Physics, Université de Genève, 24 quai Ansermet, CH-1211 Genève 4, Switzerland
[b] Departamento de Física, ICE, Universidade Federal de Juiz de Fora, Campus Universitário, Juiz de Fora, 36036-330, MG, Brazil
[c] Tomsk State Pedagogical University and Tomsk State University, Tomsk, Russia

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ABSTRACT

In the recently proposed non-local theory of quantum gravity one can avoid massive tensor ghosts at the tree level by introducing an exponential form factor between the two Ricci tensors. We show that at the quantum level this theory has an infinite amount of massive unphysical states, mostly corresponding to complex poles.

1. Introduction

The general relativity (GR) is a very successful theory of gravity, but it is perhaps not an ultimate theory. One of the reasons is that the fourth derivative terms in the action of gravity become necessary as the UV completion of the theory at the semiclassical level [1] (see also [2,3] for the introduction and [4] for a recent pedagogical review). The same fourth-derivative terms make the theory of quantum gravity (QG) renormalizable [5]. On the other side, fourth derivatives lead to the massive ghosts in the physical spectrum of the theory, leading to the violation of unitarity.

The consistency of the fourth derivative quantum gravity (QG) can be, in principle, achieved by dealing with the dressed propagator instead of the classical one [6–8]. The main expectation is that the massive ghost poles become unstable and decay in the far future, such that the asymptotic *out*-state becomes free of ghosts. Unfortunately, the final conclusion concerning this approach requires a complete non-perturbative knowledge of the dressed propagator [9], which is unavailable.

Some years ago a completely different approach was proposed by Tomboulis [10]. The action of this new theory of QG has an infinite amount of derivatives. It was discovered a few years earlier by Tseytlin [11] that for some specially tuned form of the non-local action such a theory is free of ghosts at the tree level while the exponential form factors remove Newtonian singularity, similar to the much simpler fourth derivative gravity [5]. The approach of [11] (see also [12]) was to use this action in the framework of string theory, as an alternative of the Zwiebach ghost-killing transformation of the background fields [13–15]. In string theory the ghost-free non-local action is a kind of a "final product", which is not supposed to gain further quantum corrections.[1] On the contrary, if one takes the same model as a basis of quantum gravity [10], the following three important questions should be answered:

- First, how to quantize the non-local theory?
- Second, what is the power counting in a theory with infinite amount of derivatives?
- The third and most difficult question is what happens with the ghost-free structure of the theory after the quantum corrections are taken into account?

Concerning the first point, the quantization of non-local theories has been discussed in the literature [17] and is relatively well-understood. The second issue has been explored in [10] and

* Correspondence to: Departamento de Física, ICE, Universidade Federal de Juiz de Fora, Campus Universitário, Juiz de Fora, 36036-330, MG, Brazil.
E-mail address: shapiro@fisica.ufjf.br.

[1] However, this does not make it free of ambiguity related to the third and higher powers of curvature, similar to the one discussed in [16].

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in the more recent publications [18–20]. The main conclusion is that the power counting in the non-local theory of [10] is the same as in the local higher derivative superrenormalizable QG suggested earlier in [21]. Moreover, in both cases there is a chance to make such a QG theory finite. This can be certainly achieved in the local case [21] and very likely in the non-local one.[2]

In the present work we will mainly address the third question. There are strong arguments that at least the most simple example of the non-local theory suggested in [10] does not remain free of ghost-like states at the quantum level. The last means that the quantum corrections lead to an infinite amount of the ghost-like states in the dressed propagator. The relation between ghosts in the classical (naked) and dressed propagators is almost opposite to what was expected in the fourth-derivative renormalized theory of QG [5–8].

The paper is organized as follows. In Section 2 we present a brief review of the non-local gravity which is ghost-free at the tree-level. In Section 3 we explain the power counting in the non-local model, here our consideration mainly follows previous publications [10,18] and [20], but we try to make it more transparent, especially by comparing to the local superrenormalizable QG case [21]. Some relevant details concerning Lagrangian quantization of the non-local theory are settled in Appendix A. In Section 4 it is shown how the ghost-free structure is violated by quantum corrections to the propagator. In Section 5 we discuss the modified Newtonian limit in the non-local theory and the possible role played by the "hidden" ghosts. Finally, in the last section we draw our conclusions.

2. Non-local ghost-free models

The simplest way to count degrees of freedom in QG is based on the analysis of the tree-level propagator on the flat background. In most of the theories this procedure gives the same result as canonical quantization [22,3]. In order to explore the flat-space propagator, the relevant part of the classical action is at most bilinear in the curvature tensor,

$$S = \int d^4x\sqrt{-g}\,\Big\{-\frac{1}{\kappa^2}\,R + R\,F_1(\Box)\,R + R_{\mu\nu}\,F_2(\Box)\,R^{\mu\nu} + R_{\mu\nu\alpha\beta}\,F_3(\Box)\,R^{\mu\nu\alpha\beta}\Big\}. \qquad (1)$$

Here $\kappa^2 = 16\pi G$ and $F_{1,2,3}$ are functions of d'Alembertian operator. The cosmological constant term is set to zero, following the standard treatment [5]. In order to simplify the action, let us note that the difference between the term $R_{\mu\nu\alpha\beta}F_3(\Box)R^{\mu\nu\alpha\beta}$ and the combination $4R_{\mu\nu}F_3(\Box)R^{\mu\nu} - RF_3(\Box)R$ is proportional to the term of the third power in curvature, $\mathcal{O}(R^3_{...})$ (see, e.g., [21,23]). Therefore one can cast the relevant part of the action (1) in the form

$$S = \int d^4x\sqrt{-g}\,\Big\{-\frac{1}{\kappa^2}\,R + \frac{1}{2}\,C_{\mu\nu\alpha\beta}\,\Phi(\Box)\,C_{\mu\nu\alpha\beta} + \frac{1}{2}\,R\,\Psi(\Box)\,R\Big\}, \qquad (2)$$

where $C_{\mu\nu\alpha\beta}$ is the Weyl tensor. The function Ψ is responsible for the spin-0 part of the propagator and the function Φ for the spin-2 part. For the sake of simplicity, we can mainly concentrate on the spin-2 sector. The consideration for the Ψ-part would be very similar. After the Fourier transformation, the relevant equation for defining the poles of the propagator is [10]

$$p^2\left[1 + \kappa^2 p^2\Phi(-p^2)\right] = 0\,. \qquad (3)$$

One can see that there is always a massless pole corresponding to gravitons. For a constant Φ there is also a massive pole corresponding to a spin-2 ghost, which may be also a tachyon. For a non-constant polynomial function Φ there are always ghost-like poles, real or complex [21]. However, one can choose the function Φ in such a way that there will not be any other spin-2 pole, except the graviton $p^2 = 0$. The simplest example of this sort is [11]

$$1 + \kappa^2 p^2\Phi(-p^2) = e^{\alpha p^2}\,, \qquad (4)$$

where α is some constant of the dimension $mass^{-2}$. One can find other entire functions which have the same features [10,18], but for the sake of simplicity we consider only (4).

Let us remember that the exponential function has two remarkable properties. The equation $\exp z = 0$ has no real solutions and only one very peculiar solution

$$z = -\infty + i\times 0 \qquad (5)$$

on the extended complex plane. At the same time, already the equation $\exp z = A \neq 0$ has infinitely many complex solutions, the same is true for

$$e^z = Az^2\log z\,, \qquad (6)$$

which is the typical case for the exponential theory with logarithmic quantum corrections. These well-known features of exponential function mean, in our case, that the absence of massive ghosts in the spin-2 part of the propagator of the theory (4) is the result of an absolutely precise tuning of the function $\Phi(-p^2)$. If this tuning is violated by the loop corrections, then the ghosts-like states will emerge in an infinite number. For instance, any polynomial addition to the exponential function produce infinitely many complex solutions.

One important note is in order. The expression "ghosts-like states" means that these states are not exactly the "classical" massive ghosts, that means states with positive square of mass and negative kinetic energy. In the present case there are mostly complex poles, that means a complex "square of mass" and complex "kinetic energy". This situation makes the particle interpretation of these states rather complicated. We postpone the discussion of this issue until another publication and will call these states simply ghosts in what follows.

If the theory with more ghosts should be qualified worst, then the exponential gravity (4) with violated absolute tuning is worse than the polynomial version of superrenormalizable QG [21] (see also the next section), because the last has only finite amount of ghosts. So, the main question concerning the theory of exponential gravity (4) is whether one can preserve an absolute tuning of (4) at the quantum level. In the next sections we consider this issue starting from the strongest effect related to the UV divergences and related logarithmic running. For comparison, we also present considerations for the mentioned polynomial model of QG.

3. Power-counting in local and non-local QG

Before discussing the dressed propagator and possible violation of the absolute tuning in (4), let us shortly review the renormalization properties of the theory (2) and some its natural extensions. A brief survey of the Lagrangian quantization of the theories such as (2) or (7) with some details related to non-local versions of the theory can be found in Appendix A.

[2] In the odd space–time dimensions this can be easily proved in [18].