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On the dualization of Born–Infeld theories

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ABSTRACT

We construct a general Lagrangian, quadratic in the field strengths of n abelian gauge fields, which interpolates between BI actions of n abelian vectors and actions, quadratic in the vector field-strengths, describing Maxwell fields coupled to non-dynamical scalars, in which the electric–magnetic duality symmetry is manifest. Depending on the choice of the parameters in the Lagrangian, the resulting BI actions may be inequivalent, exhibiting different duality groups. In particular we find, in our general setting, for different choices of the parameters, a $\mathrm{U}(n)$-invariant BI action, possibly related to the one in [4], as well as the recently found $\mathcal{N}=2$ supersymmetric BI action [11].

1. Introduction

The Born–Infeld (BI) theory [1] describes a non-linear electrodynamics in four dimensional space-time enjoying remarkable features, among which electric–magnetic duality symmetry. Such a peculiarity, which has been generalized to the case of n abelian field strengths, where the duality group is contained in $\mathrm{Sp}(2n,\mathbb{R})$ [2–4], hints to a connection of BI with extended supersymmetric theories, which also have the electric–magnetic duality invariance [5] as a characteristic property. The supersymmetric version of the BI Lagrangian was constructed in [6,7], while in [8–10] it was identified as the invariant action of the Goldstone multiplet in an $\mathcal{N}=2$ supersymmetric theory spontaneously broken to $\mathcal{N}=1$. Recently, the results of [9] have been generalized to the case of n vector multiplets in $\mathcal{N}=2$ supersymmetry [11,12], with explicit solutions for the cases $n=2$ and $n=3$.

In this letter we provide a linear (in the squared field strengths) realization of the bosonic BI Lagrangian in terms of a redundant Lagrangian containing two couples of non-dynamical scalars. The classical BI Lagrangian is recovered solving the field-equation constraints when varying our Lagrangian with respect to one of the two couples of scalars, while variation with respect to the other couple of Lagrange multipliers leads to a version of linear electromagnetism with generalized (scalar dependent) couplings and a positive scalar potential, in which the duality symmetry is manifest. Remarkably, the properties of the resulting theory fit very well with the bosonic sector of the $\mathcal{N}=2$ supersymmetric Lagrangian for a vector multiplet in the presence of a complex Fayet–Iliopoulos term, in the limit where the masses of the scalar sector are dominant with respect to their kinetic term. By appropriate choice of the normalization of the fields, we recover indeed, in a component form, the results of [9].

Let us remark that in our approach the possibility of dualization to BI is due to the presence of a scalar function $f(\Lambda)\propto\sqrt{1+\Lambda}$, Λ being one of the Lagrange multipliers. After implementing the proper normalization of the fields corresponding to the supersymmetric case, the coefficient in front of $f(\Lambda)$ turns out to be twice the product of an electric and a magnetic charge. In the absence of either the electric or the magnetic charge, our Lagrangian would reduce to linear electrodynamics coupled to scalars and it would not be able to implement the dualization to BI. On the other hand, the need for both electric and magnetic charges is in fact a necessary condition for partial supersymmetry breaking $\mathcal{N}=2\to\mathcal{N}=1$, as shown in [13,15]. Our formalism, recalling the results in [9], makes the relation between partial supersymmetry breaking and BI manifest. Not surprisingly, the presence of $f(\Lambda)$ in our Lagrangian is also necessary to obtain, in the other version of the theory, a scalar potential manifestly invariant under electric–magnetic duality symmetry.

In our framework, the generalization to more than one vector fields, at the purely bosonic level, is straightforward by promoting scalar fields to matrices. We write a general Lagrangian which also includes some constant matrices η^{IJ}, $\tilde{\eta}_{IJ}$. In the generic case where η^{IJ}, $\tilde{\eta}_{IJ}$ are invertible, the extension of our approach to any number of vectors is straightforward and leads to the definition of an abelian multi-field BI action which comprises, for

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a suitable choice of parameters, a $U(n)$-invariant BI action, possibly related to the one of [4] in the absence of extra scalar fields. However, we show that we can relax the invertibility condition on the two constant matrices η^{IJ}, $\tilde{\eta}_{IJ}$, allowing for an $\mathcal{N} \geq 2$ supersymmetric extension. For specific choices of η^{IJ}, $\tilde{\eta}_{IJ}$ in terms of the electric and magnetic Fayet–Iliopoulos charges we reproduce the $\mathcal{N}=2$ supersymmetric BI action found in [11]. Therefore, we show that, starting from our unifying description, different choices of the constant matrices η^{IJ}, $\tilde{\eta}_{IJ}$ may lead, upon integrating out the non-dynamical fields, to inequivalent theories which exhibit different global symmetries.

2. Linear realization of the Born–Infeld Lagrangian

Let us consider the Born–Infeld Lagrangian in four dimensions:

$$\mathcal{L} = \frac{1}{\lambda}\left\{1 - \sqrt{\left|\det\left[\eta_{\mu\nu} + \sqrt{\lambda}F_{\mu\nu}\right]\right|}\right\} = \\ = \frac{1}{\lambda}\left(1 - \sqrt{1 + \frac{\lambda}{2}F^2 - \frac{\lambda^2}{16}(F\tilde{F})^2}\right), \tag{2.1}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is an abelian field strength, $\tilde{F}_{\mu\nu} = \frac{1}{2}F^{\rho\sigma}\epsilon_{\mu\nu\rho\sigma}$ its Hodge dual and

$$F^2 \equiv F_{\mu\nu}F^{\mu\nu}, \tag{2.2}$$

$$F\tilde{F} \equiv \frac{1}{2}F_{\mu\nu}F_{\rho\sigma}\epsilon^{\mu\nu\rho\sigma}. \tag{2.3}$$

We are going to show that it can be written as the standard Lagrangian of a gauge field-strength in a theory whose field content is enlarged to include two couples of scalar fields which play the role of Lagrange multipliers $\tilde{g}$, $\tilde{\theta}$, Λ, Σ:

$$\mathcal{L}' = \frac{\tilde{g}}{2\lambda}\left(\Lambda + \Sigma^2 - \frac{\lambda}{2}F^2\right) + \tilde{\theta}\left(\frac{1}{4}F\tilde{F} - \frac{\Sigma}{\lambda}\right) \\ + \frac{1}{\lambda}\left(1 - \sqrt{1+\Lambda}\right). \tag{2.4}$$

Indeed, variation of $\mathcal{L}'$ in (2.4) with respect to $\tilde{g}$, $\tilde{\theta}$:

$$\frac{\delta\mathcal{L}'}{\delta\tilde{g}} = 0 \Rightarrow \Lambda = \frac{\lambda}{2}F^2 - \Sigma^2, \tag{2.5}$$

$$\frac{\delta\mathcal{L}'}{\delta\tilde{\theta}} = 0 \Rightarrow \Sigma = \frac{\lambda}{4}F\tilde{F}, \tag{2.6}$$

yields the BI Lagrangian (2.1), while variation with respect to Λ, Σ allows to express them in terms of $\tilde{g}$, $\tilde{\theta}$:

$$\frac{\delta\mathcal{L}'}{\delta\Lambda} = 0 \Rightarrow \bar{\Lambda} = \tilde{g}^{-2} - 1, \tag{2.7}$$

$$\frac{\delta\mathcal{L}'}{\delta\Sigma} = 0 \Rightarrow \bar{\Sigma} = \frac{\tilde{\theta}}{\tilde{g}}, \tag{2.8}$$

leading to the "dual" expression

$$\mathcal{L}' = -\frac{\tilde{g}}{4}F^2 + \frac{\tilde{\theta}}{4}F\tilde{F} - \mathcal{V}(\tilde{g},\tilde{\theta}), \tag{2.9}$$

where

$$\mathcal{V}(\tilde{g},\tilde{\theta}) = -\frac{1}{\lambda}\left[\frac{\tilde{g}}{2}(\Lambda+\Sigma^2) - \tilde{\theta}\Sigma - \sqrt{1+\Lambda} + 1\right]_{\Lambda=\bar{\Lambda};\,\Sigma=\bar{\Sigma}} = \\ = \frac{1}{2\lambda}\left(\tilde{g} + \tilde{\theta}^2\tilde{g}^{-1} + \tilde{g}^{-1}\right) - \frac{1}{\lambda}. \tag{2.10}$$

Two properties of Eq. (2.10) allow to embed Eq. (2.9) into a supersymmetric theory: If we assume $\tilde{g} > 0$, which gives the correct sign to the gauge-field kinetic term in (2.9), the potential $\mathcal{V}$ is positive definite (apart for an irrelevant additive constant). Furthermore, it can be written as

$$\mathcal{V}(\tilde{g},\tilde{\theta}) = \frac{1}{2\lambda}\mathrm{Tr}[\mathcal{M}] - \frac{1}{\lambda}, \tag{2.11}$$

where we introduced the matrix

$$\mathcal{M}_{MN}[\tilde{g},\tilde{\theta}] = \begin{pmatrix} \tilde{g} + \tilde{\theta}\tilde{g}^{-1}\tilde{\theta} & -\tilde{\theta}\tilde{g}^{-1} \\ -\tilde{\theta}\tilde{g}^{-1} & \tilde{g}^{-1} \end{pmatrix}, \tag{2.12}$$

which is familiar to supersymmetry and supergravity users, since it is the symplectic matrix encoding the scalar-couplings to the gauge field-strengths in extended supersymmetric theories.

As shown below, (2.9) can be thought of as the bosonic sector of the Lagrangian of an $\mathcal{N}=2$ vector multiplet with a supersymmetry-breaking scalar potential, in a limit where the scalar-field kinetic term is negligible with respect to the potential term in the action. It will in fact turn out to coincide with the result of [11].

The definition of (2.11) in terms of an invariant quantity (the trace of the symplectic matrix $\mathcal{M}$) allows to define an extension of the BI Lagrangian to n abelian vectors. This will be discussed in Section 3.

2.1. Embedding of the 4D Born–Infeld action in $\mathcal{N}=2$ supersymmetry

Let us consider an $\mathcal{N}=2$ vector multiplet, consisting of a gauge-vector A_μ, a complex scalar z and a couple of Majorana spinors λ^A $(A=1,2)$. The bosonic Lagrangian is

$$\mathcal{L} = -\frac{g(z,\bar{z})}{4}F^2 + \frac{\theta(z,\bar{z})}{4}F\tilde{F} + G_{z\bar{z}}\partial_\mu z\partial^\mu\bar{z} - \mathcal{V}_{\mathcal{N}=2}(z,\bar{z}) \tag{2.13}$$

where g and θ are functions of the complex scalars z, $\bar{z}$ and $G_{z\bar{z}}$ is the metric of the sigma-model. In this case, and in the absence of the hypermultiplet sector, the scalar potential $\mathcal{V}_{\mathcal{N}=2}$ is due to the presence of a (electric and magnetic) FI term $\mathcal{P}^x_M$ ($x=1,2,3$ is an $SU(2)$ index, $M=1,2$ is a symplectic one) such that the supersymmetry transformation-law of the (chiral) gaugino acquires the shift $W^{z|AB} = \mathrm{i}(\sigma^x)^{AB}G^{z\bar{z}}\bar{U}^M_{\bar{z}}\mathcal{P}^x_M$, where $U^M_z = (f_z, h_z)$ is the symplectic section, $G^{z\bar{z}}$ the inverse of $G_{z\bar{z}}$, and

$$\mathcal{V}_{\mathcal{N}=2} = \frac{1}{2}W^{z|AB}G_{z\bar{z}}\bar{W}^{\bar{z}}_{AB} = \frac{1}{2}\mathcal{P}^x_M\mathcal{M}^{MN}\mathcal{P}^x_N, \tag{2.14}$$

where we used the special-geometry relation $U^M_z G^{z\bar{z}}\bar{U}^N_{\bar{z}} = \frac{1}{2}\left(\mathcal{M}^{MN} - \mathrm{i}\Omega^{MN}\right)$, having defined the symplectic metric $\Omega = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$.

The fermion shift generally fully breaks supersymmetry in the vacuum. However, by setting one of the three FI terms, say $\mathcal{P}^3$, to zero, thus breaking $SU(2) \to U(1)$, it is possible to preserve $\mathcal{N}=1$ supersymmetry. In this case, considered in [11], the spontaneously broken theory has a scalar potential which can be written in terms of a complex FI term $P = \frac{1}{\sqrt{2}}\Omega(\mathcal{P}^1 + \mathrm{i}\mathcal{P}^2)$ as:

$$\mathcal{V}_{FPS} = \bar{P}^M\left(\mathcal{M}_{MN} + \mathrm{i}\,\Omega_{MN}\right)P^N = \\ = m^2\left[g + (\theta - \frac{e_1}{m})^2 g^{-1}\right] + e_2^2 g^{-1} - 2me_2, \tag{2.15}$$

where, by fixing the $U(1)$ R-symmetry, we chose $P^M = \begin{pmatrix} m \\ e_1 + \mathrm{i}\,e_2 \end{pmatrix}$. Let us denote by $\mathcal{L}_{FPS}$ the Lagrangian of [11], with scalar potential (2.15). The $\mathcal{N}=1$ scalar potential (2.15) differs from the $\mathcal{N}=2$