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Relating quarks and leptons with the T_7 flavour group

Cesar Bonilla [a,*], Stefano Morisi [b], Eduardo Peinado [c], J.W.F. Valle [a]

[a] Instituto de Física Corpuscular (CSIC-Universitat de València), Apdo. 22085, E-46071 Valencia, Spain
[b] DESY, Platanenallee 6, D-15735 Zeuthen, Germany
[c] Instituto de Física, Universidad Nacional Autónoma de México, A.P. 20-364, México D. F. 01000, Mexico

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ABSTRACT

In this letter we present a model for quarks and leptons based on T_7 as flavour symmetry, predicting a canonical mass relation between charged leptons and down-type quarks proposed earlier. Neutrino masses are generated through a Type-I seesaw mechanism, with predicted correlations between the atmospheric mixing angle and neutrino masses. Compatibility with oscillation results leads to lower bounds for the lightest neutrino mass as well as for the neutrinoless double beta decay rates, even for normal neutrino mass hierarchy.

1. Introduction

Ever since the discovery of the muon in the thirties particle physicists have wondered about a possible simple understanding of fermion mass and mixing patterns. The experimental confirmation of neutrino oscillations [1–4] has brought again the issue into the spotlight. Yet despite many attempts, so far the origin of neutrino mass and its detailed flavour structure remains one of the most well-kept secrets of nature. In particular the observed values of neutrino oscillation parameters [5] pose the challenge to figure out why lepton mixing angles are so different from those of quarks. Indeed the sharp differences between the flavour mixing parameters characterising the quark and lepton sectors escalate the complexity of the flavour problem. Many extensions of the Standard Model (SM) have been proposed in order to induce nonzero neutrino masses [6] and to predict the oscillation parameters such as the neutrino mass ordering, the octant of the atmospheric mixing angle and the value of the CP-violating phase in the lepton sector.

A popular approach to tackle these issues is the use of discrete non-Abelian flavour symmetries which are known to be far more restrictive than Abelian ones [7]. In the literature there are many models based on, for instance, A_4 (the group of even permutations of a tetrahedron) whose simplest realisations predict zero reactor mixing angle and maximal atmospheric angle [8–10]. However, this nice prediction has now been experimentally ruled out [1–4] so that the corresponding models need to be revamped in order to account for observations [11].

A variety of possible predictions of flavour symmetry based models can be found, for instance [12]:

i) *neutrino mass sum rules* leading to restrictions on the effective mass parameter $|m_{ee}|$ characterising neutrinoless double beta decay ($0\nu\beta\beta$) processes [13–16];
ii) *neutrino mixing sum rules* [17].

Here we concentrate on the alternative possibility of having *mass relations* in the charged fermion sector. For definiteness we focus on the relation in Eq. (1),

$$\frac{m_b}{\sqrt{m_d m_s}} \approx \frac{m_\tau}{\sqrt{m_e m_\mu}}. \tag{1}$$

* Corresponding author.
E-mail addresses: cesar.bonilla@ific.uv.es (C. Bonilla), stefano.morisi@gmail.com (S. Morisi), epeinado@fisica.unam.mx (E. Peinado), valle@ific.uv.es (J.W.F. Valle).

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Table 1
Matter assignments of the model where $a^7 = 1$.

	$\bar{L}$	ℓ_R	N_R	ν_R	$\overline{Q}$	d_R	u_{R_i}	H	φ_ν	φ_u	φ_d	ξ_ν
T_7	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}_0$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}_i$	$\mathbf{1}_0$	$\mathbf{3}$	$\bar{\mathbf{3}}$	$\mathbf{3}$	$\mathbf{1}_0$
$\mathbb{Z}_7$	a^3	a^3	a^5	a^2	a^3	a^3	a^2	1	a^4	a^2	a^1	a^3

Table 2
Vacuum expectation value alignments.

Flavon	VEV alignment in T_7	Model
φ_ν	$(1, 1, 0)$	$(1 + \delta_{\nu_1}, 1, \delta_{\nu_2})$
φ_u	$(0, 0, 1)$	$(\delta_{u_1}, \delta_{u_2}, 1)$
φ_d	$(1, 0, 0)$	$(1, \delta_{d_1}, \delta_{d_2})$

This relation was suggested in [18–21] and can hold at the electroweak scale.[1] First we note that such a relation between down-type quark and charged lepton masses can be understood because of group structure, when there are three vacuum expectation values and only two invariant contractions (Yukawas) in the product, $\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3}$. For example, such relation can be obtained with other groups containing three-dimensional irreducible representations (irreps) such as, for example, $T_n \cong Z_n \rtimes Z_3$ (with $n = 7, 13, 19, 31, 43, 49$; [23]) as well as T'.

In what follows we build a flavour model for quarks and leptons based upon the smallest non-Abelian group after A_4, namely the flavour group T_7 [24–29] leading to the mass relation in Eq. (1). Neutrino masses are generated by implementing a Type-I seesaw [30] in contrast to the dimensional-five Weinberg operator approach used in previous Refs. [18–20]. We discuss the resulting phenomenological predictions, namely, a correlation between the lightest neutrino mass and the atmospheric angle, as well as lower bounds for the effective mass parameter $|m_{ee}|$ characterising $0\nu\beta\beta$ decay for both neutrino mass orderings.

2. The model

Here we consider a model with the multiplet content in Table 1 where the SM electroweak gauge symmetry is crossed with a global flavour symmetry group T_7. The down-type quarks and leptons (left- and right-handed) transform as triplets, RH up-type quarks transform as singlets while the SM Higgs is blind, as shown in Table 1. Then the Yukawa Lagrangian for the charged sector is given by,

$$\mathcal{L} = \frac{Y^\ell}{\Lambda}\bar{L}\ell_R H_d + \frac{Y^d}{\Lambda}\overline{Q}d_R H_d + \frac{Y^u}{\Lambda}\overline{Q}u_R H_u + h.c. \tag{2}$$

Here for simplicity we have omitted the flavour indices, and have defined $H_d \equiv H\varphi_d$, $H_u \equiv \tilde{H}\varphi_u$ and $\tilde{H} \equiv i\sigma_2 H^*$, where φ_a are T_7 flavon triplets and Λ is the scale at which these fields get their vacuum expectation values (vevs), $\langle\varphi_a\rangle$.

On the other hand, let us assume the existence of four RH-neutrinos accommodated as $\mathbf{3} \oplus \mathbf{1}_0$ under T_7 so that the Lagrangian for the neutrino sector becomes,

$$\mathcal{L}_\nu = \frac{Y_1^\nu}{\Lambda}\bar{L}N_R\tilde{H}_d + \frac{Y_2^\nu}{\Lambda}\bar{L}\nu_R H_u + \kappa_1 N_R N_R \varphi_\nu + \kappa_2 \nu_R \nu_R \xi_\nu \tag{3}$$

where, $\tilde{H}_d \equiv \tilde{H}\overline{\varphi}_d$. Notice that the additional Abelian symmetry $\mathbb{Z}_7$ couples each T_7 flavon triplet with only one fermion sector (down-type, up-type or neutrino sector), so that, flavons transform non-trivially under the discrete Abelian group and their charges are unrelated to each other by conjugation. Therefore, in some sense, the order of the $\mathbb{Z}_n$ symmetry is fixed by the Yukawa sector.

In what follows we will study the flavon potential for three distinct triplets under T_7. The second column of Table 2 shows the vacuum expectation value alignments allowed in T_7 [24,31], with small deviations from those alignments shown in the third column.

2.1. Flavon potential

The Higgs scalar potential for a single T_7 flavon triplet, i.e. $\varphi \simeq \mathbf{3}$, is given by [24,31]

$$V_s = -\mu_s^2 \sum_{i=1}^{3} \varphi_i^\dagger \varphi_i + \lambda_s \left(\sum_{i=1}^{3} \varphi_i^\dagger \varphi_i \right)^2 + \kappa_s \sum_{i=1}^{3} \varphi_i^\dagger \varphi_i \varphi_i^\dagger \varphi_i, \tag{4}$$

where the possible vacuum expectation value alignments are, see Appendix A,

$$\langle\varphi\rangle \sim \frac{1}{\sqrt{3}}(1,1,1) \quad \text{for } \kappa_s > 0 \quad \text{and} \quad \langle\varphi\rangle \sim (1,0,0), (0,1,0), (0,0,1) \quad \text{for } \kappa_s < 0. \tag{5}$$

In our case, ignoring the singlet ξ_ν, there are three triplets, φ_u, φ_d and φ_ν, with an additional $\mathbb{Z}_7$ charge so that the flavon potential is given as

$$V' = V_\nu + V_d + V_u + V_{\text{mix}}, \tag{6}$$

[1] In an early paper [22] Wilczek and Zee found, by using an $SU(2)_H$ symmetry, an extended mass relation $\frac{m_b}{\sqrt{m_d m_s}} = \frac{m_\tau}{\sqrt{m_e m_\mu}} = \frac{m_t}{\sqrt{m_u m_c}}$ which is now evidently ruled out.