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Causal structure in the scalar–tensor theory with field derivative coupling to the Einstein tensor

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We investigate the causal structure in the scalar–tensor theory with the field derivative coupling to the Einstein tensor, which is a class of the Horndeski theory in the four-dimensional spacetime. We show that in general the characteristic hypersurface is non-null, which admits the superluminal propagations. We also derive the conditions that the characteristic hypersurface becomes null and show that a Killing horizon can be the causal edge for all the propagating degrees of freedom, if the additional conditions for the scalar field are satisfied. Finally, we find the position of the characteristic hypersurface in the dynamical spacetime with the maximally symmetric space, and that the fastest propagation can be superluminal, especially if the coupling constant becomes positive. We also argue that the superluminality itself may not lead to the acausality of the theory.

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1. Introduction

After the investigation of many models [1], it has turned out that the successful modification of the Einstein gravity can be rewritten into a class of the Horndeski scalar–tensor theory [2,3], which is known as the most general scalar–tensor theory with the second order equations of motion despite the existence of the various derivative interactions. On the other hand, it is also well known that in the spacetime with more than five dimensions the most general tensor gravitational theory is not the Einstein gravity, but the gravitational theory with the correction of the Lovelock terms [4], for example, in the five-dimensional spacetime the Einstein gravity with the correction of the Gauss–Bonnet term, which does not give rise to the higher derivative terms in the gravitational equations of motion. In superstring/M theories the Lovelock terms appear as a typical form of the quantum corrections [5]. The relation between the Horndeski and Lovelock theories has been argued in the recent works [6] and essentially the Horndeski theory can be derived via the dimensional reduction from the higher-dimensional Lovelock theory. Thus, to find the fundamental aspects of quantum gravity, the investigation of the general properties of the Horndeski theory will be very important.

The causality and well-posedness of the initial value problem are the fundamental issues in any gravitational theory. For example, the well-posedness of the initial value problem in the Einstein gravity has been proven (see e.g. [7]). In the Einstein gravity coupled to the fields with the canonical kinetic terms, it is well known that all the speeds of propagation are less than or equal to the speed of light. On the other hand, if a gravitational theory admits a superluminal degree of freedom, its propagation can become spacelike and hence the discussion on the causality based on the metric does not make sense, because the Cauchy development is fixed by this fastest propagation.

A superluminal propagation is a typical feature in the theory with noncanonical kinetic terms [8–10]. In the case where all the fields have canonical kinetic terms, taking the high frequency mode, the equation of motion of the I-th canonical field ψ_I in the Fourier space reduces to $g^{\mu\nu}k_\mu k_\nu \hat{\psi}_I = 0$, where $g^{\mu\nu}$ is the (inverse) gravitational metric, $\hat{\psi}_I$ is the Fourier component of ψ_I and k_μ is the covariant momentum vector, which gives the solution that k_μ is a null vector. Thus the fastest propagation speeds are the same and coincide with the speed of light. On the other hand, if the degrees of freedom have noncanonical kinetic terms, the above equation is modified as $\mathcal{G}^{\mu\nu}_{(I)} k_\mu k_\nu \hat{\psi}_I = 0$, where $\mathcal{G}^{\mu\nu}_{(I)}$ represents the effective metric for the I-th field, which in general nonlinearly depends on the fields and differs from $g^{\mu\nu}$. Thus the fastest propagation may not be along the null hypersurface but along the spacelike one. If Σ is the hypersurface beyond which the evolution is not unique, Σ is called the characteristic hypersurface (see e.g. [11]). The characteristic hypersurface gives the edge

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of the Cauchy development and the fastest propagation must be tangent to it. The superluminality does not necessarily mean acausality, but the fastest propagation could characterize the Cauchy development. The causal structure should be defined as the chronological past set by this fastest propagation. The Cauchy problem in the modified gravity theories has been investigated, so far in the Lovelock theory [8,9,12,13], the scalar–tensor theories [14,15], the $f(R)$ gravity [16], the nonlinear massive gravity [10] and the string-inspired gravitational theories [17].

In this letter, we investigate the causal properties in a class of the scalar–tensor theory with the field derivative coupling to the Einstein tensor:

$$S=\frac{1}{2}\int d^Dx\sqrt{-g}\Big[M_D^{D-2}R-\Big(g^{\mu\nu}-\frac{z}{M_D^2}G^{\mu\nu}\Big)\partial_\mu\phi\partial_\nu\phi-2V(\phi)\Big], \tag{1}$$

where the indices $\mu,\nu=0,1,2\cdots,D-1$ run the D-dimensional spacetime, $g_{\mu\nu}$ is the metric tensor, $g:=\det(g_{\mu\nu})$ is its determinant, and R and $G_{\mu\nu}$ are the Ricci scalar and the Einstein tensor computed from the metric $g_{\mu\nu}$, respectively. M_D and z represent the D-dimensional Planck mass and the dimensionless parameter characterizing the field derivative coupling to the Einstein tensor, respectively. The scalar field ϕ has the mass dimension $\frac{D-2}{2}$ and $V(\phi)$ is its potential. Despite the existence of the derivative coupling, the highest derivative terms in the equations of motion are of the second order because of the contracted Bianchi identities $\nabla_\mu G^{\mu\nu}=0$, where ∇_μ is the covariant derivative with respect to the metric $g_{\mu\nu}$. In this letter, we do not consider the matter sector, as we focus on the causal properties of the pure gravity sector which is composed of the metric and the scalar field. Also, we will set $M_D=1$, unless it should be given explicitly.

Before starting with the explicit analyses, we should add more explanations about why we focus on the field derivative coupling $G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$. Among the field derivative couplings to the curvature which are of the quadratic order with respect to ϕ, argued in the earlier works [18], $G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$ is the unique coupling that gives the second order equations of motion. In the four-dimensional spacetime, in addition to the fact that the theory (1) corresponds to a class of the Horndeski theory, the coupling $G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$, which is of the quadratic order with respect to ϕ, provides the simplest class of the derivative couplings to the curvature, because the other derivative couplings in the Horndeski theory are typically of higher order with respect to ϕ. Moreover, from the cosmological point of view, Ref. [19] argued that among the various couplings in the Horndeski theory, the coupling $\mathcal{F}_1(\phi)G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$ is one of the special ones which could exhibit the self-tuning mechanism of the cosmological constant. The other couplings obtained in [19] are the nonminimal coupling to the Ricci scalar $\mathcal{F}_2(\phi)R$, that to the Gauss–Bonnet term $\mathcal{F}_3(\phi)(R^2-4R_{\mu\nu}R^{\mu\nu}+R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu})$ and also the field derivative coupling to the double-dual of the Riemann tensor $\mathcal{F}_4(\phi)P^{\mu\nu\alpha\beta}\big(\partial_\mu\phi\,\partial_\alpha\phi\big)\nabla_\nu\nabla_\beta\phi$, where $P^{\mu\nu\alpha\beta}:=-\frac{1}{4}\epsilon^{\mu\nu\lambda\sigma}R_{\lambda\sigma\gamma\delta}\epsilon^{\alpha\beta\gamma\delta}$ ($\epsilon^{\alpha\beta\gamma\delta}$ is the Levi-Civita tensor), which is divergence-free $\nabla_\mu P^{\mu\nu\alpha\beta}=0$ and has the same symmetries with the Riemann tensor [20]. Then, Ref. [19] also argued that among them the field derivative couplings to the spacetime curvature, $\mathcal{F}_1(\phi)G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$ and/or $\mathcal{F}_4(\phi)P^{\mu\nu\alpha\beta}\big(\partial_\mu\phi\,\partial_\alpha\phi\big)\nabla_\nu\nabla_\beta\phi$, should always be included into the theory for obtaining the phenomenologically viable self-tuning mechanism. As the operator $P^{\mu\nu\alpha\beta}\big(\partial_\mu\phi\,\partial_\alpha\phi\big)\nabla_\nu\nabla_\beta\phi$ is typically higher-dimensional than $G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$, $P^{\mu\nu\alpha\beta}\big(\partial_\mu\phi\,\partial_\alpha\phi\big)\nabla_\nu\nabla_\beta\phi$ would be less important at the low energy scales. Therefore, among the couplings argued in [19], as the first step it is reasonable to focus on $G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$ in (1). On the other hand, also in the proxy theory of the nonlinear massive gravity [21] both the couplings $G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$ and $P^{\mu\nu\alpha\beta}\big(\partial_\mu\phi\,\partial_\alpha\phi\big)\nabla_\nu\nabla_\beta\phi$ appear. Again, the coupling $G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$ would be less suppressed than $P^{\mu\nu\alpha\beta}\big(\partial_\mu\phi\,\partial_\alpha\phi\big)\nabla_\nu\nabla_\beta\phi$ by the inverse powers of the strong coupling scale. Furthermore, we should also stress that the coupling $G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$ appears in the low energy effective action of string theory [22] and can be embedded into supergravity [23]. Finally, from the phenomenological points of view, the theory (1) has been extensively applied to cosmology [18,24,25] and black hole physics [26–28]. In the context of the inflationary cosmology, the coupling $G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$ could realize the inflationary expansion and a graceful exit from inflation in the early universe without introducing a potential (for $z<0$) [18]. The shift symmetry and the modified scalar field dynamics with the enhanced friction term due to this kinetic coupling could also provide a UV protected framework for slow-roll inflation (for $z>0$) [25], which could give the predictions consistent with the observational data more easily. In the context of the black hole physics, the exact solution found in the theory (1) represents the stealth accretion of the scalar field onto a Schwarzschild black hole [27], which can circumvent the no-hair arguments and may provide an interesting playground to test the Horndeski theory in the astrophysical environment (see also [26–28], for the other black hole solutions). In summary, $G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$ is one of the most important derivative couplings in the Horndeski theory, in the sense that it could be dominant at the low energy scales and is motivated very well by the various aspects of more fundamental physics, and has very interesting applications to the problems in cosmology and black hole physics. It has also been reported that the perturbation could exhibit the superluminal propagation in the inflationary and black hole backgrounds [25,29]. Therefore, as the next step, it will be very important to clarify more general properties of the theory (1) beyond the particular background solutions, and in this letter we will focus on the causal properties in the theory (1).

Our purpose is to clarify the general conditions that the fastest propagation speed can be superluminal and also all the propagation speeds coincide with the speed of light. We believe that our results can reveal some of the essential causal properties in the Horndeski theory, and also the similarity/difference between the Horndeski and the Lovelock theories studied in [8,9,13]. While the Lovelock terms can be nontrivial in the spacetime with more than five dimensions, the theory (1) becomes nontrivial even in the four-dimensional spacetime and hence the properties pointed out here may also be important in the problems in astrophysics and cosmology.

2. The dynamical equations and characteristics

Varying the action (1) with respect to the metric $g_{\mu\nu}$ gives the gravitational equation of motion

$$G_{\mu\nu}=T_{\mu\nu}+zE_{\mu\nu}, \tag{2}$$

where we defined

$$T_{\mu\nu}:=\nabla_\mu\phi\nabla_\nu\phi-\frac{1}{2}g_{\mu\nu}\nabla^\lambda\phi\nabla_\lambda\phi-V(\phi)g_{\mu\nu}, \tag{3}$$

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