Physics Letters B 742 (2015) 183–188

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Bridging a gap between continuum-QCD and *ab initio* predictions of hadron observables

Daniele Binosi [a], Lei Chang [b], Joannis Papavassiliou [c], Craig D. Roberts [d,*]

[a] *European Centre for Theoretical Studies in Nuclear Physics and Related Areas (ECT*) and Fondazione Bruno Kessler, Villa Tambosi, Strada delle Tabarelle 286, I-38123 Villazzano (TN), Italy*
[b] *CSSM, School of Chemistry and Physics, University of Adelaide, Adelaide, SA 5005, Australia*
[c] *Department of Theoretical Physics and IFIC, University of Valencia and CSIC, E-46100, Valencia, Spain*
[d] *Physics Division, Argonne National Laboratory, Argonne, IL 60439, USA*

ARTICLE INFO

Article history:
Received 15 December 2014
Received in revised form 9 January 2015
Accepted 21 January 2015
Available online 23 January 2015
Editor: A. Ringwald

Keywords:
Dyson–Schwinger equations
Confinement
Dynamical chiral symmetry breaking
Fragmentation
Gribov copies

ABSTRACT

Within contemporary hadron physics there are two common methods for determining the momentum-dependence of the interaction between quarks: the top-down approach, which works toward an *ab initio* computation of the interaction via direct analysis of the gauge-sector gap equations; and the bottom-up scheme, which aims to infer the interaction by fitting data within a well-defined truncation of those equations in the matter sector that are relevant to bound-state properties. We unite these two approaches by demonstrating that the renormalisation-group-invariant running-interaction predicted by contemporary analyses of QCD's gauge sector coincides with that required in order to describe ground-state hadron observables using a nonperturbative truncation of QCD's Dyson–Schwinger equations in the matter sector. This bridges a gap that had lain between nonperturbative continuum-QCD and the *ab initio* prediction of bound-state properties.

1. Introduction

The last two decades have seen significant progress and phenomenological success in the formulation and use of symmetry preserving methods in continuum-QCD for the computation of observable properties of hadrons [1–8]. A large part of that work is based on the rainbow-ladder (RL) truncation of QCD's Dyson–Schwinger equations (DSEs), which is the leading-order term in a symmetry preserving approximation scheme [9,10]. The RL truncation is usually employed with a one-parameter model for the infrared behaviour of the quark–quark interaction produced by QCD's gauge-sector [11,12]. It is accurate for ground-state vector- and isospin-nonzero pseudoscalar-mesons constituted from light quarks and also for nucleon and Δ properties because corrections in all these channels largely cancel owing to parameter-free preservation of the Ward–Green–Takahashi (WGT) identities [13–16]. Corrections do not cancel in other channels, however; and hence studies based on the RL truncation, or low-order improvements thereof [17,18], have usually provided poor results for all other systems.

A recently developed truncation scheme [19] overcomes the weaknesses of RL truncation in all channels considered thus far. This new strategy, too, is symmetry preserving but it has an additional strength; namely, the capacity to express dynamical chiral symmetry breaking (DCSB) nonperturbatively in the integral equations connected with bound-states. That is a crucial advance because, like confinement, DCSB is one of the most important emergent phenomena within the Standard Model: it may be considered as the origin of more than 98% of the visible mass in the Universe. Owing to this feature, the new scheme is described as the "DCSB-improved" or "DB" truncation. It preserves successes of the RL truncation but has also enabled a range of novel nonperturbative features of QCD to be demonstrated [20–23].

The widespread phenomenological success of this bottom-up approach to the calculation of hadron observables raises an important question; viz., are the one-parameter RL or DB interaction models, used in those equations relevant to colour-singlet bound-states, consistent with modern analyses of QCD's gauge sector and the solutions of the gluon and ghost gap equations they yield [24–34]? An answer in the affirmative will grant significant additional credibility to the claim that these predictions are firmly grounded in QCD.

* Corresponding author.
E-mail address: cdroberts@anl.gov (C.D. Roberts).

http://dx.doi.org/10.1016/j.physletb.2015.01.031

2. Quark gap equation

In order to expose the computational essence of the bottom-up DSE studies, it is sufficient to consider the gap equation for the dressed quark Schwinger function, $S(p) = Z(p^2)/[i\gamma \cdot p + M(p^2)]$:

$$S^{-1}(p) = Z_2\big(i\gamma \cdot p + m^{\rm bm}\big) + \Sigma(p), \tag{1a}$$

$$\Sigma(p) = Z_1 \int_{dq}^{\Lambda} g^2 D_{\mu\nu}(p-q)\frac{\lambda^a}{2}\gamma_\mu S(q)\frac{\lambda^a}{2}\Gamma_\nu(q,p), \tag{1b}$$

where: $D_{\mu\nu}$ is the gluon propagator[1]; Γ_ν, the quark–gluon vertex; $\int_{dq}^{\Lambda}$, a symbol representing a Poincaré invariant regularisation of the four-dimensional integral, with Λ the regularisation mass-scale; $m^{\rm bm}(\Lambda)$, the current-quark bare mass; and $Z_{1,2}(\zeta^2, \Lambda^2)$, respectively, the vertex and quark wave-function renormalisation constants, with ζ the renormalisation point, which is $\zeta = \zeta_2 :=$ 2 GeV here. Eqs. (1) are the starting point for all DSE predictions of hadron properties.

Significantly, owing to asymptotic freedom, there is no model dependence in the behaviour of the gap equation's kernel on the domain $\mathcal{A} = \{(p^2, q^2) \mid k^2 = (p-q)^2 \simeq p^2 \simeq q^2 \gtrsim 2\ \text{GeV}^2\}$ because perturbation theory and the renormalisation group can be used to show [38–40]:

$$g^2 D_{\mu\nu}(k) Z_1 \Gamma_\nu(q,p) \overset{k^2 \gtrsim 2\,\text{GeV}^2}{=} 4\pi\alpha_s\big(k^2\big) D^{\rm free}_{\mu\nu}(k) Z_2^2 \gamma_\nu, \tag{2}$$

where $D^{\rm free}_{\mu\nu}(k)$ is the free-gauge-boson propagator and $\alpha_s(k^2)$ is QCD's running coupling on this domain. Kindred results follow immediately for the kernels in the two-body Bethe–Salpeter equations relevant for meson bound-states [9,10,19].

Eq. (2) entails that the model input in realistic DSE studies is expressed in a statement about the nature of the gap equation's kernel on $\mathcal{A}$; i.e., at infrared momenta. One writes

$$Z_1 g^2 D_{\mu\nu}(k)\Gamma_\nu(q,p) = k^2 \mathcal{G}\big(k^2\big) D^{\rm free}_{\mu\nu}(k) Z_2 \Gamma^A_\nu(q,p) \tag{3a}$$

$$= \big[k^2 \mathcal{G}_{\rm IR}\big(k^2\big) + 4\pi\tilde{\alpha}_{\rm pQCD}\big(k^2\big)\big] \times D^{\rm free}_{\mu\nu}(k) Z_2 \Gamma^A_\nu(q,p), \tag{3b}$$

where $\tilde{\alpha}_{\rm pQCD}(k^2)$ is a bounded, monotonically-decreasing regular continuation of the perturbative-QCD running coupling to all values of spacelike-k^2; $\mathcal{G}_{\rm IR}(k^2)$ is an assumed form for the interaction at infrared momenta, with $k^2\mathcal{G}_{\rm IR}(k^2) \ll 4\pi\tilde{\alpha}_{\rm pQCD}(k^2)$ $\forall k^2 \gtrsim$ 2 GeV^2; and $\Gamma^A_\nu(q,p)$ is an *Ansatz* for the dressed-gluon–quark vertex, with $\Gamma^A_\nu(q,p) = Z_2\gamma_\nu$ on $\mathcal{A}$.

As reviewed elsewhere [5,6,8], successful explanations and predictions of numerous hadron observables are obtained with

$$\mathcal{I}\big(k^2\big) = k^2 \mathcal{G}\big(k^2\big), \tag{4a}$$

$$\mathcal{G}\big(k^2\big) = \frac{8\pi^2}{\omega^4} D e^{-k^2/\omega^2} + \frac{8\pi^2\gamma_m \mathcal{E}(k^2)}{\ln[\tau + (1 + k^2/\Lambda^2_{\rm QCD})^2]}, \tag{4b}$$

where: $\gamma_m = 12/(33 - 2N_f)$ [typically, $N_f = 4$], $\Lambda_{\rm QCD} = 0.234$ GeV; $\tau = e^2 - 1$; and $\mathcal{E}(k^2) = [1 - \exp(-k^2/[4m_t^2])]/k^2$, $m_t = 0.5$ GeV.

[1] Landau gauge is typically used because it is, *inter alia* [35–37]: a fixed point of the renormalisation group; that gauge for which sensitivity to model-dependent differences between *Ansätze* for the fermion-gauge-boson vertex are least noticeable; and a covariant gauge, which is readily implemented in numerical simulations of lattice regularised QCD. Importantly, capitalisation on the gauge covariance of Schwinger functions obviates any question about the gauge dependence of gauge invariant quantities.

The origin and features of Eq. (4b) are detailed in Ref. [11] so here we only highlight two key aspects: the *Ansatz* is consistent with the constraints described above and it involves just one free parameter.

The last point deserves further attention. At first glance there appear to be two free parameters in Eq. (4b): D, ω. However, computations show [11,12,41] that a large body of observable properties of ground-state vector- and isospin-nonzero pseudoscalar-mesons are practically insensitive to variations of $\omega \in [0.4, 0.6]$ GeV, so long as

$$(\varsigma_G)^3 := D\omega = \text{constant}. \tag{5}$$

(The midpoint $\omega = 0.5$ GeV is usually employed in calculations.) This feature also extends to numerous properties of the nucleon and Δ resonance [4,7]. The value of ς_G is typically chosen in order to obtain the measured value of the pion's leptonic decay constant, f_π. It is striking that fitting just one parameter in a *Gaußian Ansatz* for the gap equation's kernel is sufficient to achieve an efficacious description of a wide range of hadron observables. It provides *prima facie* evidence that Eqs. (3), (4) are correct in principle; and translates the question posed at the end of Section 1 into the following: "How does $k^2\mathcal{G}_{\rm IR}(k^2)$ in Eq. (4a) compare with today's understanding of QCD's gauge sector?"

That question has a subtext, however, because the fitted value of ς_G depends on the form of $\Gamma^A_\nu(q,p)$. We consider two choices herein: RL and DB. The RL truncation is obtained with

$$\Gamma^A_\nu(q,p) = Z_2\gamma_\nu. \tag{6}$$

It is summarised in Appendix A.1 of Ref. [42] and provides the most widely used DSE computational scheme in hadron physics. In this case one has [23]

$$\varsigma_G^{\rm RL} = 0.87\ \text{GeV}. \tag{7}$$

The form of $\Gamma^{\rm DB}_\nu(q,p)$ is detailed in Appendix A.2 of Ref. [42]. It is consistent with constraints imposed by both the longitudinal and transverse WGT identities [43]. The DB kernel is connected with the most refined nonperturbative truncation that is currently available. It is therefore expected to be the most realistic. With this vertex, one has [23]

$$\varsigma_G^{\rm DB} = 0.55\ \text{GeV}. \tag{8}$$

Following upon this discussion, we arrive at a pair of simple questions. Does an analysis of QCD's gauge sector produce a running interaction-strength that is generally consistent with the form in Eqs. (4); and, if so, does it more closely resemble the function obtained with ς_G in Eq. (7) or (8)?

3. RGI interaction kernel

In order to expose the quantity with which Eq. (4b) should be compared, we must provide some background. The Landau-gauge dressed-gluon propagator has the simple form

$$D_{\mu\nu}(k) = D^{\rm free}_{\mu\nu}(k)\Delta\big(k^2\big) = \left[\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right]\frac{\Delta(k^2)}{k^2} =: \mathcal{T}^k_{\mu\nu}\mathcal{D}\big(k^2\big); \tag{9}$$

and since we are interested in QCD's gauge sector, the dressed-ghost propagator will also be relevant:

$$\mathcal{F}\big(k^2\big) = -\frac{F(k^2)}{k^2}. \tag{10}$$

As we now explain, the scalar function in Eq. (10) is connected with the following gluon-ghost vacuum-polarisation: