Physics Letters B 743 (2015) 189–197

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Ghosts in classes of non-local gravity

Antonio De Felice *, Misao Sasaki

Yukawa Institute for Theoretical Physics, Kyoto University, 606-8502, Kyoto, Japan

A R T I C L E   I N F O

Article history:
Received 20 December 2014
Received in revised form 19 February 2015
Accepted 19 February 2015
Available online 23 February 2015
Editor: M. Trodden

A B S T R A C T

We consider a class of non-local gravity theories where the Lagrangian is a function of powers of the inverse d'Alembertian operator acting on the Ricci scalar. We take an approach in which the non-local Lagrangian is made local by introducing auxiliary scalar fields, and study the degrees of freedom of the localized Lagrangian. We find that among the auxiliary scalar fields introduced, some of them are always ghost-like. That is, in the Einstein frame they develop a negative kinetic term. Because of this, except for a particular case already known in the literature, in general, it is not clear how to quantize these models and how to interpret this theory in the light of standard field theory.

1. Introduction

Among the theories introduced to describe the late-time acceleration of the universe, the modified-gravity paradigm has attracted much interest, because it explicitly states that the reason for the acceleration of the universe is due to a modified gravity law which is mostly felt at very large scales. The exploration of different ways of modifying gravity have started since the pioneeristic works in the so-called $f(R)$ gravity. Many other theories have been proposed since then. Among others, let us mention a few of them here: the extension of $f(R)$ theories to $f(R,G)$ theories where G stands for the Gauss–Bonnet term, the DGP model motivated by the possible existence of spatial extra-dimensions, Galileon theories and general scalar–tensor theories of the Horndeski Lagrangian with second order differential equations. All these theories generalize the Einstein–Hilbert Lagrangian by introducing second order Lagrangians (or Lagrangians which reduce to them, as in the $f(R)$ case) for gravity and some extra scalar degrees of freedom. More recently a new class of modifications of gravity has been introduced, so-called non-local theories of gravity. The Lagrangian of these theories consists of terms which are non-local in the form $f(\cdots,\Box^{-1}R,\cdots)$ [1]. These theories have attracted some attention both theoretically [2–20] and phenomenologically [21–26], as a possible alternative to dark energy that renders the universe accelerated at late times.

How to deal with this kind of Lagrangian is a non-trivial topic. We will consider here the case of a general function studied recently in the literature [27]

$$\mathcal{L}=\sqrt{-g}\,f(R,\Box^{-1}R,\cdots,\Box^{-n}R)\,,\qquad \text{with}\qquad n<+\infty\,,\tag{1}$$

and we will try to understand its content. The meaning of such terms in the Lagrangian is obscure, and not very well understood. Some people take the point of view (see e.g. [4]) that, in order to make it sensible, the $\Box^{-1}$ operator must be replaced by the operator $\Box^{-1}_{\rm ret}$ where the subscript "ret" means the retarded boundary condition. This point of view is non-standard in the conventional context of variational calculus where setting initial data is a defining constituent of a theory at level of the action. Further it is unconformable to the usual quantization procedure known for known theories based on the Lagrangian formalism.

In this paper, we take a different approach. Pursuing the standard picture of classical/quantum field theory, we interpret the non-local Lagrangian (1) as equivalent to another, local Lagrangian which can be derived by introducing auxiliary fields. The resulting Lagrangian can be studied with the usual tools of field theory. Namely we consider the Lagrangian,

$$\mathcal{L}=\sqrt{-g}\Bigg[f(\sigma,U_1,U_2,\cdots,U_n)+\frac{\partial f}{\partial\sigma}(R-\sigma)+\lambda_1(R-\Box U_1)\\+\lambda_2(U_1-\Box U_2)+\cdots+\lambda_n(U_{n-1}-\Box U_n)\Bigg].\tag{2}$$

Having the new local Lagrangian (2), we can perform the usual study of the degrees of freedom in the theory. We then find that such a Lagrangian contains in general n ghost-like propagating degrees of freedom in any background. Special cases are also studied, such as the case $\partial^2 f/\partial\sigma^2=0$, separately. In all these subcases we find a finite number of ghost degrees of freedom except for the $n=1$ case. These ghosts are unavoidable, in the sense that they cannot be gauged away. Therefore their presence would make

* Corresponding author.
E-mail address: antonio.defelice@yukawa.kyoto-u.ac.jp (A. De Felice).

http://dx.doi.org/10.1016/j.physletb.2015.02.045

these models, in general, unviable, unless one tunes the mass of these modes to values larger than the cut-off of the theory.

This paper is organized as follows. In Section 2 we rewrite the general non-local Lagrangian in the form of a localized Lagrangian as given by Eq. (2) and analyze its physical degrees of freedom. In Section 3, we focus on a special case where the Lagrangian is linear in the Ricci scalar, that is, the case $\partial^2 f/\partial\sigma^2 = 0$ in Eq. (2). Section 4 is devoted to discussions and conclusion.

2. General non-local gravity action

Let us consider the general action,

$$S = \int d^4x\sqrt{-g}\, f\,; \qquad f \equiv f_1(R, \Box^{-1}R, \Box^{-2}R, \cdots, \Box^{-n}R) + f_2(\Box^{-1}R, \Box^{-2}R, \cdots, \Box^{-m}R)\,, \tag{3}$$

where f is a general function of $\Box^{-k}R$ ($k = 0, 1, 2, \cdots, \max(n, m)$), where n and m are positive integers, i.e. $1 \le (m, n) < \infty$, and the function f_1 is chosen by the condition that it satisfies

$$\frac{\partial^2 f}{\partial R\,\partial(\Box^{-n}R)} = \frac{\partial^2 f_1}{\partial R\,\partial(\Box^{-n}R)} \neq 0\,. \tag{4}$$

Thus n is the largest integer for which this inequality holds. Note that the choice of f_1 is not unique, given the function f, but this ambiguity does not affect our discussion below.

As already mentioned in the Introduction, by allowing ourselves to interpret the action (1) as a model which can be redefined in terms of a local action (without e.g. assuming the d'Alembertian operators restricted on particular or prior-given boundary conditions, which would result in considering different theories), we can rewrite the action as

$$S_{m\le n} = \int d^4x\sqrt{-g}\Big[f_1(\sigma, U_1, U_2, \cdots, U_n) + \frac{\partial f_1}{\partial\sigma}(R-\sigma) + f_2(U_1, \cdots, U_m) + \lambda_1(R - \Box U_1) + \lambda_2(U_1 - \Box U_2) + \cdots + \lambda_n(U_{n-1} - \Box U_n)\Big], \tag{5}$$

or

$$S_{m>n} = \int d^4x\sqrt{-g}\Big[f_1(\sigma, U_1, U_2, \cdots, U_n) + \frac{\partial f_1}{\partial\sigma}(R-\sigma) + f_2(U_1, \cdots, U_m) + \lambda_1(R - \Box U_1) + \lambda_2(U_1 - \Box U_2) + \cdots + \lambda_n(U_{n-1} - \Box U_n) + \cdots + \lambda_m(U_{m-1} - \Box U_m)\Big]. \tag{6}$$

By taking the equations of motion for the fields σ, and λ_i ($i = 1, \cdots, n$), we find

$$\frac{\partial^2 f_1}{\partial\sigma^2}(R-\sigma) = 0\,, \tag{7}$$

$$R = \Box U_1\,, \tag{8}$$

$$U_1 = \Box U_2\,, \tag{9}$$

$$\cdots$$

$$U_{n-1} = \Box U_n\,, \tag{10}$$

for $m \le n$, and the additional equations,

$$U_n = \Box U_{n+1}\,, \tag{11}$$

$$\cdots$$

$$U_{m-1} = \Box U_m\,, \tag{12}$$

for $m > n$. Therefore provided that $\partial^2 f_1/\partial\sigma^2 \neq 0$, we obtain

$$\sigma = R\,, \tag{13}$$

$$U_1 = \Box^{-1}R\,, \tag{14}$$

$$U_2 = \Box^{-1}U_1 = \Box^{-2}R\,, \tag{15}$$

$$\cdots$$

$$U_n = \Box^{-1}U_{n-1} = \Box^{-n}R\,, \tag{16}$$

for $m \le n$, and additionally

$$U_{n+1} = \Box^{-1}U_n = \Box^{-n-1}R\,, \tag{17}$$

$$\cdots$$

$$U_m = \Box^{-1}U_{m-1} = \Box^{-m}R\,, \tag{18}$$

for $m > n$. We regard the original non-local Lagrangian (3) as equivalent to the new one, (5) or (6).

The importance of the new action, (5) or (6), is that it is now clear how many degrees of freedom are present, and their scalar nature. In fact, we can rewrite them as

$$S_{m\le n} = \int d^4x\sqrt{-g}\Big[\Big(\frac{\partial f_1}{\partial\sigma} + \lambda_1\Big)R + g^{\alpha\beta}(\partial_\alpha\lambda_1\partial_\beta U_1 + \partial_\alpha\lambda_2\partial_\beta U_2 + \cdots + \partial_\alpha\lambda_n\partial_\beta U_n) + f_1(\sigma, U_1, U_2, \cdots, U_n) - \sigma\frac{\partial f_1}{\partial\sigma} + \lambda_2 U_1 + \cdots + \lambda_n U_{n-1} + f_2(U_1, \cdots, U_m)\Big], \tag{19}$$

and

$$S_{m>n} = \int d^4x\sqrt{-g}\Big[\Big(\frac{\partial f_1}{\partial\sigma} + \lambda_1\Big)R + g^{\alpha\beta}(\partial_\alpha\lambda_1\partial_\beta U_1 + \partial_\alpha\lambda_2\partial_\beta U_2 + \cdots + \partial_\alpha\lambda_m\partial_\beta U_m) + f_1(\sigma, U_1, U_2, \cdots, U_n) - \sigma\frac{\partial f_1}{\partial\sigma} + \lambda_2 U_1 + \cdots + \lambda_m U_{m-1} + f_2(U_1, \cdots, U_m)\Big]. \tag{20}$$

Let us make a field redefinition as

$$\frac{\partial f_1}{\partial\sigma} + \lambda_1 = \Phi\,, \tag{21}$$

which can be solved for U_n provided

$$\frac{\partial^2 f_1}{\partial\sigma\,\partial U_n} \neq 0\,, \tag{22}$$

which is guaranteed by definition, as given by Eq. (4). Notice that Eq. (22), or, in our approach, its equivalent form (4), excludes General Relativity in this class of theories. Therefore the set of theories considered here, are those ones for which it is possible to solve Eq. (21) in terms of the field U_n. In fact, the field U_n becomes a function of the other $n+2$ fields as

$$U_n = U_n(\sigma, U_j, \Phi - \lambda_1)\,; \quad j = 1, \cdots, n-1\,. \tag{23}$$