Physics Letters B 744 (2015) 244–249

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

An irreducible massive superspin one half action built from the chiral dotted spinor superfield

John A. Dixon

Mathematical Institute, Oxford University, Oxford, England, United Kingdom

ARTICLE INFO

Article history:
Received 2 March 2015
Accepted 26 March 2015
Available online 31 March 2015
Editor: M. Cvetič

ABSTRACT

Although the chiral dotted spinor superfield should describe a Massive Superspin One Half multiplet, it has not been obvious how to derive this from an action. In this paper this is done by including a chiral undotted spinor superfield, finding the BRST transformations that govern both of these, and then finding the action as an invariant of the transformations. It turns out that both kinds of spinor superfields are needed. Moreover, the BRST transformations for the two kinds of chiral spinor superfields are generated from each other by a special involution that exchanges Grassmann odd (even) sources with Grassmann even (odd) fields.

1. In some suitable limit for the superstring [21], the massive modes might be described by massive supersymmetric actions, coupled in some way to each other. As noted in [9] and as emphasized rather recently by Gates and Koutrolikos [7], massive supersymmetric theories possess a rich off shell structure and there is still much to learn about them. In [1] the authors posed the 'Off Shell Susy Problem' in a simple and general way, and pointed out that the answer is not likely to be simple, but that it is probably important. This has generated the adinkra approach which is making progress on this complicated problem [2].

Assembling interacting actions is a significant problem when one has only 'on-shell closure' since this necessarily implies some particular action of course, and that makes it tricky to generalize the action to include other couplings. It is usually taken for granted that the best way to approach the problem of generating actions and couplings is to look for auxiliary fields, and actions expressed in terms of superfields, so that the SUSY algebra closes on shell and the SUSY transformations are then obvious from superspace theory. Progress using these ideas has been reported in [3,4] and [6]. Of course, the general problem, as noted in [1], is complicated by the existence of other symmetries, such as those that must exist in the Standard Model of particle theory.

2. The intent in this paper, and its sequel, is to show that the BRST approach, using cohomology, offers a different approach to some of these problems. The present paper will illustrate some of the issues here by constructing a massive superspin $\frac{1}{2}$ action out of superfields with spin $\frac{1}{2}$. We will not use superfields here, except at the start, but the algebra is closed in the sense that the BRST operator is nilpotent. The nilpotence of the BRST operator arises as though the auxiliaries have been integrated out. This can happen even when no auxiliaries exist.[1] If there is a nilpotent BRST operator, one can use the spectral sequence to discover the cohomology [13]. The cohomology then points out where there are new invariants.

This BRST approach singles out the physical fields, and any remaining auxiliary fields get eliminated from consideration early on in the analysis [13]. So the BRST approach generates a different set of insights and problems, and it is not simple to sort out the relationship between the superfield approach and the BRST cohomology approach. They are complementary.

3. For a number of reasons to do with BRST cohomology,[2] it is of some interest to construct a well-behaved action starting with a chiral dotted spinor superfield $\widehat{\phi}_{\dot\alpha}$. Chirality means that $\overline{D}_{\dot\beta}\widehat{\phi}_{\dot\alpha} = 0$. It is well known [5] that this superfield can be

[1] This is probably the case for the present action, and also for 10-D Super Yang–Mills theory [2,15], for example. This feature is related to the Batalin–Vilkovisky method, see for example [8] for a simple exposition of the latter.

[2] It has been evident for a long time [10,11,14,12] that the BRST cohomology of the chiral scalar superfield $\widehat{A}$ couples naturally to a chiral dotted spinor superfield $\widehat{\phi}^{\dot\alpha}$. The simplest example is $\int d^6z \widehat{\phi}^{\dot\alpha}\widehat{A}\overline{C}_{\dot\alpha}$. Here $\overline{C}_{\dot\alpha}$ is a spacetime constant supersymmetry ghost. However the superfield $\widehat{\phi}^{\dot\alpha}$ here needs further constraints, which is the progress reported in this paper.

E-mail addresses: dixon@maths.ox.ac.uk, jadixg@gmail.com.

http://dx.doi.org/10.1016/j.physletb.2015.03.059

subjected to a 'reality constraint' in addition to chirality, and that on-shell it should represent superspin $\frac{1}{2}$. But the problem is that no action has been found, until now, that is consistent with both the chirality and the reality constraints and which then yields superspin $\frac{1}{2}$.

4. Trying to write down an action for a chiral dotted spinor superfield $\widehat{\phi}_{\dot\alpha}$ meets a problem right at the start, because the most obvious action is

$$\mathcal{A} = \int d^8 z \widehat{\phi}_{\dot\alpha} \partial^{\alpha\dot\alpha} \overline{\widehat{\phi}}_\alpha + m^2 \int d^6 z \widehat{\phi}^{\dot\alpha} \widehat{\phi}_{\dot\alpha} + m^2 \int d^6 \bar{z} \overline{\widehat{\phi}}^\alpha \overline{\widehat{\phi}}_\alpha \tag{1}$$

but this immediately leads to higher derivative equations of motion, and there are tachyons in the spectrum too:

$$\left(\Box^2 - m^4\right) \widehat{\phi}_{\dot\alpha} = \left(\Box - m^2\right)\left(\Box + m^2\right) \widehat{\phi}_{\dot\alpha} = 0 \tag{2}$$

This is certainly not a promising start for a model that is supposed to be phenomenologically viable.

5. The chiral undotted spinor superfield $\overline{\widehat{\chi}}_\alpha$ should also yield massive superspin $\frac{1}{2}$ on shell [5] and this is also puzzling. Here chirality means that $D_\beta \widehat{\chi}_{\dot\alpha} = 0$. $\overline{\widehat{\chi}}_\alpha$ does appear in gauge theory [16], but as a massive matter representation it poses a difficulty, because the only known way to give it mass, until the present action, involves the spontaneous breaking of gauge symmetry together with the introduction of Higgs scalars. Such a method to construct a massive representation of superspin $\frac{1}{2}$ is clearly not irreducible, because it mixes $\widehat{\chi}_{\dot\alpha}$ with the components of chiral scalar Higgs multiplets.

6. Here we also want to add another feature, which is phase invariance, corresponding to some conserved quantity like Lepton or Baryon number. So we add a chirality index L, R to the superfields. Then the chirality constraints have the form:

$$\overline{D}_{\dot\beta} \widehat{\phi}_{L\dot\alpha} = \overline{D}_{\dot\beta} \widehat{\phi}_{R\dot\alpha} = D_\beta \widehat{\chi}_{L\dot\alpha} = D_\beta \widehat{\chi}_{R\dot\alpha} = 0 \tag{3}$$

So the Complex Conjugates satisfy:

$$D_\alpha \overline{\widehat{\phi}}_{L\beta} = D_\alpha \overline{\widehat{\phi}}_{R\beta} = \overline{D}_{\dot\alpha} \overline{\widehat{\chi}}_{L\beta} = \overline{D}_{\dot\alpha} \overline{\widehat{\chi}}_{R\beta} = 0 \tag{4}$$

Next, in order to get an irreducible representation of supersymmetry, along with a phase invariance, along the lines of the textbook [5], we want to also impose the additional 'reality constraints':

$$D_\alpha \widehat{\phi}_{L\dot\alpha} = \overline{D}_{\dot\alpha} \overline{\widehat{\phi}}_{R\alpha};\ \overline{D}^{\dot\alpha} \widehat{\chi}_{R\dot\alpha} = D^\alpha \overline{\widehat{\chi}}_{L\alpha} \tag{5}$$

These are designed so that there is a global U(1) phase invariance that is conserved by the action.

7. In the context of BRST [22,8], a theory is defined by its BRST transformations, which can be derived from its BRST Poisson Bracket. Here is the BRST Poisson Bracket of the present theory:

$$\mathcal{P}_{\rm Total}[\mathcal{A}] = \mathcal{P}_\chi[\mathcal{A}] + \mathcal{P}_\phi[\mathcal{A}] + \mathcal{P}_{\rm SUSY}[\mathcal{A}] \tag{6}$$

$$\mathcal{P}_\chi[\mathcal{A}] = \int d^4x \left\{ \frac{\delta\mathcal{A}}{\delta U_{R\dot\alpha}} \frac{\delta\mathcal{A}}{\delta \chi_L^{\dot\alpha}} + \frac{\delta\mathcal{A}}{\delta U_{L\dot\alpha}} \frac{\delta\mathcal{A}}{\delta \chi_R^{\dot\alpha}} \right. \tag{7}$$

$$+ \frac{\delta\mathcal{A}}{\delta \overline{\Omega}_{\alpha\dot\alpha}} \frac{\delta\mathcal{A}}{\delta V^{\alpha\dot\alpha}} + \frac{\delta\mathcal{A}}{\delta \overline{\Xi}} \frac{\delta\mathcal{A}}{\delta B} \tag{8}$$

$$\left. + \frac{\delta\mathcal{A}}{\delta \overline{K}} \frac{\delta\mathcal{A}}{\delta \omega} + \frac{\delta\mathcal{A}}{\delta \overline{J}} \frac{\delta\mathcal{A}}{\delta \eta} + \frac{\delta\mathcal{A}}{\delta \overline{\Delta}} \frac{\delta\mathcal{A}}{\delta L} + * \right\} \tag{9}$$

$$\mathcal{P}_\phi[\mathcal{A}] = \int d^4x \left\{ \frac{\delta\mathcal{A}}{\delta Z_L^{\dot\alpha}} \frac{\delta\mathcal{A}}{\delta \phi_{R\dot\alpha}} + \frac{\delta\mathcal{A}}{\delta Z_R^{\dot\alpha}} \frac{\delta\mathcal{A}}{\delta \phi_{L\dot\alpha}} \right. \tag{10}$$

$$+ \frac{\delta\mathcal{A}}{\delta \overline{\Sigma}^{\alpha\dot\alpha}} \frac{\delta\mathcal{A}}{\delta W_{\alpha\dot\alpha}} + \frac{\delta\mathcal{A}}{\delta \overline{\Upsilon}} \frac{\delta\mathcal{A}}{\delta E} \tag{11}$$

$$\left. + \frac{\delta\mathcal{A}}{\delta \overline{J}'} \frac{\delta\mathcal{A}}{\delta \eta'} + \frac{\delta\mathcal{A}}{\delta \overline{K}'} \frac{\delta\mathcal{A}}{\delta \omega'} + \frac{\delta\mathcal{A}}{\delta \overline{\Delta}'} \frac{\delta\mathcal{A}}{\delta L'} + * \right\} \tag{12}$$

$$\mathcal{P}_{\rm SUSY}[\mathcal{A}] = \frac{\partial\mathcal{A}}{\partial h_{\alpha\dot\alpha}} \frac{\partial\mathcal{A}}{\partial \xi^{\alpha\dot\alpha}} \tag{13}$$

As emphasized above, in this paper we do not try to keep manifest supersymmetry. We decompose the superfields into components and look for nilpotent BRST transformations, which then generate the action. All the above fields and sources are components, not superfields.

We note that each term in the above, such as the first one $\frac{\delta\mathcal{A}}{\delta U_{R\dot\alpha}} \frac{\delta\mathcal{A}}{\delta \chi_L^{\dot\alpha}}$, contains one Zinn source derivative (here it is $U_{R\dot\alpha}$) and one Field derivative (here it is $\chi_L^{\dot\alpha}$), and one of them is Grassmann even ($U_{R\dot\alpha}$ here), and the other odd ($\chi_L^{\dot\alpha}$ here). We will discuss the meaning of the symbols more fully below after Eq. (24), where we write down the action.

The Action $\mathcal{A}$ of the theory contains two parts:

$$\mathcal{A} = \mathcal{A}_{\rm Zinn} + \mathcal{A}_{\rm Fields} \tag{14}$$

To start with, one must find an action $\mathcal{A}_{\rm Zinn}$ such that the related BRST Poisson Bracket vanishes identically. This action generates the transformations. We can define a sort of square root of the BRST Poisson Bracket, by

$$\delta_{\rm First} = \delta_{\rm Fields} + \delta_{\rm Zinns} \tag{15}$$

where

$$\delta_{\rm Fields} = \sum_i \int d^4x \frac{\delta\mathcal{A}_{\rm Zinn}}{\delta {\rm Zinn}_i} \frac{\delta}{\delta {\rm Field}_i} \tag{16}$$

$$= \int d^4x \left\{ \frac{\delta\mathcal{A}}{\delta U_{R\dot\alpha}} \frac{\delta}{\delta \chi_L^{\dot\alpha}} + \frac{\delta\mathcal{A}}{\delta U_{L\dot\alpha}} \frac{\delta}{\delta \chi_R^{\dot\alpha}} + \cdots \right) + * \tag{17}$$

and where

$$\delta_{\rm Zinns} = \sum_i \int d^4x \frac{\delta\mathcal{A}_{\rm Zinn}}{\delta {\rm Field}_i} \frac{\delta}{\delta {\rm Zinn}_i} \tag{18}$$

$$= \int d^4x \left\{ \frac{\delta\mathcal{A}_{\rm Zinn}}{\delta \chi_L^{\dot\alpha}} \frac{\delta}{\delta U_{R\dot\alpha}} + \frac{\delta\mathcal{A}_{\rm Zinn}}{\delta \chi_R^{\dot\alpha}} \frac{\delta}{\delta U_{L\dot\alpha}} + \cdots \right) + * \tag{19}$$

If it is true that

$$\mathcal{P}[\mathcal{A}_{\rm Zinn}] = 0 \tag{20}$$

then[3] it follows that

$$\delta^2_{\rm First} = 0. \tag{21}$$

Next one looks for an action that satisfies the invariance identity

$$\delta_{\rm Fields} \mathcal{A}_{\rm Fields} = 0 \tag{22}$$

[3] In the present case this further reduces to suboperators: $\delta_{\rm Fields} = \delta_{\chi\ \rm Fields} + \delta_{\phi\ \rm Fields}$ and $\delta_{\rm Zinns} = \delta_{\chi\ \rm Zinns} + \delta_{\phi\ \rm Zinns}$ and all these suboperators are nilpotent or they anticommute: $\delta^2_{\rm Fields} = \delta^2_{\rm Zinns} = \{\delta_{\rm Fields}, \delta_{\rm Zinns}\} = \delta^2_{\chi\ \rm Fields} = \delta^2_{\phi\ \rm Fields} = \delta^2_{\chi\ \rm Zinns} = \delta^2_{\phi\ \rm Zinns} = 0$ etc.