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Einstein static Universe in non-minimal kinetic coupled gravity

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ABSTRACT

We study the stability of Einstein static Universe, with FLRW metric, by considering linear homogeneous perturbations in the kinetic coupled gravity. By taking linear homogeneous perturbations, we find that the stability of Einstein static Universe, in the kinetic coupled gravity with quadratic scalar field potential, for closed ($K = 1$) isotropic and homogeneous FLRW Universe depends on the coupling parameters κ and ε. Specifically, for $\kappa = L_P^2$ and $\varepsilon = 1$ we find that the stability condition imposes the inequality $a_0 > \sqrt{3}L_P$ on the initial size a_0 of the closed Einstein static Universe before the inflation. Such inequality asserts that the initial size of the Einstein static Universe must be greater than the Planck length L_P, in consistency with the quantum gravity and quantum cosmology requirements. In this way, we have determined the non-minimal coupling parameter κ in the context of Einstein static Universe. Such a very small parameter is favored in the inflationary models constructed in the kinetic coupled gravity. We have also studied the stability against the vector and tensor perturbations and discussed on the acceptable values of the equation of state parameter.

1. Introduction

Inflationary scenario can address most of the problems in the standard cosmology, however, in spite of the interesting prosperities of inflationary scenario, the existence of a big bang singularity at the beginning of Universe is the major problem of standard cosmology. In attempt to remove the initial singularity, several theories have been proposed to address this issue, such as the string/M-theory, the pre-big bang theory [1] and ekpyrotic/cyclic [2].

In the static closed Friedmann–Lemaître–Robertson–Walker model, the Einstein static Universe is one of the exact solutions of Einstein's equations coupled to a perfect fluid and a cosmological constant [3]. The stability conditions of Einstein static Universe have been widely studied in the literature indicating that this solution is not usually stable against the homogeneous perturbations [4]. In addition, it has been shown that this solution has neutral stability against the adiabatic scalar inhomogeneities with high enough sound speed, as well as the small inhomogeneous vector and tensor perturbations [5]. Nevertheless, it was shown that the Einstein static Universe is unstable against Bianchi type-IX spatially homogeneous perturbations in the presence of perfect fluids with $\rho + 3P > 0$ [6], and for various sources of matter fields [7]. Regardless of historical importance of the Einstein static Universe, the reiterated interest to this solution comes from the "Emergent Universe" scenario, an inflationary cosmological model in which Einstein static Universe plays an incisive role as an initial state.

In the context of general relativity, this model was proposed in 2004 by Ellis et al. to solve the problem of initial singularity in the standard cosmological model [8]. Moreover, the Einstein static Universe has been discussed in several modified gravitational theories and quantum gravity models. Actually, when we are working with the modified cosmological equations, it is possible to find many new static solutions, essentially different from that of classical Einstein static solution of GR, in which the stability properties depend on the details of the studied theory or family of theories taken into account. Basically, due to the existence of neutral stable solutions, the fine-tuning problem of cosmological constant is so improved. But, in fact a mechanism is needed to finish the phase of infinite expansions and collapses, and to operate the expanding phase of the Universe [9]. Such a mechanism has been known as "inflation" [10].

In the context of inflationary cosmology, the role of scalar field potential to establish an inflation is unavoidable. In general, the slowly varying potentials behave like a large effective cosmological constant suitable for driving an inflation. The question that "Is it possible to recover the cosmological constant and the inflationary phase "without" considering any effective potential" led some

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authors to try for constructing an effective cosmological constant starting from extended gravity theories such as non-minimally coupled or higher order theories [11]. In [12], the author considered some types of coupling between curvature and the scalar field, called non-minimal derivative coupling. The authors in [13] studied this kind of couplings and connected them with inflation. Realistic cosmological scenario was introduced based on non-minimal kinetic coupling and it was shown that at early Universe the domination of coupling term in the field equation predicts a quasi-de Sitter expansion [14]. In the background of cosmological scenarios, the non-minimal kinetic coupling gravity has been considered by choosing zero and constant potentials, for the quintessence and the phantom cases [15]. Also, in [15,16] the authors have considered some cosmological aspects of the non-minimal kinetic coupling gravity such as Big Bang, an expanding Universe with no beginning, an eternally contracting Universe, a Big Crunch, a Big Rip avoidance and a cosmological bounce in the absence of the matter. In general, the scalar tensor theory of gravity with scalar field non-minimally coupled to gravity reveals interesting cosmological and astrophysical behaviors [17,18].

According to the above approach to the issue of inflation, it is interesting to study the Einstein static Universe in the inflationary Universe based on the non-minimal kinetic coupled gravity. We show that an asymptotically Einstein static Universe in such inflationary Universe may result due to the terms in the field equations of the non-minimal kinetic coupled gravity. In fact, we find that at early Universe these terms could be dominating and the cosmological evolution could have started around an Einstein static Universe with a size $a_0 > \sqrt{\frac{3\kappa}{\varepsilon}}$, where κ and ε are coupling parameters (see below). We try to remove the initial singularity problem in the standard cosmological model by studying Einstein static Universe and its stability in the non-minimal kinetic coupled gravity theory. Actually, the stability of Einstein static state has been studied in various theories: in GR with a non-constant pressure [19], in brane world scenarios [20], in Einstein–Cartan gravity [21], in loop quantum cosmology [22], in $f(R)$ gravity [23–25], in Gauss–Bonnet gravity [26], in IR modified Hořava gravity [27], in massive gravity [28], and induced matter Brane Gravity [29].

This paper is organized as follows. In Section 2, we briefly review the formalism of the kinetic coupled gravity theory, in particular the action and field equations. In Section 3, we present the modified Friedman equations within the kinetic coupled gravity. In Section 4, we consider linear homogeneous perturbations and study the stability of Einstein static Universe in the kinetic coupled gravity. In Section 5, we study the stability against the vector and tensor perturbations. We summarize our results in Section 6.

2. Non-minimal kinetic coupling gravity

Let us consider a gravitational theory with non-minimal derivative coupling given by the action [30]

$$S = \int d^4x\sqrt{-g}\left\{\frac{R}{8\pi} - \left[\varepsilon g_{\mu\nu} + \kappa G_{\mu\nu}\right]\phi^{,\mu}\phi^{,\nu} - 2V(\phi)\right\} + S_m, \tag{1}$$

where S_m stands for the action of matter, $V(\phi)$ is a scalar field potential, $G_{\mu\nu}$ is the Einstein tensor, ε takes the value $+1$ for the canonical field and -1 for the phantom one and κ is the coupling parameter with dimension of $(length)^2$. Varying the action (1) with respect to $g_{\mu\nu}$ and ϕ gives the field equations, respectively:

$$G_{\mu\nu} = 8\pi\left[T^{(m)}_{\mu\nu} + T^{(\phi)}_{\mu\nu} + \kappa\Theta_{\mu\nu}\right], \tag{2a}$$

$$[\varepsilon g^{\mu\nu} + \kappa G^{\mu\nu}]\nabla_\mu\nabla_\nu\phi = V'(\phi), \tag{2b}$$

where $V'(\phi) \equiv dV(\phi)/d\phi$, $T^{(m)}_{\mu\nu}$ is a stress-energy tensor of ordinary matter, and

$$T^{(\phi)}_{\mu\nu} = \varepsilon[\nabla_\mu\phi\nabla_\nu\phi - \tfrac{1}{2}g_{\mu\nu}(\nabla\phi)^2] - g_{\mu\nu}V(\phi), \tag{3}$$

$$\begin{aligned}\Theta_{\mu\nu} = &-\tfrac{1}{2}\nabla_\mu\phi\,\nabla_\nu\phi\,R + 2\nabla_\alpha\phi\,\nabla_{(\mu}\phi R^\alpha_{\nu)} \\ &+ \nabla^\alpha\phi\,\nabla^\beta\phi\,R_{\mu\alpha\nu\beta} + \nabla_\mu\nabla^\alpha\phi\,\nabla_\nu\nabla_\alpha\phi \\ &- \nabla_\mu\nabla_\nu\phi\,\Box\phi - \tfrac{1}{2}(\nabla\phi)^2 G_{\mu\nu} \\ &+ g_{\mu\nu}\big[-\tfrac{1}{2}\nabla^\alpha\nabla^\beta\phi\,\nabla_\alpha\nabla_\beta\phi + \tfrac{1}{2}(\Box\phi)^2 \\ &- \nabla_\alpha\phi\,\nabla_\beta\phi\,R^{\alpha\beta}\big].\end{aligned} \tag{4}$$

By imposing the Bianchi identity $\nabla^\mu G_{\mu\nu} = 0$ and the matter conservation law $\nabla^\mu T^{(m)}_{\mu\nu} = 0$, Eq. (2a) reduces to

$$\nabla^\mu\left[T^{(\phi)}_{\mu\nu} + \kappa\Theta_{\mu\nu}\right] = 0. \tag{5}$$

Note that by inserting Eqs. (3) and (4) into (5) the differential equation (5) reduces to (2b). Simply, Eq. (2b) is a differential consequence of Eq. (2a).

The authors in [16], have established an inflation model without scalar field potential for the kinetic coupled gravity with spatially flat ($K = 0$) FLRW metric and a cosmological constant, where the cosmological evolution of Universe at the vacuum dominated state $p_v = -\rho_v$ is described by

$$a(t) \propto \exp(H_\kappa t), \tag{6}$$

and

$$\dot\phi(t) \propto \exp(-3H_\kappa t), \tag{7}$$

where

$$H \simeq \sqrt{\frac{1}{9\kappa}}, \tag{8}$$

$$\dot H \simeq 0. \tag{9}$$

As is seen in (8), the role of coupling parameter κ in this inflationary behavior is important such that small value of κ results in a sufficiently large value of the Hubble parameter. Although the present model is different from [16], regarding the scalar field potential and the curvature parameter K, but it is interesting to study the Einstein static Universe and its stability in the context of kinetic coupled gravity and investigate the possible impact of stability requirement on the coupling parameter κ.

3. Einstein static Universe and modified Friedmann equations

3.1. Effective Friedmann equations

The cosmological studies of the non-minimal kinetic coupling gravity have been sufficiently investigated [14–16]. Especially, it was shown that the inflation and any cosmological behavior, explicitly depends on the non-minimally kinetic term of a scalar field ϕ with the curvature. However, it is important to notice that the non-minimally kinetic term of a scalar field ϕ with the curvature describes further degrees of freedom of the gravitational field resulting from modified gravities.

We apply the Friedmann–Lemaître–Robertson–Walker (FLRW) line element as follows

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-Kr^2} + r^2(d^2\theta + \sin^2\theta d^2\phi)\right], \tag{10}$$

where $K = +1, 0, -1$ denotes a closed, flat, and open Universe, respectively.