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Variation of entanglement entropy in scattering process

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ABSTRACT

In a scattering process, the final state is determined by an initial state and an S-matrix. We focus on two-particle scattering processes and consider the entanglement between these particles. For two types initial states, *i.e.*, an unentangled state and an entangled one, we calculate perturbatively the change of entanglement entropy from the initial state to the final one. Then we show a few examples in a field theory and in quantum mechanics.

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1. Introduction

Entanglement is a characteristic feature in a quantum theory. The entanglement in quantum field theories has been studied extensively in the past decade. When one considers a sub-system A and its complement $\overline{A}$, the entanglement entropy between A and $\overline{A}$ is defined by the von Neumann entropy $S_E = -\mathrm{tr}_A \rho_A \log \rho_A$ with the reduced density matrix ρ_A. Calabrese and Cardy have systematically studied it in a conformal field theory with the use of a replica trick [1]. The other remarkable recent progress is the holographic derivation of entanglement entropy suggested by Ryu and Takayanagi [2,3]. Following it, one can obtain an entanglement entropy by calculating $S_E = \mathcal{A}/(4G_N)$, where $\mathcal{A}$ is the area of a minimal surface whose boundary is the boundary of the subsystem A. In other words, the holographic entanglement entropy provides us with a geometric understanding of entanglement.

Then there is the other geometric interpretation of entanglement entropy conjectured recently by Maldacena and Susskind [4]. Its original purpose was to resolve the firewall paradox [5].[1] This conjecture states that an Einstein–Rosen–Podolski pair, *i.e.*, a pair of entangled objects, is connected by an Einstein–Rosen bridge (or a wormhole). Therefore the conjecture is symbolically called the ER=EPR conjecture. From the point of view of the AdS/CFT correspondence, some examples supporting the ER=EPR conjecture have been shown. An entangled pair of accelerating quark and anti-quark was studied in Ref. [7]. Investigating the causal structure on the world-sheet minimal surface that is the holographic bulk dual of such a quark and anti-quark on the AdS boundary, Ref. [7] has found that there exists a wormhole on the minimal surface and that any open strings connecting the quark and anti-quark must go through the wormhole. Therefore the entanglement of the accelerating quark and anti-quark coincides with the existence of the wormhole. Furthermore, Ref. [8] considered Schwinger pair creation of a quark and an anti-quark and confirmed that there is a wormhole on the string world-sheet of their bulk dual. Ref. [9] focused on a pair of scattering gluons as an EPR pair. Since Ref. [10] had shown the minimal surface solution corresponding to the gluon scattering, Ref. [9] calculated the induced metric on the minimal surface and found a wormhole connecting the gluon pair. One can then naturally guess that, in a scattering process,[2] an interaction induces the variation of entanglement from an incoming state to an outgoing one. We know these states are associated with each other by an S-matrix. So the question is how the variation of entanglement entropy and the S-matrix are related. In this paper we attack this problem by a perturbative analysis in a weak coupling λ. In order to evaluate the entanglement entropy, it is useful to calculate Rényi entropy by the replica trick when one can calculate it exactly. For instance, Ref. [12] explicitly calculated the time evolution of the entanglement entropy between two free scalar field theories with a specific interaction. However, this method is often unavailable for a perturbative analysis. Therefore we apply the method developed by Refs. [13,14], in which the

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[1] See Ref. [6] for an earlier work. It has predicted an energetic curtain, which is similar to the firewall, on the assumptions different from Ref. [5].

[2] Ref. [11] has studied the entanglement entropy in a decay process in terms of the Wigner–Weisskopf method.

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entanglement between two divided momentum spaces was studied perturbatively.

In Section 2, we consider the variations of entanglement entropy from two kinds of initial states; one is an unentangled initial state and the other is an entangled one. In Section 3, we evaluate the variation of entanglement entropy in the field theory with a ϕ^4-like interaction. We also consider the time-dependent interaction in quantum mechanics. Section 4 is devoted to conclusion and discussion.

2. Perturbative calculation of entanglement entropy

Since we are interested in a scattering process of two particles, A and B, and their entanglement, let us consider the Hamiltonian with an interaction:

$$H = H_0 + \lambda H_{\rm int}, \quad H_0 = H_A \otimes \mathbf{1} + \mathbf{1} \otimes H_B. \tag{2.1}$$

It is usually difficult to divide the total Hilbert space $\mathcal{H}$ to $\mathcal{H}_A \otimes \mathcal{H}_B$ due to the interaction. However an initial state far in the past and a final state far in the future in a scattering process can be regarded as states generated by an asymptotically free Hamiltonian. Furthermore, although a field theory in general includes arbitrary multi-particle states in its Hilbert space, we concentrate only on an elastic scattering of two particles such as $\mathrm{A}+\mathrm{B} \to \mathrm{A}+\mathrm{B}$ with a weak coupling. That is to say, we restrict the Hilbert space to the $(1+1)$-particle Fock space, in which the initial and final states are. Since such a restriction usually violates unitarity for local interaction terms, we assume in this paper specific theories that do not produce states of more than $1+1$ particles at lower orders of perturbation (see an example in Section 3.1). Then the unitarity is approximately recovered at a weak coupling. Under this assumption, we can divide the Hilbert space of the initial and final states to $\mathcal{H}_A \otimes \mathcal{H}_B$, and these states are denoted by a $(1+1)$-particle state generated by the free Hamiltonian H_0, namely, a state of a particle A and B with momentum p and q:

$$|p,q\rangle := |p\rangle_A \otimes |q\rangle_B. \tag{2.2}$$

One can express the infinite time evolution from the initial state to the final one in terms of S-matrix by definition,

$$\lim_{t\to\infty} \langle \mathrm{fin}|e^{-iHt}|\mathrm{ini}\rangle = \langle \mathrm{fin}|\mathbf{S}|\mathrm{ini}\rangle, \quad \mathbf{S} := \mathbf{1} + i\mathbf{T}. \tag{2.3}$$

$\mathbf{T}$ is a transition matrix in $\mathcal{O}(\lambda)$ which is induced by the interaction. Then the final state is described as

$$|\mathrm{fin}\rangle = \int dkdl\, |k,l\rangle\langle k,l|\mathbf{S}|\mathrm{ini}\rangle, \tag{2.4}$$

in which we used the completeness relation of $(1+1)$-particles' states, *i.e.*, $(\mathbf{1})_{(1+1)\text{-particles}} = \int dkdl\, |k,l\rangle\langle k,l|$, and an inner product of states, *i.e.*, $\langle k,l|p,q\rangle = \delta(k-p)\delta(l-q)$. Although the norm $\langle p,q|p,q\rangle =: V$ has an infinite volume, we shall fix a normalization at the stage of a reduced density matrix. Here we comment that one can easily formulate the case of discrete spectra by replacing $\int dkdl$ with $\sum_{k,l}$. As an example we shall show in Section 3.2 the theory with a time-dependent interaction in non-relativistic quantum mechanics.

The total density matrix of the final state is $\rho^{(\mathrm{fin})} = |\mathrm{fin}\rangle\langle\mathrm{fin}|$, and we obtain the reduced density matrix $\rho_A^{(\mathrm{fin})}$ by taking trace of $\rho^{(\mathrm{fin})}$ with respect to the particle B, *i.e.*, $\rho_A^{(\mathrm{fin})} = \mathrm{tr}_B\, \rho^{(\mathrm{fin})}$ up to normalization. In the case of (2.4) we can write down the reduced density matrix as

$$\rho_A^{(\mathrm{fin})} = \frac{1}{\mathcal{N}} \int dkdk' \left(\int dl \langle k,l|\mathbf{S}|\mathrm{ini}\rangle\langle \mathrm{ini}|\mathbf{S}^\dagger|k',l\rangle \right) |k\rangle\langle k'|, \tag{2.5}$$

where $\mathcal{N}$ is a normalization constant determined by $\mathrm{tr}_A\, \rho_A^{(\mathrm{fin})} = 1$, namely,

$$\mathcal{N} = \int dkdl\, |\langle k,l|\mathbf{S}|\mathrm{ini}\rangle|^2. \tag{2.6}$$

Then the entanglement entropy between A and B in the final state is

$$S_E^{(\mathrm{fin})} = -\,\mathrm{tr}\, \rho_A^{(\mathrm{fin})} \log \rho_A^{(\mathrm{fin})}, \tag{2.7}$$

and the variation of entanglement entropy from the initial state to the final one is

$$\Delta S_E = S_E^{(\mathrm{fin})} - S_E^{(\mathrm{ini})}, \tag{2.8}$$

where $S_E^{(\mathrm{ini})}$ is the entanglement entropy of the initial state. We shall calculate these entanglement entropies perturbatively.

The replica trick allows us to calculate a Rényi entropy, $S(n) = \frac{1}{1-n} \log \mathrm{tr}_A\, \rho_A^n$. The entanglement entropy is given by the $n \to 1$ limit of Rényi entropy, namely, $S_E = \lim_{n\to 1} S(n) = -\lim_{n\to 1} \frac{\partial}{\partial n} \mathrm{tr}_A\, \rho_A^n$. Therefore the method to derive an entanglement entropy via a Rényi entropy is often useful. However, we are confronted with a problem when we analyze a quantum theory with a coupling λ in terms of perturbation. When one obtains a perturbative expansion of $\mathrm{tr}_A\, \rho_A^n$, the term of order λ^n relevantly contributes to the entanglement entropy because the operation $\lim_{n\to 1} \frac{\partial}{\partial n}$ acts on λ^n and yields a term of $\lambda \log \lambda$ order. In other words, the higher order terms in the Rényi entropy are responsible for the convergence of the entanglement entropy under the $n \to 1$ limit. Hence any λ^n-order terms in $\mathrm{tr}_A\, \rho_A^n$ are necessary in order to obtain a meaningful entanglement entropy. In this paper, instead of the replica trick, we apply the perturbative method developed by Ref. [13] for calculating an entanglement entropy.

2.1. Unentangled initial state

Let us consider the simplest single state with fixed momenta p_1 and q_1 for the initial state of particle A and B,

$$|\mathrm{ini}\rangle \sim |p_1, q_1\rangle. \tag{2.9}$$

The normalization of states will be properly fixed later in normalizing a density matrix so that $\mathrm{tr}_A\, \rho_A^{(\mathrm{fin})} = 1$. This initial state is obviously unentangled, *i.e.*, $S_E^{(\mathrm{ini})} = 0$. Then we can describe the final state (2.4) as

$$\begin{aligned} |\mathrm{fin}\rangle &= \int dkdl\, |k,l\rangle \mathcal{S}_{kl;p_1q_1} \\ &= \frac{\mathcal{S}_{p_1q_1;p_1q_1}}{V^2} |p_1,q_1\rangle + i\lambda \int_{k\neq p_1} dk\, \frac{\mathcal{T}_{kq_1;p_1q_1}}{V} |k,q_1\rangle \\ &\quad + i\lambda \int_{l\neq q_1} dl\, \frac{\mathcal{T}_{p_1l;p_1q_1}}{V} |p_1,l\rangle \\ &\quad + i\lambda \int_{\substack{k\neq p_1 \\ l\neq q_1}} dkdl\, \mathcal{T}_{kl;p_1q_1} |k,l\rangle, \end{aligned} \tag{2.10}$$

where we introduced an infinite spacial volume $V := \int dx\, e^{ix\cdot 0} = \delta(0)$ due to the divergence of norms, *i.e.*, $\langle p|p\rangle_A = \langle q|q\rangle_B = \delta(0)$. The integral $\int_{k\neq p} dk$ means $\int dk(1 - V^{-1}\delta(k-p))$. $\mathcal{S}_{kl;pq}$ and $\mathcal{T}_{kl;pq}$ denote S- and T-matrix elements,

$$\mathcal{S}_{kl;pq} := \langle k,l|\mathbf{S}|p,q\rangle, \quad \mathcal{T}_{kl;pq} := \frac{1}{\lambda}\langle k,l|\mathbf{T}|p,q\rangle. \tag{2.11}$$

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