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Quantum corrections in classicalon theories

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ABSTRACT

We use the heat kernel in order to compute the one-loop effective action on a classicalon background. We find that the UV divergences are suppressed relative to the predictions of standard perturbation theory in the interior of the classicalon. There is a strong analogy with the suppression of quantum fluctuations in Galileon theories, within the regions where the Vainshtein mechanism operates (discussed in arXiv:1401.2775). Both classicalon and Galileon theories display reduced UV sensitivity on certain backgrounds.

The scenario of classicalization [1] suggests that high-energy scattering in certain classes of nonrenormalizable scalar field theories can take place at length scales much larger than the typical scale associated with the nonrenormalizable terms in the Lagrangian. It has been argued that the reason for this behavior is that the UV completion of the theory is achieved not through the inclusion of arbitrarily hard modes, but through collective states, which are composed of a large number of soft quanta and display classical properties [2]. The crucial ingredient is the presence of a semiclassical configuration, the classicalon, generated by a point-like source. Classicalons generally exist in theories of Goldstone bosons, or other higher-derivative theories [1,3]. In spite of several studies, a complete picture of classicalization is not available yet. It has been shown that a collapsing spherical wavepacket can be deformed significantly at the so-called classicalization radius, which can be much larger than the fundamental length scale of the theory [4,5]. However, in some theories the classical scattering problem may not have real solutions over the whole space at late times,[1] while in others the maximum of the collapsing wavepacket can reach distances of the order of the fundamental scale. It seems that classicalization is not a generic phenomenon, but appears in theories with particular properties. It has been suggested that such theories cannot be extended through the inclusion of new degrees of freedom at short scales, but generate a physical UV cutoff through their own dynamics [7]. The “wrong-sign” DBI theory is a possible candidate. It can display some undesirable features, such as superluminality on certain nontrivial backgrounds [8,9] (see, however, [7,10]). On the other hand, it has been argued that quantum fluctuations are suppressed in this theory, as well as in all theories that admit classicalons [11]. This is consistent with the notion that hard modes do not play a role in high-energy scattering. In this letter we would like to address this issue through an explicit calculation of quantum corrections on classicalon backgrounds.

We shall follow the steps of a similar calculation, performed in the context of the cubic Galileon theory on a background that realizes the Vainshtein mechanism [12]. The Galileon theory describes the dynamics of the scalar mode that survives in the decoupling limit of the DGP model [13]. It contains a dimensionful coupling that sets the scale Λ at which the theory becomes strongly coupled [14]. This scale can be identified with the UV cutoff. In the presence of a point-like source, the theory has a spherically symmetric solution with a characteristic radius r_V, usually refer to as the Vainshtein radius [12]. At distances much larger than r_V classical fluctuations on top of the background propagate as free waves, while at distances smaller than r_V they are suppressed. In [15] it was argued that, at the scales at which the Vainshtein mechanism operates, quantum fluctuations could be suppressed as well. Quantum corrections in Galileon theories were studied in Refs. [16,17] on a trivial background. In [17] the one-loop corrections were calculated on the Vainshtein background in the presence of an explicit UV cutoff. Through an appropriate modification of the heat-kernel formalism, it was shown that the background reduces the magnitude of the divergent terms. It must be emphasized that the theory remains nonrenormalizable. However, the sensitivity to the physical UV cutoff is much smaller than what would have been expected through naive perturbative arguments.

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[1] An interesting possibility is that the absence of a real classical solution in the scattering problem may indicate the presence of a tunnelling solution in the quantum theory [6], so that classicalization is a quantum process.

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The similar features of Galileon and classicalon theories make it plausible that a mechanism of suppression of quantum fluctuations could operate on classicalon backgrounds. In order to examine this possibility, we repeat the calculation of Ref. [17] for theories that can support classicalons. We consider a class of actions of the form

$$S = \int d^4x \mathcal{K}(X)\,, \tag{1}$$

with $X = \partial_\mu \pi \partial^\mu \pi/2$. Our convention for the Minkowski metric is $\eta_{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1)$. The equation of motion has a one-parameter, static, spherically-symmetric solution given by

$$\mathcal{K}_X \pi'(w) = -\frac{c}{2w^{\frac{3}{2}}}, \tag{2}$$

with c an integration constant (positive or negative), $w = r^2$, $X = 2w\pi'^2$, and $\mathcal{K}_X = \mathcal{K}'(X)$, $\mathcal{K}_{XX} = \mathcal{K}''(X)$ etc. The primes indicate derivatives with respect to the indicated arguments of the various functions: $\pi'(w) = d\pi/dw$, $\mathcal{K}'(X) = d\mathcal{K}/dX$. The factor of 2 in the denominator and the minus sign have been added in order to simplify formulae in the following. When these configurations extend over the whole space, they can be identified as classicalons.

For specific calculations we concentrate on variations of the DBI action. The standard DBI action has

$$\mathcal{K}_1 = \frac{1}{\mu}\sqrt{1 - 2\mu X} \tag{3}$$

with $\mu < 0$, while the "wrong-sign" theory corresponds to $\mu > 0$. The solution (2) becomes

$$\pi_1'(w) = \frac{1}{2}\frac{c}{\sqrt{w^3 + \mu c^2 w}}, \tag{4}$$

with c positive or negative. For $\mu < 0$ the two branches can be joined at the location of the square-root singularity in order to obtain the catenoidal solution that has been studied in [18]. This solution does not extend over the whole space and it is not possible to characterize it as a classicalon. On the other hand, the "wrong-sign" DBI theory with $\mu > 0$ leads to configurations that span the whole space. These are the classicalons considered in [1]. The discontinuity of the first derivative at the origin requires the presence of a δ-function source at this point. Similar solutions can be obtained for a theory with

$$\mathcal{K}_2 = -X - \mu X^2/2 \tag{5}$$

and $\mu > 0$ (keeping only the first two terms in the expansion of the square root in the DBI theory). They are given by

$$\pi_2'(w) = \frac{2 \times 3^{\frac{1}{3}}\mu w^4 - \left(-9\mu^2 c w^5 + \sqrt{\mu^3 w^{10}\left(24w^2 + 81\mu c^2\right)}\right)^{\frac{2}{3}}}{2 \times 3^{\frac{2}{3}}\mu w^{\frac{5}{2}}\left(-9\mu^2 c w^5 + \sqrt{\mu^3 w^{10}\left(24w^2 + 81\mu c^2\right)}\right)^{\frac{1}{3}}}. \tag{6}$$

In Fig. 1 we depict the classicalon solutions $\pi_1'(w)$ (blue lines) and $\pi_2'(w)$ (red lines) for various values of the integration constant c. We express all dimensionful quantities in terms of the fundamental scale of the theory, so that $\mu = 1$. For large w, both solutions are approximately given by $\pi'(w) \simeq c/(2w^{3/2})$, so that $\pi(r) = -c/r$. On the other hand, for $\mu > 0$ and small w we have $\pi_1'(w) \simeq \mathrm{sign}(c)/(2\sqrt{\mu w})$ and $\pi_2'(w) \simeq \mathrm{sign}(c)|c|^{1/3}/(2^{2/3}\mu^{1/3}w^{5/6})$. The transition between the two regimes occurs at the classicalization radius $r_{cl} = \sqrt{w_{cl}} \sim (c^2\mu)^{1/4}$. We do not consider the structure of the classicalons at distances from the origin smaller than $\sim \mu^{1/4}$ because we assume that the theory contains a physical UV cutoff $\Lambda \sim \mu^{-1/4}$. For $|c| \gg 1$ there is a hierarchy between the scales $\mu^{1/4}$ and r_{cl}, and the classicalons are well defined classical objects.

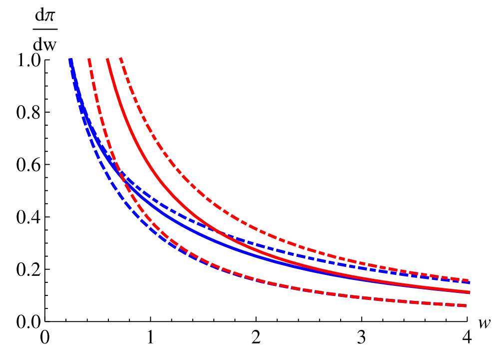

Fig. 1. The classicalon solutions $\pi_1'(w)$ (blue lines) and $\pi_2'(w)$ (red lines) for $\mu = 1$ and $c = 1$ (dashed lines), $c = 2$ (solid lines), $c = 3$ (dot-dashed lines). (For interpretation of the references to colour in this figure, the reader is referred to the web version of this article.)

Our aim is to evaluate the one-loop effective action

$$\Gamma_1 = \frac{1}{2}\mathrm{tr}\log\Delta_E, \tag{7}$$

where Δ_E is the fluctuation operator on the classicalon configuration. The calculation of the effective action (7) requires the transition to Euclidean signature though the definition $t = -ix^0$. For this reason the derivative operators appearing in Δ_E are assumed to act on fields in four-dimensional Euclidean space. The second variation of the action (1) around the solution (2) gives

$$\Delta_E = -G_{\mu\nu}\partial^\mu\partial^\nu - E_\mu\partial^\mu \tag{8}$$

$$G_{\mu\nu} = -\mathcal{K}_X g_{\mu\nu} - \mathcal{K}_{XX}\partial_\mu\pi\partial_\nu\pi \tag{9}$$

$$E_\mu = -2\mathcal{K}_{XX}\partial_\mu\partial_\nu\pi\partial^\nu\pi - \mathcal{K}_{XXX}\partial_\nu\partial_\rho\pi\partial^\rho\pi\partial^\nu\pi\partial_\mu\pi - \mathcal{K}_{XX}\Box\pi\partial_\mu\pi\,. \tag{10}$$

Here $g_{\mu\nu}$ stands for the Euclidean metric.

We would like to compute the effective action (7) using the heat kernel [19]. The calculation of $\mathrm{tr}\log\Delta_E$ for the fluctuation operator (8) can be mapped onto the calculation for a similar operator with covariant derivatives involving both a Riemann and a gauge part [19], for which known results exist [20]. However, the correspondence between the two pictures is very complicated. We find it more efficient to follow the approach of [21], as applied to the case of Galileon theories in [17]. The heat kernel of Δ_E can be computed through the relation

$$h(x, x', \epsilon) = \int \frac{d^4k}{(2\pi)^4} e^{-ikx'} e^{-\epsilon\Delta_E} e^{ikx}. \tag{11}$$

The effective action can then be obtained from its diagonal part as

$$\Gamma_1 = -\frac{1}{2}\int_{1/\Lambda^2}^{\infty} \frac{d\epsilon}{\epsilon}\int d^4x\, h(x, x, \epsilon). \tag{12}$$

A lower limit has been introduced for the ϵ-integration in order to regulate the possible UV divergences. In our case, the UV cutoff is assumed to be $\Lambda \sim \mu^{-1/4}$. The divergent terms in the effective action are generated through the expansion of the exponential in Eq. (11). In order to determine the UV divergences, which appear for $\epsilon \to 0$, it is useful to rescale k by $\sqrt{\epsilon}$, as was done in Ref. [21]. (For details, see [17].) The diagonal part of the heat kernel becomes