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Characterizing flow fluctuations with moments

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ABSTRACT

We present a complete set of multiparticle correlation observables for ultrarelativistic heavy-ion collisions. These include moments of the distribution of the anisotropic flow in a single harmonic and also mixed moments, which contain the information on correlations between event planes of different harmonics. We explain how all these moments can be measured using just two symmetric subevents separated by a rapidity gap. This presents a multi-pronged probe of the physics of flow fluctuations. For instance, it allows to test the hypothesis that event-plane correlations are generated by non-linear hydrodynamic response. We illustrate the method with simulations of events in A MultiPhase Transport (AMPT) model.

1. Introduction

Large anisotropic flow has been observed in ultra-relativistic nucleus–nucleus collisions at the Relativistic Heavy-Ion Collider (RHIC) and the Large Hadron Collider (LHC) [1]. Anisotropic flow is an azimuthal (φ) asymmetry of the single-particle distribution [2]:

$$P(\varphi) = \frac{1}{2\pi} \sum_{n=-\infty}^{+\infty} V_n e^{-in\varphi}, \qquad (1)$$

where V_n is the (complex) anisotropic flow coefficient in the nth harmonic. One usually uses the notation v_n for the magnitude: $v_n \equiv |V_n|$. Anisotropic flow is understood as the hydrodynamic response to spatial deformation of the initial density profile. This profile fluctuates event to event, which implies that the flow also fluctuates [3,4]. The recognition of the importance of flow fluctuations has led to a wealth of new flow observables, among which are triangular flow [5] and higher harmonics, as well as correlations between different Fourier harmonics [6].

Flow fluctuations provide a window [7] into both the early stage dynamics and the transport properties of the quark–gluon plasma. Specifically, the magnitudes of higher-order harmonics (V_3 to V_6) are increasingly sensitive to the shear viscosity to entropy density ratio [8]. The distributions of V_2 and V_3 carry detailed information about the initial density profile [9,10], while V_4 and higher harmonics are understood as superpositions of linear and nonlinear responses, through which they are correlated with lower-order harmonics [11,12]. Ideally, one would like to measure the full probability distribution $p(V_1, V_2, \cdots, V_n)$ [13]. So far, only limited information has been obtained, concerning either the distribution of a single V_n [14] or specific angular correlations between different harmonics [6].

We propose to study the distribution $p(V_1, V_2, \cdots, V_n)$ via its moments in various harmonics [15,16], either single or mixed, and illustrate our point with realistic simulations using the AMPT model [17]. In Section 2, we recall how moments can be measured simply with a single rapidity gap [18]. This procedure is less demanding in terms of detector acceptance than the one based on several rapidity windows separated pairwise by gaps [6], and can be used to study two, three and even four-plane correlators. In Section 3, we list standard measures of flow fluctuations which have been used in the literature and express them in terms of moments. In Section 4, we introduce new observables which shed additional light on the origin of event-plane correlations. For instance, a correlation between $(V_2)^2$ and V_4 has been observed, which increases with impact parameter [6]. This correlation is usually understood [19] as an effect of the non-linear hydrodynamic response which creates a V_4 proportional to $(V_2)^2$ [11,20,21]: the increase in the correlation is thus assumed to result from the increase of elliptic flow [22]. We show that this hypothesis can be tested directly by studying how the correlation between $(V_2)^2$ and V_4 is correlated with the magnitude of V_2. We also investigate in

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a similar way the origin of the three-plane correlation between V_2, V_3 and V_5 [6].

2. Measuring moments

The statistical properties of V_n are contained in its moments, which are average values of products of V_n, of the form

$$\mathcal{M} \equiv \left\langle \prod_n (V_n)^{k_n} (V_n^*)^{l_n} \right\rangle, \tag{2}$$

where k_n and l_n are integers, and angular brackets denote an average value over events. Note that $V_n^* = V_{-n}$ and $V_0 = 1$. Azimuthal symmetry implies that the only nontrivial moments satisfy [23]

$$\sum_n n k_n = \sum_n n l_n. \tag{3}$$

We now describe a simple procedure for measuring these moments, which applies to harmonics $n \geq 2$, i.e., $k_1 = l_1 = 0$. (We do not study here moments involving directed flow V_1 [23].) We define in each collision the flow vector [24] by

$$Q_n \equiv \frac{1}{N} \sum_j e^{in\varphi_j}, \tag{4}$$

where the sum runs over N particles seen in a reference detector, and φ_j are their azimuthal angles.[1] One typically measures Q_n in two different parts of the detector ("subevents" [28]) A and B, which are symmetric around midrapidity and separated by a gap in pseudorapidity (i.e., polar angle) [29]. The moment (2) is then given by

$$\mathcal{M} \equiv \left\langle \prod_n (V_n)^{k_n} (V_n^*)^{l_n} \right\rangle = \left\langle \prod_n (Q_{nA})^{k_n} (Q_{nB}^*)^{l_n} \right\rangle, \tag{5}$$

which one can symmetrize over A and B to decrease the statistical error. This configuration, with all factors of Q_n on one side and all factors of Q_n^* on the other side [18], suppresses nonflow correlations and self correlations as long as only harmonics $n \geq 2$ are involved. An alternative procedure, where self correlations are explicitly subtracted, is described in [15].

In order to illustrate the validity of the method, we perform calculations using the AMPT model [17]. AMPT reproduces quite well LHC data for anisotropic flow (v_2 to v_6) at all centralities [30–32]. The implementation adopted in this paper [33] uses initial conditions from the HIJING 2.0 model [34], which contains nontrivial event-by-event fluctuations. Flow in AMPT is produced by elastic scatterings in the partonic phase. In addition, the model contains resonance decays, and thus nontrivial nonflow effects. In the present work, subevent A consists of all particles in the pseudorapidity range $0.4 < \eta < 4.8$, and subevent B is symmetric around mid-rapidity, so that there is an η gap of 0.8 between A and B [35].

The thumb rule for measuring moments is that smaller values of n are easier to measure because v_n decreases with n for $n \geq 2$. Lower order moments, corresponding to smaller values of k_n and l_n, are also easier because higher-order moments are plagued with large variances, which entail large statistical errors.

3. v_n fluctuations, event-plane correlations, standard candles

We first list observables which have been previously studied in the literature and explain how they can be measured using the method outlined in Section 2. Fluctuations of v_n have been studied using cumulants [37–40], which are linear combinations of even moments of the distribution of v_n, that is, $\langle (v_n)^{2k} \rangle$. These moments are obtained by keeping only one value of n and setting $k_n = l_n = k$ in Eq. (5). Fig. 1 displays the scaled moments

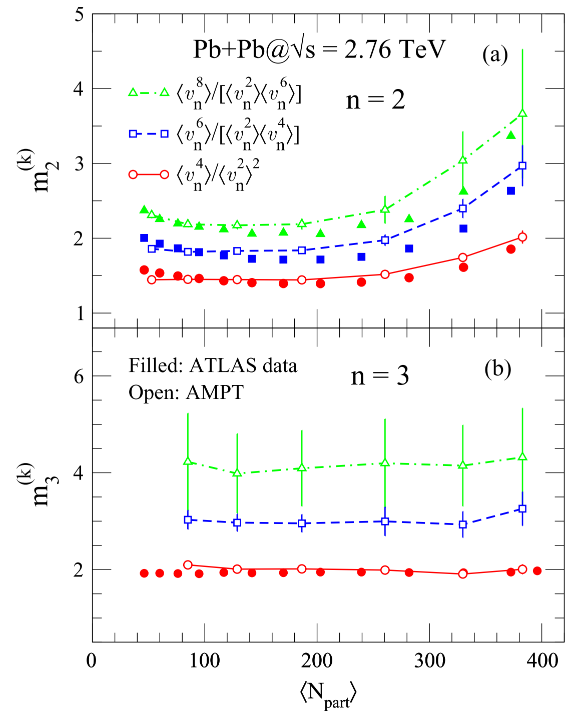

Fig. 1. (Color online.) Scaled moments of the distribution of v_n, (see Eq. (6)) for $k = 2, 3, 4$, as a function of centrality, measured with the number of participant nucleons. Results are for (a) elliptic flow, $n = 2$, and (b) triangular flow, $n = 3$, in Pb–Pb collisions at $\sqrt{s} = 2.76$ TeV. Open symbols represent AMPT calculations and closed symbols are obtained from ATLAS data [36].

$$m_n^{(k)} \equiv \frac{\langle v_n^{2k} \rangle}{\langle v_n^{2(k-1)} \rangle \langle v_n^2 \rangle}, \tag{6}$$

for $k = 2, 3, 4$ as a function of centrality for $n = 2$ and $n = 3$, obtained by using the subevent method of Section 2. The scaled moment $m_n^{(k)}$ thus defined is invariant if one multiplies v_n by a constant, therefore it reflects the statistics of v_n and should be essentially independent of the detector acceptance. AMPT calculations are in fair agreement with the ATLAS data [36], but tend to slightly overpredict $m_n^{(k)}$, i.e., overestimate flow fluctuations.

If flow is solely created by fluctuations and if the statistics of these fluctuations is a 2-dimensional Gaussian [41], then $m_n^{(k)} = k$. As can be seen in Fig. 1(b), $m_3^{(k)} \simeq k$ for all centralities, as expected since v_3 is only from fluctuations in Pb–Pb collisions.[2] Similarly, as seen in Fig. 1(a), $m_2^{(k)}$ is roughly equal to k for central collisions where v_2 is mostly from Gaussian fluctuations, but decreases for mid-central collisions, corresponding to the emergence of a mean elliptic flow in the reaction plane [43].

Event-plane correlations [6] can also be expressed in terms of moments which can be measured using the method outlined in Section 2, as already discussed in Ref. [18]. Specifically, two-plane correlations are Pearson correlation coefficients between

[1] The factor $1/N$ in Eq. (4) means that we choose to average over particles in each event [25], rather than summing [24,26] or dividing by $1/\sqrt{N}$ [27]. This choice is discussed at the end of Section 3.

[2] Small deviations from Gaussian statistics are actually seen experimentally and result in a non-zero cumulant $v_3\{4\}$ [42]. Our simulation does not have enough statistics to detect this small non-Gaussianity.