Physics Letters B 728 (2014) 63–67

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

On transverse spin sum rules

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ARTICLE INFO

Article history:
Received 30 September 2013
Received in revised form 16 November 2013
Accepted 19 November 2013
Available online 25 November 2013
Editor: A. Ringwald

ABSTRACT

In this work we show that (i) both the form factors A_i and $\bar{C}_i$ contribute to the matrix element of the energy–momentum tensor T_i^{+-} in a transversely polarized state, (ii) there is no relative suppression factor between these two contributions and (iii) the contribution to the matrix element of the Pauli–Lubanski operator $W_i^{\perp}$ from that of T_i^{++} contains only the form factor B_i and not the form factor A_i. These results support our criticism and the conclusions as stated in Ref. [13]. Comparing and contrasting the spin sum rules in two different approaches, one advocated by us and the one proposed by Jaffe and Manohar, we point out that the physical content of the sum rules is very transparent in our approach, whereas, in the second approach details of the dynamics remain hidden and the separation into orbital and intrinsic spin parts is not visible.

Introduction

At present, understanding the helicity and the transverse spin structure of the proton in the context of Deep Inelastic Scattering (DIS) is of great interest. Intense experimental and theoretical research activities have been going on in this field for more than a decade. It is well-known that since DIS is a light cone dominated process, the most appropriate theoretical tool to study it is provided by Light Front Quantization (for a review, see Ref. [1]). In order to understand the spin structure of the proton which is a composite object and investigate any sum rule associated with it, one should start from the intrinsic spin operators $\mathcal{J}^i$, $i = 1, 2, 3$, which can be constructed from the Pauli–Lubanski operator. Among the Poincare group generators, the intrinsic spin operators on the light front commute with the generators of translations and boosts (which are kinematical as well in the light front dynamics) in the longitudinal and transverse directions. As a result, the light front intrinsic spin operators are boost and translation invariant and, further, they obey the angular momentum algebra [2–4]. On the other hand, instant form intrinsic spin operators do not commute with boost operators which are dynamical [5]. Any angular momentum sum rule, solely based on the matrix elements of rotation operators that are part of Poincare generators, will have frame dependence. The same is also true, in general, if one starts with the Pauli–Lubanski operators as we discuss below. As already stated, the solution to this problem is to start from the intrinsic spin operators $\mathcal{J}^i$.

The helicity operator $\mathcal{J}^3$ (whose explicit construction and a perturbative analysis in light front QCD is carried out in Ref. [6] in the total transverse momentum zero frame) is kinematical (interaction free). On the other hand, it is well known that the transverse rotation operators and hence the transverse spin operators in light front theory are dynamical (interaction dependent). Construction and analysis of $\mathcal{J}^i$ $(i = 1, 2)$ in light front QCD is carried out in Refs. [7,8].

Recently, the matrix element of the transverse component of the Pauli–Lubanski operator has been formally analyzed in Refs. [9] and [10] (hereafter referred to as Ji et al.) following the approach of Ref. [11] and using the parameterizations of the off-forward matrix elements of the energy–momentum tensor. These authors are partly inspired by Ref. [12] in which a relation between the expectation value of equal time transverse rotation generator J_q^i and the form factors $A_q(0)$ and $B_q(0)$ is obtained using delocalized wave packet states that are transversely polarized in the rest frame of the nucleon.

We have pointed out in Ref. [13] that many of the statements in Ji et al. appear unsupported by explicit calculations. In this work we present explicit calculations supporting our statements in Ref. [13]. We also compare and contrast our approach [6–8] with the approach presented in [11] to derive sum rules.

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0370-2693
http://dx.doi.org/10.1016/j.physletb.2013.11.042

General outline of the calculation

The starting point in Ji et al. is the Pauli–Lubanski operator which is defined in terms of energy–momentum tensor in a very standard way as follows.

$$W^{\mu} = -\frac{1}{2}\epsilon^{\mu\nu\alpha\beta}M_{\nu\alpha}P_{\beta},$$
$$M^{\mu\nu} = \frac{1}{2}\int dx^{-}\,d^2x^{\perp}\,\big[x^{\mu}T^{+\nu} - x^{\nu}T^{+\mu}\big]. \tag{1}$$

Whereas, starting point in Refs. [7,8] is the intrinsic spin operators which, for a massive particle like nucleon, are related to Pauli–Lubanski operators.

$$M\mathscr{J}^{i} = W^{i} - P^{i}\mathscr{J}^{3}$$
$$= \epsilon^{ij}\left(\frac{1}{2}F^{j}P^{+} + K^{3}P^{j} - \frac{1}{2}E^{j}P^{-}\right) - P^{i}\mathscr{J}^{3},$$
$$\mathscr{J}^{3} = \frac{W^{+}}{P^{+}} = J^{3} + \frac{1}{P^{+}}\big(E^{1}P^{2} - E^{2}P^{1}\big). \tag{2}$$

In Eqs. (2), $F^{i} = M^{-i}$ are the light front transverse rotation operators and are interaction dependent or dynamical; while $E^{i} = M^{+i}$ are light front transverse boost operators and are interaction independent or kinematical. Longitudinal boost $K^{3} = M^{+-}$ and helicity $J^{3} = M^{12}$ are also kinematical. Note that the light front transverse rotation and the boost operators were mis-identified in Ji et al. This was already pointed out in Ref. [14]. Moreover, Ji et al. did not consider longitudinal boost operator $K^{3} = M^{+-}$ for working explicitly in $P^{\perp} = 0$ frame and only for such a choice of frame, both the starting points appear to be the same. In the following, we kept this term to show an example in the course of our explicit calculations that, in general, for a frame with non-zero $P^{\perp}$ both are not the same. We also assume that the various Poincare generators can be separated to quark and gluon parts.

Next, to compare with the results of Ji et al., we need to calculate the transverse component of the Pauli–Lubanski operator corresponding to species i formally defined as

$$W_i^1 = \frac{1}{2}F_i^2P^{+} + K_i^3P^2 - \frac{1}{2}E_i^2P^{-} \tag{3}$$

and its matrix element in a transversely polarized state

$$\frac{\langle PS^{(1)}|W_i^1|PS^{(1)}\rangle}{(2\pi)^3 2P^{+}\delta^3(0)} \tag{4}$$

where i denotes either the quark or gluon part. Note that, in the rest of the Letter, we always deal with only one component, namely, W_i^1, while calculation with W_i^2 is trivially the same and unnecessary for our purpose.

The transverse rotation operator is

$$F_i^2 = \frac{1}{2}M_i^{-2} = \frac{1}{4}\int dx^{-}\,d^2x^{\perp}\,\big[x^{-}T_i^{+2} - x^2T_i^{+-}\big]. \tag{5}$$

We note that,

$$K_i^3 = \frac{1}{2}M_i^{+-} = \frac{1}{4}\int dx^{-}\,d^2x^{\perp}\,\big[x^{+}T_i^{+-} - x^{-}T_i^{++}\big]$$
$$= \frac{1}{2}x^{+}P^{-} + \tilde{K}_i^3,$$
$$E_i^2 = M_i^{+2} = \frac{1}{2}\int dx^{-}\,d^2x^{\perp}\,\big[x^{+}T_i^{+2} - x^2T_i^{++}\big]$$
$$= x^{+}P^2 + \tilde{E}_i^2. \tag{6}$$

In writing the last equalities in both the above expressions, we note that light front time x^{+} can be taken out of the integral in the first terms and simplified. Putting them back in Eq. (3), we see that only the second terms in these expressions contribute to W_i^1. Thus we find that

$$W_i^1 = \frac{1}{2}F_i^2P^{+} + \tilde{K}^3P^2 - \frac{1}{2}\tilde{E}_i^2P^{-} \tag{7}$$

with no explicit x^{+} dependence. Lastly, the light front helicity operator is given by

$$J_i^3 = M_i^{12} = \frac{1}{2}\int dx^{-}\,d^2x^{\perp}\,\big[x^1T_i^{+2} - x^2T_i^{+1}\big]. \tag{8}$$

According to the procedure prescribed in Ref. [11], rest of the calculation relies on defining the Fourier transform of the off-forward matrix elements of relevant component of energy–momentum tensor and then consider the forward limit. Since W_i^1 is independent of x^{+} explicitly, we consider three dimensional Fourier transform of the off-forward matrix element. In general, we define

$$\big\langle P'S^{(1)}\big|\hat{\mathscr{O}}^{\alpha}(k_{-},k_i)\big|PS^{(1)}\big\rangle$$
$$= \frac{1}{2}\int dx^{-}\,d^2x^{\perp}\,e^{i(k_{-}x^{-}+k_ix^i)}x^{\alpha}\big\langle P'S^{(1)}\big|\mathscr{O}(x)\big|PS^{(1)}\big\rangle \tag{9}$$

where $\alpha = -, 1, 2$. Using translational invariance, we find

$$\big\langle P'S^{(1)}\big|\hat{\mathscr{O}}^{\alpha}(k)\big|PS^{(1)}\big\rangle$$
$$= -i(2\pi)^3\frac{\partial}{\partial k_{\alpha}}\big[\delta^3\big(k+P'-P\big)\big\langle P'S^{(1)}\big|\mathscr{O}(0)\big|PS^{(1)}\big\rangle\big]$$
$$= -i(2\pi)^3\delta^3\big(k+P'-P\big)\frac{\partial}{\partial k_{\alpha}}\big\langle P'S^{(1)}\big|\mathscr{O}(0)\big|PS^{(1)}\big\rangle \tag{10}$$

ignoring the term containing the derivative on the delta function [11].

Thus, with $\Delta = P' - P$, we find

$$\big\langle PS^{(1)}\big|F_i{}^2\big|PS^{(1)}\big\rangle = i(2\pi)^3\delta^3(0)\bigg[\frac{\partial}{\partial\Delta_{-}}\big\langle P'S^{(1)}\big|T_i^{+2}(0)\big|PS^{(1)}\big\rangle$$
$$- \frac{\partial}{\partial\Delta_2}\big\langle P'S^{(1)}\big|T_i^{+-}(0)\big|PS^{(1)}\big\rangle\bigg]_{\Delta=0}, \tag{11}$$

$$\big\langle PS^{(1)}\big|\tilde{K}_i^3\big|PS^{(1)}\big\rangle$$
$$= -\frac{i}{2}(2\pi)^3\delta^3(0)\bigg[\frac{\partial}{\partial\Delta_{-}}\big\langle P'S^{(1)}\big|T_i^{++}(0)\big|PS^{(1)}\big\rangle\bigg]_{\Delta=0} \tag{12}$$

and

$$\big\langle PS^{(1)}\big|\tilde{E}_i^2\big|PS^{(1)}\big\rangle$$
$$= -i(2\pi)^3\delta^3(0)\bigg[\frac{\partial}{\partial\Delta_2}\big\langle P'S^{(1)}\big|T_i^{++}(0)\big|PS^{(1)}\big\rangle\bigg]_{\Delta=0}. \tag{13}$$

Matrix elements of the energy–momentum tensor

We start from the following parameterization as used in Ji et al.,

$$\big\langle P',S'\big|T_i^{\mu\nu}(0)\big|PS\big\rangle$$
$$= \overline{U}\big(P',S'\big)\bigg[A_i\big(\Delta^2\big)\frac{1}{2}\big(\gamma^{\mu}\overline{P}^{\nu} + \gamma^{\nu}\overline{P}^{\mu}\big)$$
$$+ B_i\big(\Delta^2\big)\frac{1}{2M_N}\frac{1}{2}\big(\overline{P}^{\mu}i\sigma^{\nu\alpha}\Delta_{\alpha} + \overline{P}^{\nu}i\sigma^{\mu\alpha}\Delta_{\alpha}\big)$$
$$+ C_i\big(\Delta^2\big)\frac{1}{M_N}\big(\Delta^{\mu}\Delta^{\nu} - g^{\mu\nu}\Delta^2\big) + \overline{C}_i\big(\Delta^2\big)M_Ng^{\mu\nu}\bigg]U(P,S). \tag{14}$$

Here $\overline{P} = \frac{1}{2}(P + P')$.