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Geometric measure of quantum discord for entanglement of Dirac fields in noninertial frames

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ABSTRACT

We investigate the geometric measure of quantum discord of all possible bipartite divisions of a tripartite system of Dirac fields in noninertial frames. As a comparison, we calculate the geometric measure of entanglement. We discuss the properties of geometric measure of quantum discord and geometric measure of entanglement for three qubit–qubit subsystems with acceleration parameter and the parameter describing the degree of entanglement the system in detail. We have found a conservative relationship involving two of three geometric discords in some condition and another conservative relationship involving three geometric discords for initially maximally entangled states. By the way, we also report some conservative relationships of concurrence, mutual information and geometric measure of entanglement for two bipartite subsystems.

1. Introduction

The theory of relativity and quantum theory form the cornerstones of modern physics. In addition, the combination of quantum theory and information theory further yields quantum information theory. Furthermore, the integration of quantum information and relativity theory creates the theory of relativistic quantum information [1–3], which combines general relativity, quantum field theory and quantum information theory. Evidently, the study on the theory of quantum information in a relativistic framework is not only helpful for understanding some fundamental questions in quantum information theory, but also is practical, because many contemporary experiments on quantum-information processing use photons or other particles that have relativistic velocities. In recent years, the theory of relativistic quantum information has become a focus of research in quantum information science for both conceptual and experimental reasons.

Recently, much effort has been made in the study of entanglement shared between inertial and noninertial observers by discussing how the Unruh effect and Hawking effect will influence the degree of entanglement. Following some seminal work performed in this regard [4,5], many authors focus on the study of entanglement between quantum field modes as observed by relatively accelerating observers. For example, Q. Pan and J. Jing studied the degradation of non-maximal entanglement of scalar and Dirac fields in non-inertial frames [6]. Mi-Ra Hwang et al. studied tripartite entanglement in a noninertial frame using π-tangle [7]. J. Wang and J. Jing investigated multipartite entanglement of fermionic systems in noninertial frames also using π-tangle [8]. Due to the resemblance between the Unruh effect [9] and Hawking radiation [10], some authors also studied the degradation of entanglement occurred in black-hole physics. Q. Pan and J. Jing have investigated the effect of the Hawking temperature on the entanglement and teleportation for the scalar field in a most general, static and asymptotically flat black hole with spherical symmetry [11]. They also studied entanglement redistribution in the Schwarzschild spacetime [12].

Though most studies in noninertial systems have focused on the entanglement, however, it has been found that the entanglement is not the only characteristic of a quantum system, and it has no advantage for some quantum information tasks. In some cases [13–15], although there is no entanglement, certain quantum information processing tasks can still be done efficiently by using quantum correlation [16–18], which is believed to be more workable than the entanglement. Quantum correlation may be quantified by quantum discord rather than by entanglement. The quantum discord, initially introduced by Ollivier and Zurek [16] and by Henderson and Vedral [17], is a measure of quantum correlations that extends beyond entanglement. M. Ali et al. found the Quantum discord for two-qubit X states [19]. J. Wang et al. studied

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classical correlation and quantum discord sharing of Dirac fields in noninertial frames [20]. C.C. Rulli et al. studied the global quantum discord in multipartite systems [21]. A. Datta investigated the quantum discord between relatively accelerated observers [22].

The above studies show the calculation of quantum discord involves a difficult optimization procedure: it is difficult to obtain analytical results except for a few families of two-qubit states [19,23,24]. Therefore, Dakić et al. proposed a geometric measure of quantum discord [25], which also was called a geometric discord [26–28].

Before we start discussing our subject, we notice that recently there was a debate on the geometric measure of quantum discord [29]. M. Piani argued that the geometric measure of quantum discord is not a good measure for the quantum correlations. A detailed discussion about this issue in detail is beyond the scope of this paper. Here, we will still use the geometric discord to study bipartite correlations presented in three two-qubit subsystems of the tripartite system in a noninertial frame.

In this paper we are going to consider the following situation. Alice and Rob share an entangled state initially when they are not moving relatively. Subsequently, Rob moves with a uniform acceleration with respect to Alice. This system is a bipartite from an inertial perspective, but from a noninertial perspective an extra set of complementary modes in Rindler region II becomes relevant. Therefore, we calculate the geometric measure of quantum discord in all possible bipartite divisions of the tripartite system: the mode A described by Alice, the mode I in Rindler region I (described by Rob), and the complementary mode II in Rindler region II. For comparison, we also derived the analytic expressions of geometric measure of entanglement [30,31] as a function of Rob's acceleration for the same system. Our results revealed that geometric measure of quantum discord gives the similar global properties as geometric measure of entanglement does for the system under consideration, but in some respect, the description of the system using geometric discord are more detailed than using other measures. More important, we report some conservative relationship of geometric discord, concurrence and mutual information in noninertial frame.

This paper is organized as follows. In the next section, we give a short review of geometric measure of quantum discord and geometric measure of entanglement. In Section 3 we derive the analytic expressions of geometric discord. Section 4 devotes to calculate the geometric measure of entanglement and compares two kinds of geometric measures. A detailed discussion and summary are given in Section 5.

2. Brief review of geometric measure of quantum discord and geometric measure of entanglement

For convenience of later use, we give a brief review of geometric measure of quantum discord and geometric measure of entanglement, respectively.

Quantum discord is a quantum-versus-classical paradigm for correlations [32–34] and is not in the entanglement-versus-separability framework [35,36]. The quantum discord of a bipartite state ρ on a system $H^a \otimes H^b$ with marginals ρ^a and ρ^b can be expressed as

$$Q(\rho) = \min_{\Pi^a}\{I(\rho) - I(\Pi^a(\rho))\}. \tag{1}$$

Here the minimum is over von Neumann measurements (one-dimensional orthogonal projectors summing up to the identity) $\Pi^a = \{\Pi^a_k\}$ on subsystem a, and

$$\Pi^a(\rho) = \sum_k (\Pi^a_k \otimes I^b)\rho(\Pi^a_k \otimes I^b) \tag{2}$$

is the resulting state after the measurement. $I(\rho) = S(\rho^a) + S(\rho^b) - S(\rho)$ is the quantum mutual information, $S(\rho) = -\operatorname{tr}\rho\ln\rho$ is the von Neumann entropy, and I^b is the identity operator on H^b. Then, Dakić et al. proposed the following geometric measure of quantum discord [25]:

$$D(\rho) = \min_{\chi}\|\rho - \chi\|_2^2, \tag{3}$$

where the minimum is over the set of zero-discord states [i.e., $Q(\chi) = 0$] and $\|A\|_2 := \sqrt{\operatorname{tr}(A^\dagger A)}$ is the Frobenius or Hilbert–Schmidt norm. The density operator of any two-qubit state can be expressed as

$$\rho = \frac{1}{4}\left(\mathbf{I}^A \otimes \mathbf{I}^B + \sum_{i=1}^{3}(x_i\sigma_i \otimes \mathbf{I}^B + \mathbf{I}^A \otimes y_i\sigma_i) + \sum_{i,j=1}^{3} t_{ij}\sigma_i \otimes \sigma_j\right), \tag{4}$$

where $\{\sigma_i,\ i = 1, 2, 3\}$ denote the Pauli spin matrices. Then, the geometric measure of quantum discord of any two-qubit state is evaluated as

$$D(\rho) = \frac{1}{4}(\|\mathbf{x}\|^2 + \|\mathbf{T}\|^2 - k_{max}), \tag{5}$$

where $\mathbf{x} := (x_1, x_2, x_3)^t$ is a column vector, $\|\mathbf{x}\|^2 := \sum_i x_i^2$, $x_i = \operatorname{tr}(\rho(\sigma_i \otimes \mathbf{I}^b))$, $T := (t_{ij})$ is a matrix and $t_{ij} = \operatorname{tr}(\rho(\sigma_i \otimes \sigma_j))$, k_{max} is the largest eigenvalue of matrix $\mathbf{x}\mathbf{x}^t + \mathbf{T}\mathbf{T}^t$.

Since Dakić et al. proposed the geometric measure of quantum discord, many authors extended Dakić's results to the general bipartite states. Luo and Fu evaluated the geometric measure of quantum discord for an arbitrary state and obtained an explicit formula

$$D(\rho) = \operatorname{tr}(\mathbf{C}\mathbf{C}^t) - \max_A \operatorname{tr}(\mathbf{A}\mathbf{C}\mathbf{C}^t\mathbf{A}^t), \tag{6}$$

where $\mathbf{C} = (c_{ij})$ is an $m^2 \times n^2$ matrix, given by the expansion $\rho = \sum c_{ij} X_i \otimes Y_j$ in terms of orthonormal operators $X_i \in L(H^a)$, $Y_j \in L(H^b)$ and $A = (a_{ki})$ is an $m \times m^2$ matrix given by $a_{ki} = \operatorname{tr}|k\rangle\langle k|X_i = \langle k|X_i|k\rangle$ for any orthonormal basis $|k\rangle$ of H^a. They also gave a tight lower bound for geometric discord of arbitrary bipartite states [37]. Recently, a different tight lower bound for geometric discord of arbitrary bipartite states was given by S. Rana et al. [26], and Ali Saif M. Hassan et al. [38] independently. Alternatively, D. Girolami et al. found an explicit expression of geometric discord for two-qubit system and extended it to $(2\bigotimes d)$-dimensional systems [27]. T. Tufarelli et al. also gave another formula of geometric discord for qubit–qudit system, which is available to $(2\bigotimes d)$-dimensional systems including $d = \infty$ [28].

On the other hand, geometric measure of entanglement was first proposed by T.C. Wei et al. [30,31]. For pure states it is defined as follows:

$$E_g(|\psi\rangle) = 1 - \Lambda^2_{\max} = 1 - \max_{\phi}|\langle\psi|\phi\rangle|^2 \tag{7}$$

where $|\phi\rangle$ is an arbitrary separable pure state and the maximization is done over the set of $|\phi\rangle$. For mixed states ρ, the geometric measure of entanglement was originally defined via the convex roof construction, in the same way as was done for the entanglement of formation:

$$E_g(\rho) = \min\sum_i p_i E_g(|\psi_i\rangle) \tag{8}$$

with minimization over all pure state decompositions of ρ. Calculation of geometric measure needs to find the entanglement