



# The method of regions and next-to-soft corrections in Drell–Yan production

D. Bonocore<sup>a</sup>, E. Laenen<sup>a,b,c</sup>, L. Magnea<sup>d</sup>, L. Vernazza<sup>d,e</sup>, C.D. White<sup>f,\*</sup>

<sup>a</sup> Nikhef, Science Park 105, NL-1098 XG Amsterdam, The Netherlands

<sup>b</sup> ITFA, University of Amsterdam, Science Park 904, Amsterdam, The Netherlands

<sup>c</sup> ITF, Utrecht University, Leuvenlaan 4, Utrecht, The Netherlands

<sup>d</sup> Dipartimento di Fisica Teorica, Università di Torino, and INFN, Sezione di Torino, Via P. Giuria 1, I-10125 Torino, Italy

<sup>e</sup> Higgs Centre for Theoretical Physics, School of Physics and Astronomy, The University of Edinburgh, Edinburgh EH9 3JZ, Scotland, UK

<sup>f</sup> SUPA, School of Physics and Astronomy, University of Glasgow, Glasgow G12 8QQ, Scotland, UK

## ARTICLE INFO

### Article history:

Received 7 November 2014

Received in revised form 28 January 2015

Accepted 3 February 2015

Available online 9 February 2015

Editor: A. Ringwald

## ABSTRACT

We perform a case study of the behaviour of gluon radiation beyond the soft approximation, using as an example the Drell–Yan production cross section at NNLO. We draw a careful distinction between the eikonal expansion, which is in powers of the soft gluon energies, and the expansion in powers of the threshold variable  $1 - z$ , which involves important hard-collinear effects. Focusing on the contribution to the NNLO Drell–Yan K-factor arising from real–virtual interference, we use the method of regions to classify all relevant contributions up to next-to-leading power in the threshold expansion. With this method, we reproduce the exact two-loop result to the required accuracy, including  $z$ -independent non-logarithmic contributions, and we precisely identify the origin of the soft-collinear interference which breaks simple soft-gluon factorisation at next-to-eikonal level. Our results pave the way for the development of a general factorisation formula for next-to-leading-power threshold logarithms, and clarify the nature of loop corrections to a set of recently proposed next-to-soft theorems.

© 2015 The Authors. Published by Elsevier B.V. This is an open access article under the CC BY license (<http://creativecommons.org/licenses/by/4.0/>). Funded by SCOAP<sup>3</sup>.

## 1. Introduction

It is well known that singularities arise in perturbative scattering amplitudes due to low-energy (soft) emission of massless gauge bosons, and to collinear splittings of massless particles. These *infrared* (IR) singularities cancel for suitably defined inclusive cross sections, once real and virtual diagrams are combined [1–3]; more generally, they are known to factorise at the level of scattering amplitudes [4], and their general structure in the case of multi-parton non-abelian gauge amplitudes has been the subject of much recent activity (for a recent summary, see for example [5, 6], and the references therein).

Even for finite, infrared-safe cross sections, residual contributions persist after the cancellation of singularities, taking the form of potentially large kinematic logarithms at all orders in perturbation theory, which may need to be resummed. In the generic case of multi-scale processes, these logarithms can have a variety of arguments, such as transverse momenta which vanish at Born level,

or event shape variables which vanish in the two-jet limit. In this note, we will concentrate on *threshold* logarithms, which arise in inclusive cross sections when real radiation is forced to be soft or collinear by the properties of the selected final state. Examples are: electroweak annihilation processes, such as Drell–Yan production or Higgs production via gluon fusion, where the threshold variables are  $z = Q^2/\hat{s}$  and  $z = M_H^2/\hat{s}$ , respectively, with  $\hat{s}$  the partonic center-of-mass energy; Deep Inelastic Scattering (DIS), where the threshold variable is the partonic version of Bjorken  $x$ ; and  $t\bar{t}$  production, where the threshold variable is  $z = 4m_t^2/\hat{s}$ . In all of these cases the cancellation of infrared singularities leaves behind logarithms of the general form  $\alpha_s^n (1 - z)^m \log^p(1 - z)$ , with  $0 \leq p \leq 2n - 1$ , and  $m \geq -1$ .

Contributions with  $m = -1$ , which we describe as *leading power* (LP) threshold logarithms, have been extensively studied, and successfully resummed to very high logarithmic accuracy using a variety of formalisms [7–12]. It is however known that also logarithms accompanied by subleading powers of the threshold variable, most notably those with  $m = 0$ , which we call *next-to-leading power* (NLP) threshold logarithms, can give numerically significant contributions [13]. In recent years, a number of studies have ap-

\* Corresponding author.

E-mail address: [Christopher.White@glasgow.ac.uk](mailto:Christopher.White@glasgow.ac.uk) (C.D. White).

peared [14–27] developing our understanding of certain classes of NLP threshold logarithms. A full-fledged resummation formalism for NLP logarithms is however still not available.

An important class of NLP threshold logarithms, which is the best studied so far, is generated by contributions to scattering amplitudes that arise from the emission of soft gluons, at next-to-leading power in the soft gluon energy. We call these contributions *next-to-eikonal* (NE), or *next-to-soft*. It has been known for many years, at least in the abelian case [28–30], that next-to-soft emissions share many of the universal features that characterise leading-power soft radiation, which is described by the eikonal approximation. This understanding, to some extent, has been generalised to non-abelian theories in [15,21,31], where it was shown that the eikonal approximation can be generalised to take into account next-to-soft effects, while preserving many of the nice universality and factorisation properties which are present at leading power. Ultimately, however, in order to organise all NLP threshold logarithms, one must include also the effects of collinear emissions. The importance of collinear emissions is evident in the case of processes with final-state jets, for example DIS, where some threshold logarithms are directly associated with the mass of the current jet. It is crucial to realise, however, that collinear emissions will also contribute to NLP logarithms for processes, like Drell–Yan or Higgs production, where real radiation is forced to be soft by phase space constraints. In such cases the soft expansion breaks down because singularities arising from virtual hard collinear gluons interfere with the soft approximation. This issue was first tackled, in the abelian case, in Ref. [30], and similar effects were noted in Refs. [15,16]. The analysis of the present paper will precisely identify the origin of these interfering contributions in an example involving real–virtual interference for the Drell–Yan cross section at NNLO.

Quite interestingly, next-to-soft corrections to scattering amplitudes have been the focus of intense recent research also from a more formal point of view. It is well known that leading-power soft radiation can be studied with eikonal methods both in gauge theories and in gravity [32–36]. Recently, Ref. [37] conjectured that next-to-soft behaviour at tree-level is universal in gravity, based on the observation that the known universal soft behaviour [32] can be obtained via a Ward identity associated with the Bondi–Metzner–Sachs (BMS) symmetry at past and future null infinity [38]. Ref. [39] generalised this to Yang–Mills theory, and there have been a number of follow-up studies [40–52]. In particular, Ref. [53] pointed out the relationship between this body of work and the more phenomenological results of Refs. [15,21,28–30]. A key point of contention in the current literature is whether next-to-soft theorems receive corrections at loop level. As Ref. [52] makes clear, this is related to the sequential order in which the expansions in soft momentum and the dimensional regularisation parameter  $\epsilon$  (in  $4 - 2\epsilon$  dimensions) are carried out. The authors of Ref. [52] state that the soft expansion should be carried out first (with  $\epsilon$  kept non-zero). Loop corrections were further explored in Refs. [45,48,51], with Ref. [48] advocating that the soft expansion be carried out *after* the  $\epsilon$ -expansion, which would correspond to how complete amplitudes are usually calculated.

Our aim in this Letter is to perform a case study of NLP threshold logarithms at loop level in Drell–Yan production, including in particular those that originate from next-to-soft corrections to the corresponding scattering amplitude. There are a number of motivations for doing so. First, our ultimate aim (building on the work of Refs. [15,21]), is to develop a fully general resummation prescription for NLP threshold logarithms. Our investigation here will provide crucial data in this regard, although we postpone a detailed discussion of factorisation at NLP accuracy to a subsequent paper [54]. Secondly, by explicitly characterising contributions in

Drell–Yan according to their soft and/or collinear behaviour, we will be able to concretely examine the issue of loop corrections to next-to-soft theorems, including the interplay between the dimensional regularisation and soft expansions. We will verify explicitly that performing the  $\epsilon$  expansion *before* the soft expansion correctly reproduces known results that are sensitive to this ordering. The reason is, as might be expected, the fact that there are collinear singularities arising from virtual exchanges of hard collinear gluons, which are not correctly taken into account if one performs a soft expansion before loop integrations.

More specifically, we will examine the  $K$ -factor for Drell–Yan production at NNLO, concentrating on those terms which arise from having one real and one virtual gluon emission, which are ideally suited to examine the questions posed above. Indeed, logarithms arising from double real emission were already understood from an effective next-to-soft approach in Ref. [21], using the fact that, for electroweak annihilation processes, real radiation near threshold is forced to be soft. Double virtual corrections, on the other hand, have a trivial dependence on the threshold variable  $z$ , and do not influence the present considerations. In this Letter, we will further concentrate on terms proportional to the colour prefactor  $C_F^2$ , which are the same as those that would be obtained in an abelian theory, as considered in the earlier work of [28–30]. This is sufficient to illustrate our main points, and a complete analysis will be given in forthcoming work [54]. Our task here will be to perform a detailed momentum-space analysis of the selected contributions, and trace the origin of all NLP threshold logarithms to hard, soft, or collinear configurations. To this end, we will use the *method of regions*, as developed in [55]. A similar analysis has recently been performed in the case of Higgs production via gluon fusion in the large top mass limit, to an impressive  $N^3$ LO accuracy [56], as part of the complete calculation of the soft and virtual contributions to the cross section at this order. In that case, the method of regions was used as an alternative technique to check the validity of the threshold expansion, and as a method to investigate the convergence properties of the expansion itself.<sup>1</sup> Our goal is different, namely to analyse the factorisation properties of various diagrammatic contributions to the cross section. As a consequence, in Ref. [56] the method of regions was applied after reduction to scalar master integrals, while here we apply it to complete diagrams, thus making it easier to trace various sources of next-to-soft behaviour in our chosen (Feynman) gauge. Furthermore, for the specific NNLO contributions we focus on, we will be able to show how the method of regions gives an exact account of threshold contributions also at next-to-leading power.

Our results will prove useful in the development of a factorisation formula for NLP threshold logarithms, which will generalise the well-known soft-collinear factorisation formula at leading power (see, for example, Ref. [58] for a review of the latter); work in this direction is in progress [54].<sup>2</sup> Interestingly, we find that our analysis with the method of regions is able to reproduce correctly all NLP threshold corrections, including terms with  $m = 0$  and  $p = 0$ , which have no logarithms at all, and would correspond to terms of order  $1/N$  in a Mellin-space analysis, with no  $\log N$  enhancements. We think this gives evidence for the existence of a systematic organisation of threshold contributions to cross sections, order by order in  $m$ . The question then arises of how many terms a fully resummed approach would be able to control, given the progress already made in this regard by the physical evo-

<sup>1</sup> For a discussion of the limits of the threshold expansion in this process, see Ref. [57].

<sup>2</sup> Progress can also be made using effective field theory techniques [59]. One of the authors (CDW) is very grateful to Duff Neill for correspondence on this point, including sharing an early draft of Ref. [59].

Download English Version:

<https://daneshyari.com/en/article/1849178>

Download Persian Version:

<https://daneshyari.com/article/1849178>

[Daneshyari.com](https://daneshyari.com)