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Pairing in asymmetric many-fermion systems: Functional renormalisation group approach

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ABSTRACT

Functional renormalisation group approach is applied to a imbalanced many-fermion system with a short-range attractive force. We introduce a composite boson field to describe pairing effects, and assume a simple ansatz for the effective action. A set of approximate flow equations for the effective coupling including boson and fermionic fluctuations is derived and solved. We identify the critical values of particle number density mismatch when the system undergoes a normal state. We determine the phase diagram both at unitarity and around. The obtained phase diagram is in a reasonable agreement with the experimental data.

The mechanism of pairing in imbalanced many-fermion systems is nowdays a subject of the intensive theoretical and experimental studies (see Ref. [1] for review). This phenomenon occurs in many physical systems from molecular physics to quark matter at finite density. Being different in details, the underlying dynamical mechanisms share a common feature related to Cooper instability leading to a rearrangement of the ground state and associated spontaneous symmetry breaking.

In this paper we focus on the asymmetric ultracold atomic Fermi mixture of two fermion flavours, which realises a highly tunable system of strongly interacting fermions. This tunability is provided by a Feshbach resonance, which allows to control the interaction strength between two different species of fermions and explore the BEC-BCS crossover in a wide range of physical parameters. Another tunable parameter (in asymmetric systems) is the population imbalance which can be used to probe how stable the superfluid phase is. The problem was studied long ago by Clogston and Chandrasekhar [2] who found that in the BCS limit the system with the chemical potential mismatch $\delta\mu$ undergoes first order phase transition to a normal phase at $\delta\mu_c = 0.71\Delta_0$ where Δ_0 is the gap at zero temperature for balanced system. Recently, the issue has been looked at again but now in the case of strongly interacting fermions with infinite scattering length (unitary limit) [1]. Most theoretical studies have been performed in the framework of the mean-field (MF) type of approaches which are of limited use for the imbalanced many-fermion systems and may not be reliable in providing quantitative answers. In many cases the effects of quantum fluctuations turn out to be important.

The aim of the present paper is to set up a framework to study pairing phenomena in imbalanced many-fermion systems using the formalism of Functional Renormalisation Group [3] (FRG) where the effects of quantum fluctuations are included in a consistent and reliable way. The FRG approach makes use of the Legendre transformed effective action: $\Gamma[\phi_c] = W[J] - J\cdot\phi_c$, where W is the usual partition function in the presence of an external source J. The action functional Γ generates the 1PI Green's functions and reduces to the effective potential for homogeneous systems. In the FRG one introduces an artificial renormalisation group flow, generated by a momentum scale k and we define the effective action by integrating over components of the fields with $q \gtrsim k$. The RG trajectory then interpolates between the classical action of the underlying field theory (at large k) and the full effective action (at $k = 0$). This method has been successfully applied to a range of problems, from condensed matter physics [4] to particle physics [5].

The evolution equation for Γ in the ERG has a one-loop structure and can be written as

$$\partial_k\Gamma = -\frac{i}{2}\,\mathrm{Tr}\left[(\mathbf{\Gamma}^{(2)}_{BB} - \mathbf{R}_B)^{-1}\,\partial_k\mathbf{R}_B\right] + \frac{i}{2}\,\mathrm{Tr}\left[(\mathbf{\Gamma}^{(2)}_{FF} - \mathbf{R}_F)^{-1}\,\partial_k\mathbf{R}_F\right]. \quad (1)$$

Here $\mathbf{\Gamma}^{(2)}_{FF(BB)}$ is the matrix containing second functional derivatives of the effective action with respect to the fermion (boson) fields and $\mathbf{R}_{B(F)}$ is a matrix containing the corresponding boson (fermion) regulators which must vanish when the running scale

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approaches zero. A 2×2 matrix structure arises for the bosons because we treat ϕ and $\phi^\dagger$ as independent fields in order to include the number-violating condensate. A similar structure also appears for the fermions. By inserting the ansatz for Γ into this equation one can turn it into a set of coupled equations for the various couplings.

Here we study a system of fermions with population imbalancies interacting through an attractive two-body point-like potential and consider pairing between the fermions with different flavours assuming that the interaction between the identical ones is negligible. We take as our starting point the s-wave scattering of two nonidentical fermions in vacuum with a T-matrix determined by the scattering length a. A positive scattering length corresponds to a system with a two-body bound state (and hence repulsive phase-shifts for low-energy scattering) whereas a negative scattering length corresponds to one without a bound state. The binding energy gets deeper as a gets smaller, while the limit $a\to\pm\infty$ is related to a zero-energy bound state.

Since we are interested in the appearance of a gap in the fermion spectrum, we need to parametrise our effective action in a way that can describe the qualitative change in the physics when this occurs. A natural way to do this is to introduce a boson field whose vacuum expectation value (VEV) describes the gap and so acts as the corresponding order parameter. At the start of the RG flow, the boson field is not dynamical and is introduced through a Hubbard–Stratonovich transformation of the four-fermion point-like interaction. As we integrate out more and more of the fermion degrees of freedom by running k to lower values, we generate dynamical terms in the bosonic effective action.

We take the following ansatz for Γ which is a generalisation of the ansatz used in [6] for a balanced many-fermion system

$$\begin{aligned}\Gamma[\psi,\psi^\dagger,\phi,\phi^\dagger,\mu,k] &= \int d^4x\left[\phi^\dagger(x)\left(Z_\phi\, i\partial_t+\frac{Z_m}{2m}\nabla^2\right)\phi(x)-U(\phi,\phi^\dagger)\right.\\ &+\sum_{i=1}^{i=2}\psi^\dagger\left(Z_\psi(i\partial_t+\mu_i)+\frac{Z_{M_i}}{2M_i}\nabla^2\right)\psi\\ &\left.-g\left(\frac{i}{2}\,\psi^{\mathrm{T}}\psi\phi^\dagger-\frac{i}{2}\,\psi^\dagger\psi^{\dagger\mathrm{T}}\phi\right)\right].\end{aligned}\tag{2}$$

Here M_i and m are masses of fermions and composite boson. All renormalisation factors, couplings and chemical potentials run with the scale k. The term containing the boson chemical potential is quadratic in ϕ so it can be absorbed into effective potential U and the Yukawa coupling is assumed to describe the decay (creation) of a pair of nonidentical fermions. Due to $U(1)$ symmetry the effective potential depends only on the combination $\phi^\dagger\phi$. We expand the potential $U(\rho)$ near its minima and keep terms up to order ρ^3.

$$\begin{aligned}U(\phi,\phi^\dagger) &= u_0+u_1(\rho-\rho_0)+\frac{1}{2}\,u_2(\rho-\rho_0)^2\\ &+\frac{1}{6}\,u_3(\rho-\rho_0)^3+\ldots,\end{aligned}\tag{3}$$

where $\rho=\phi^\dagger\phi$. We assume $Z_{\psi_i}=Z_{M_i}=1$ and neglect running of Yukawa coupling. One notes that the expansion near minimum of the effective potential (either trivial or nontrivial), being quite reliable in the case of second order phase transition, may not be sufficient to quantitatively describe the first order one. It is worth emphasising that the CC limit related transition from the superfluid phase to a normal one is of the first order so that a reliability of the expansion needs to be verified. However, as we will discuss below, at small/moderate asymmetries even a simple ansatz for the effective action the effective potential expanded up to the third order in the field bilinears gives a reasonable description of the corresponding phase diagram and provides a clear evidences that the phase transition is indeed of first order.

At the starting scale the system is in a symmetric regime with a trivial minimum so that $u_1(k)$ is positive. At some lower scale $k=k_c$ the coupling $u_1(k)$ becomes zero and the system undergoes a transition to the broken phase with a nontrivial minimum and develops the energy gap.

In our RG evolution we have chosen the trajectory when chemical potentials run in the broken phase and the corresponding particle densities n_i remain fixed so that we define "running Fermi-momenta" for two fermionic species as $p_i=\sqrt{2M_i\mu_i}$. It is convenient to work with the total chemical potential and their difference so we define

$$\mu=\frac{\mu_1+\mu_2}{2};\qquad \delta=\frac{\mu_1-\mu_2}{2}\tag{4}$$

and assume that μ_1 is always larger then μ_2. Calculating corresponding functional derivatives, taking the trace and performing a contour integration results in the following flow equation for the effective potential

$$\begin{aligned}\partial_k U &= -\frac{1}{2Z_\psi}\int\frac{d^3\boldsymbol{q}}{(2\pi)^3}\,\frac{E_{1F}+E_{2F}}{\sqrt{(E_{1F}+E_{2F})^2+4g^2\rho}}\,(\partial_k R_{1F}+\partial_k R_{2F})\\ &+\frac{1}{2Z_\phi}\int\frac{d^3\boldsymbol{q}}{(2\pi)^3}\,\frac{E_{BR}}{\sqrt{E_{BR}^2-V_B^2}}\,\partial_k R_B,\end{aligned}\tag{5}$$

where

$$E_{BR}(q)=\frac{Z_m}{2m}\,q^2+U''\rho+U'+R_B(q,k),\qquad V_B=U''\rho,\tag{6}$$

and

$$\begin{aligned}&E_{iF}\equiv E_{iF}(q,k,p_i)=\epsilon_i(q)-\mu_i+R_{iF}(q,p_i,k),\\ &\epsilon_i(q)=q^2/2M_i.\end{aligned}\tag{7}$$

Here we denote $U'=\frac{\partial U}{\partial\rho}$ and $U''=\frac{\partial^2 U}{\partial\rho^2}$ etc.

One notes that the position of the pole in the fermion loop integral which defines the corresponding dispersion relation is given by

$$q_0=\frac{E_{2F}-E_{1F}\pm\sqrt{(E_{2F}+E_{1F})^2+4\Delta^2}}{2},\tag{8}$$

where $\Delta^2=g^2\rho$ is the square of the pairing gap.

In the physical limit of vanishing scale this dispersion relation indicates a possibility of the gapless exitation in asymmetric many-fermion systems (much discussed Sarma phase [7]). The gappless exitation occurs at $\frac{\Delta}{\delta}<1$. As we will show below, this condition is never fulfilled so that Sarma phase does not occur. We note, however, that this conclusion is valid at zero temperature case and can be altered at finite temperature where the possibility for the Sarma phase still exists [1]. The corresponding bosonic exitations are just gapless "Goldstone" bosons as it should be.

In order to follow the evolution at constant density and running chemical potential we define the total derivative

$$d_k=\partial_k+(d_k\mu)\,\frac{\partial}{\partial\mu}+(d_k\rho)\,\frac{\partial}{\partial\rho},\tag{9}$$

where $d_k\mu=d\mu/dk$, $d_k\rho=d\rho/dk$. Applying this to effective potential, demanding that n is constant ($d_k n=0$) gives the set of the flow equations