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Observations on the partial breaking of $N = 2$ rigid supersymmetry

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ABSTRACT

We study the partial breaking of $N = 2$ rigid supersymmetry for a generic rigid special geometry of n abelian vector multiplets in the presence of Fayet–Iliopoulos terms induced by the hyper-Kähler momentum map. By exhibiting the symplectic structure of the problem we give invariant conditions for the breaking to occur, which rely on a quartic invariant of the Fayet–Iliopoulos charges as well as on a modification of the $N = 2$ rigid symmetry algebra by a vector central charge.

1. Introduction

It is well known that partial breaking of rigid and local supersymmetry can occur [1,2], provided one evades [3–11] some no-go theorems [12,13,2] which are satisfied by a certain class of theories. In global supersymmetry, Hughes and Polchinski first pointed out the possibility to realize partial breaking of global supersymmetry [5] and, in four dimensional gauge theories, this was realized for a model of a self-interacting $N = 2$ vector multiplet, in the presence of $N = 2$ electric and magnetic Fayet–Iliopoulos terms [8]. This model is closely connected to the Goldstone action of partially broken $N = 2$ supersymmetry [14] by integrating out the ($N = 1$) chiral-multiplets components of the $N = 2$ vector multiplet [15], thus reproducing the supersymmetric Born–Infeld action [16,17]. Multi-field versions which generalize the supersymmetric Born–Infeld theory to an arbitrary number of vector multiplets were then obtained, preserving $N = 1$ supersymmetry [15,18], or preserving a second non-linearly realized supersymmetry [19,20].

It is the aim of the present note to further elucidate some general conditions for partial supersymmetry breaking to occur which are independent on the particular alignment of the unbroken supersymmetry with respect to the original two supersymmetries, and are also independent of the particular representative of the Fayet–Iliopoulos charge vector which, in our problem, is a triplet of the $N = 2$ $SU(2)$ R-symmetry and a symplectic vector with respect to the symplectic structure of the underlying Special Geometry: $\mathcal{P}^{Mx} = \sqrt{2}\begin{pmatrix} m^{Ix} \\ e^x_I \end{pmatrix}$. In terms of it, the Ward identities are manifestly independent of the choice of the symplectic frame. The symplectic invariance relies on the existence of a quartic invariant which is the squared norm of the $SU(2)$ triplet of symplectic singlets:

$$\xi_x = \frac{1}{2}\epsilon_{xyz}\mathcal{P}^{yM}\mathcal{P}^{zN}\mathbb{C}_{MN} = 2\left(\vec{m}^I \times \vec{e}_I\right)_x . \quad (1.1)$$

We shall give, in Section 2, the general Ward identities that the $N = 2$ scalar potential satisfies when Fayet–Iliopoulos symplectic-charge triplets $\mathcal{P}^{xM}$ are turned on, explicitly showing that they are modified by a constant traceless matrix $C^A{}_B = \xi_x(\sigma^x)^A{}_B$. Furthermore, in Section 2.1, we shall derive in a symplectic covariant manner the modifications of the supersymmetry algebra which, in the framework of $N = 2$ tensor calculus, was derived in [10].

2. The rigid Ward identity

It is a well known fact that the Supergravity Ward identity relating the scalar potential V to the shifts of the fermions in the presence of a gauging is a pure trace identity in the R-symmetry SU(N) indices, namely:

$$V\,\delta^A_B = \sum_i \alpha_i\,\delta\chi^{iA}\delta\chi_{iB} \quad (2.1)$$

where the index i in the sum runs over all the fermion-shifts of the theory (including the gravitino), α_i being constants which are

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positive for the spin 1/2-fields and negative for the gravitino. This is true also when the scalar potential is due to the presence of a Fayet–Iliopoulos (FI) term [1,2,4,21].

In the rigid supersymmetric theories with $N > 1$ the previous statement is violated, since on the right-hand-side of Eq. (2.1) a traceless term can appear related to the presence of electric and *magnetic* FI terms [5,8,10,15]. In the case of an $N = 2$ rigid theory, this can be seen either by direct computation of the fermionic shifts of the gauginos or by performing a suitable flat limit of the $N = 2$ Supergravity parent theory with gravitino and hyperinos constant non-zero shifts.

In the rigid case of a supersymmetric vector-multiplet theory, Eq. (2.1) allows for a traceless constant term C^A_B in the scalar potential Ward identity, namely (2.1) is modified as follows

$$V\,\delta^A_B + C_B{}^A = \sum_i \delta\lambda^{iA}\delta\lambda_{iB} \tag{2.2}$$

where λ^{iA} and $\lambda_{iA} \equiv g_{i\bar{k}}\lambda^{\bar{k}}_A$ denote the chiral and antichiral projections of the gauginos, respectively. According to the arguments given in [10], the additive constant matrix $C_B{}^A$ can be interpreted as a central extension in the supersymmetry algebra, which only affects the commutator of two supersymmetry transformations of the gauge field.

In the $N = 2$ case the traceless matrix C^A_B has the following expression

$$C_B{}^A = \frac{1}{2}\epsilon_{xyz}\mathcal{P}^{yM}\mathbb{C}_{MN}\mathcal{P}^{zN}\,(\sigma^x)_B{}^A \equiv \xi^x\,(\sigma^x)_B{}^A \tag{2.3}$$

where $(M, N, \dots)$ are symplectic indices in the fundamental of $\mathrm{Sp}(2n)$,

$$\mathcal{P}^x_M = \sqrt{2}\begin{pmatrix} -e^x_I \\ m^{I\,x}\end{pmatrix} = -\mathbb{C}_{MN}\,\mathcal{P}^{N\,x} \tag{2.4}$$

is a constant symplectic vector (defining the electric/magnetic Fayet–Iliopoulos term) whose upper and lower components $I = (1, 2, \dots n)$ are electric and magnetic respectively, and

$$\mathbb{C}_{MN} = \begin{pmatrix} \mathbf{0} & \mathbf{1} \\ -\mathbf{1} & \mathbf{0}\end{pmatrix} \tag{2.5}$$

is the symplectic metric.

Let us give for completeness the derivation of this result in the case of $N = 2$ supersymmetric theory with a number n of abelian vector multiplets, no hypermultiplets and in the presence of both electric and magnetic Fayet–Iliopoulos terms.

In this case, the fermion-shift in the supersymmetry transformation of the chiral gaugino fields can be written, using a symplectic formalism, as

$$\delta\lambda^{iA} = W^{iAB}\epsilon_B \tag{2.6}$$

with

$$W^{iAB} = i\,(\sigma^x)_C{}^A\epsilon^{CB}\,\mathcal{P}^x_M\, g^{i\bar{k}}\bar{U}^M_{\bar{k}} \tag{2.7}$$

where $g_{i\bar{k}}$ is the rigid Special Kähler metric and the symplectic section U^M_i is the derivative with respect to the scalar fields z^i of the fundamental symplectic section $V^M(z)$ of the rigid Special Kaehler Geometry [22,23]:

$$U^M_i = \frac{\partial}{\partial z^i}\begin{pmatrix} X^I \\ F_I\end{pmatrix} = \frac{\partial}{\partial z^i}V^M(z)\,. \tag{2.8}$$

Introducing a triplet of triholomorphic superpotentials $\mathcal{W}^x$

$$\mathcal{W}^x = \mathcal{P}^x_M\,V^M(z) = \sqrt{2}\left(F_I(z)m^{xI} - e^x_I\,X^I(z)\right) \tag{2.9}$$

Eq. (2.6) takes the form

$$\delta\lambda^{iA} = i(\sigma^x)_C{}^A\epsilon^{CB}\,\partial_{\bar{k}}\bar{\mathcal{W}}^x g^{i\bar{k}}\epsilon_B = i\,Y^{ix}(\sigma^x)_C{}^A\epsilon^{CB}\epsilon_B \tag{2.10}$$

with $Y^{ix} = g^{i\bar{k}}\,\partial_{\bar{k}}\overline{\mathcal{W}}^x$.

Let us now use special coordinates, that is $X^I = z^i$; in this frame we can write

$$g_{IJ} = \mathrm{Im}\,F_{IJ}$$

$$U^M_I = \begin{pmatrix} \delta^K_I \\ F_{IK}\end{pmatrix} \longrightarrow g^{I\bar{J}}\bar{U}^M_{\bar{J}} = \begin{pmatrix} g^{I\bar{K}} \\ g^{I\bar{J}}\,\mathrm{Re}\,F_{KJ} - i\delta^I_K\end{pmatrix} \tag{2.11}$$

A short computation then gives

$$Y^{Ix} = \sqrt{2}\left[-g^{I\bar{K}}\left(e^x_K - \mathrm{Re}\,F_{KJ}\,m^{Kx}\right) - im^{Ix}\right]. \tag{2.12}$$

This formula actually coincides with Eq. (23) of [8] and it shows that a non-zero magnetic charge m^{Ix} produces a constant imaginary part of the auxiliary field Y^{Ix}, a necessary condition for partial breaking of supersymmetry.

We now use the Special Geometry identity [24]:

$$U^{MN} = U^M_i\,g^{i\bar{k}}\,\bar{U}^N_{\bar{k}} = \frac{1}{2}\left(\mathcal{M}^{MN} - i\mathbb{C}^{MN}\right), \tag{2.13}$$

where $\mathcal{M}^{MP}\mathcal{M}_{PN} = \delta^M_N$ and

$$\mathcal{M}_{MN} = -\begin{pmatrix} I + R\cdot I^{-1}\cdot R & -R\cdot I^{-1} \\ -I^{-1}\cdot R & I^{-1}\end{pmatrix} > 0\,, \tag{2.14}$$

$I \equiv (-\mathrm{Im}(F_{IJ}))$ and $R \equiv (\mathrm{Re}(F_{IJ}))$. If we flatten the σ-model coordinate index of $\delta\lambda^{iA}$ in (2.6), we obtain

$$U^N_i\,\delta\lambda^{iA} = \frac{i}{2}\left(\mathcal{M}^{MN} - i\mathbb{C}^{MN}\right)\mathcal{P}^x_N\,(\sigma^x)_C{}^A\epsilon^{CB}\epsilon_B\,. \tag{2.15}$$

Finally we may compute the bilinear product in the gaugino shifts

$$\begin{aligned} g_{i\bar{k}}W^{iAB}\bar{W}^{\bar{k}}_{BC} &= \frac{\delta^A_C}{2}\mathcal{M}^{MN}\,\mathcal{P}^x_M\,\mathcal{P}^x_N + \frac{1}{2}\mathbb{C}^{MN}\,\mathcal{P}^x_M\,\mathcal{P}^x_N\epsilon^{xyz}(\sigma_z)_C{}^A \\ &= \delta^A_C\,V_{N=2} + C_C{}^A\,, \end{aligned} \tag{2.16}$$

which coincides with Eq. (2.2), proving our statement. In conclusion, the $N = 2$ scalar potential of the rigid theory is

$$V_{N=2} = \frac{1}{2}(\mathcal{P}^x)^T\,\mathcal{M}^{-1}\,\mathcal{P}^x \tag{2.17}$$

while, by the identification (2.4), the C^A_B term can be rewritten as

$$C_A{}^B = \frac{1}{2}\,(\sigma^x)_A{}^B\,\mathbb{C}^{MN}\,\mathcal{P}^y_M\,\mathcal{P}^z_N\epsilon_{xyz} = 2\left(\vec{m}^I \times \vec{e}_I\right)^x(\sigma^x)_A{}^B. \tag{2.18}$$

In the general case in which both the F and D-terms are present, we define the following SO(3)-vector:

$$\xi^x \equiv \frac{1}{2}\,\epsilon_{xyz}\,\mathcal{P}^y_M\,\mathcal{P}^z_N\mathbb{C}^{MN} = 2\,(\vec{m}^I \times \vec{e}_I)^x\,, \tag{2.19}$$

where $\vec{m}^I \equiv (m^{I\,x})$, $\vec{e}_I \equiv (e_I{}^x)$. In terms of this quantity Eq. (2.16) reads:

$$\begin{aligned} g_{i\bar{k}}W^{iAB}\bar{W}^{\bar{k}}_{BC} &= \delta^A_C\,V_{N=2} + C_C{}^A \\ &= \begin{pmatrix} V_{N=2} + \xi^3 & \xi^1 - i\,\xi^2 \\ \xi^1 + i\,\xi^2 & V_{N=2} - \xi^3\end{pmatrix}. \end{aligned} \tag{2.20}$$

Upon diagonalization, the above matrix reads

$$g_{i\bar{k}}W^{iAB}\bar{W}^{\bar{k}}_{BC} = \begin{pmatrix} V_{N=2} + \sqrt{|\xi|^2} & 0 \\ 0 & V_{N=2} - \sqrt{|\xi|^2}\end{pmatrix}, \tag{2.21}$$