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Checking flavour models at neutrino facilities

Davide Meloni

Dipartimento di Matematica e Fisica, Università di Roma Tre INFN, Sezione di Roma Tre, Via della Vasca Navale 84, I-00146 Rome, Italy

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A B S T R A C T

In the recent years, the industry of model building has been the subject of the intense activity, especially after the measurement of a relatively large values of the reactor angle. Special attention has been devoted to the use of non-abelian discrete symmetries, thanks to their ability of reproducing some of the relevant features of the neutrino mixing matrix. In this Letter, we consider two special relations between the leptonic mixing angles, arising from models based on S_4 and A_4, and study whether, and to which extent, they can be distinguished at superbeam facilities, namely T2K, NOνA and T2HK.

1. Introduction

The recent measurement of a non-vanishing θ_{13} by Daya Bay [1] and RENO [2] has exerted some pressure on models for neutrino mixing based on the permutation groups (like A_4 and S_4, [3]), as they are generically constructed to give at leading order very specific patterns in which $\theta_{13}=0$ and the other angles are also completely fixed. Corrections from the charged sector or next-to-leading contributions to the neutrino mass matrix have to be invoked to correct such patterns and make the models compatible with the experimental data. The usual approach to model building is that of considering a Lagrangian invariant under a flavour group G and to subsequently break G into two different subgroups in the charged lepton and neutrino sector, is such a way to create two different rotations, responsible for a non-diagonal Pontecorvo–Maki–Nakagawa–Sakata (U_{PMNS}) mixing matrix. The structure of G can also be reconstructed from the residual symmetries of the mass matrices after symmetry breaking; for example, using the criterion that a flavour group should be obtained from the neutrino mixing matrix without parameter tuning, it was shown in [4] that the minimal group containing all the symmetries of the neutrino mass matrix and leading to the tri-bimaximal mixing (TBM [5]) is S_4. The fact that the mixing angles are fixed to well defined values is the consequence of forcing all the symmetries of the mass matrix to belong to G. Moving from this consideration, in [6] a different point of view was adopted: they assumed that the residual symmetries in both the charged lepton and neutrino sectors are one-generator groups. Indicating with S_i and T_α ($\alpha=e,\mu,\tau$) the generators of the Z_2 and Z_m discrete symmetries of the neutrino and charged leptons mass matrices, the previous condition implies that $\{S_i, T_\alpha\}$ form a set of generators for the flavor group G for given i and α, with the meaning that all other symmetries appear accidentally. The structure of the generators is restricted by the additional requirements to be elements of $SU(3)$, for which $\mathrm{Det}[S_i]=\mathrm{Det}[T_\alpha]=1$, so they can be written as:

$$\begin{aligned}
&S_1=\mathrm{diag}(1,-1,-1), \qquad S_2=\mathrm{diag}(-1,1,-1),\\
&S_3=\mathrm{diag}(-1,-1,1), \qquad T_e=\mathrm{diag}\big(1,e^{2\pi ik/m},e^{-2\pi ik/m}\big),\\
&T_\mu=\mathrm{diag}\big(e^{2\pi ik/m},1,e^{-2\pi ik/m}\big),\\
&T_\tau=\mathrm{diag}\big(e^{2\pi ik/m},e^{-2\pi ik/m},1\big). \qquad (1)
\end{aligned}$$

The definition of G requires a relation linking S_i and T_α, assumed to be $(S_iT_\alpha)^p=(U_{PMNS}S_iU^\dagger_{PMNS}T_\alpha)^p=I$. The lack of additional symmetry in G has the direct consequence that the mixing angles are not all fixed (like in the TBM) but rather present some interesting correlations, or sum rules, that open the possibility to reconcile the predictions of the permutation groups with the experimental data already at leading order (see also [7] for similar sum rules obtained in the context of S_4 and [8] for sum-rules from residual Z_2 symmetries). The question we want to analyze in this Letter is whether such correlations can be tested at neutrino facilities or, in other words, if model comparison and selection can be achieved at currently taking data or planned superbeams. It is clear that if two models live in completely different regions of the parameter space (given by the spanned values of all θ_{ij} and the leptonic CP phase) the measurement of the mixing parameters with huge precision will give the answer; however, we are still away from such an idealized situation, at least for what concerns the CP phase, and it is necessary to evaluate the performance of the neutrino facilities to face this problem. In this respect, we have selected two models from [6], called $1T$ and $2T$, which have been shown

E-mail address: meloni@fis.uniroma3.it.

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to be compatible with the current experimental data in the neutrino sector and with the hypothesis of TBM, and have used their different correlations to compute and compare (in a χ^2 analysis) the expected event rates at T2K, NOνA and T2HK, with the aim of identifying the regions in the (θ_{13}, δ)-plane where the models can be distinguished at some confidence level. An interesting work along similar lines has been recently presented in [9], where the main focus was on the ability of next-generation of neutrino oscillation experiments to constraints correlations involving θ_{23}, θ_{13} and $\cos\delta$. We differ from [9] in that we consider different neutrino facilities, we use non-linear relations between the oscillation parameters and adopt a different statistical analysis with the purpose, given the lack of information on the CP phase, to present exclusion regions directly in the (θ_{13}, δ) parameter space. It is important to stress again that such correlations are *leading order* predictions, in the sense that they are derived from group theoretical considerations and do not take into account possible higher order effects into the lepton mass matrices of new-physics effects [10], otherwise model-dependent features will appear with the main effect to spoil the sum rules and introduce additional indetermination of the parameter spaces where the models live. We do not take into account this possibility, as we are mainly interested to check whether the easiest case (validity of the sum rules) can be addressed at neutrino experiments. We revise the useful neutrino transition probabilities in Section 2, where we also introduce the models $1T$ and $2T$ and discuss the parameter spaces allowed by the correlations; in Section 3 we introduce the neutrino facilities used in our numerical simulation and discuss the results of the statistical analysis performed to distinguish the models. Our conclusions are drawn in Section 4.

2. Setting the background

2.1. The relevant transition probabilities

Since we are interested in the performance of superbeam facilities, it is enough to consider the $\nu_\mu \to \nu_e$ appearance and $\nu_\mu \to \nu_\mu$ disappearance probabilities (and their CP-conjugate). Given the relatively large θ_{13}, we consider the probabilities up to first order in the small parameter $r = \Delta m^2_{sol}/\Delta m^2_{atm} \sim 0.03$ [11] while keeping their exact dependence on θ_{13}. In vacuum they read:

$$
\begin{aligned}
P_{\mu e} = {} & \sin^2 2\theta_{13} s^2_{23} \sin^2\Delta - r\big[\Delta s^2_{12} \sin^2 2\theta_{13} s^2_{23} \sin 2\Delta \\
& + \Delta \sin 2\theta_{12} s_{13} c^2_{13} \sin 2\theta_{23}\big(-2\sin\delta_{\mathrm{CP}} \sin^2\Delta \\
& + \cos\delta_{\mathrm{CP}} \sin 2\Delta\big)\big], \qquad (2)
\end{aligned}
$$

$$
\begin{aligned}
P_{\mu\mu} = {} & 1 - \sin^2\Delta\big[c^4_{13} \sin^2(2\theta_{23}) + s^2_{23} \sin^2(2\theta_{13})\big] \\
& + r\big\{\Delta \sin 2\Delta\big(c^2_{13}\big(\sin^2(2\theta_{23})\big(c^2_{12} - s^2_{12} s^2_{13}\big) \\
& - 4 s_{13} \cos\delta \sin 2\theta_{12}\big) \sin^3(\theta_{23}) \cos(\theta_{23})\big) \\
& + s^2_{12} s^2_{23} \sin^2(2\theta_{13})\big\}, \qquad (3)
\end{aligned}
$$

where $\Delta \equiv \frac{\Delta m^2_{31} L}{4E_\nu}$, $s_{ij} = \sin\theta_{ij}$ and $c_{ij} = \cos\theta_{ij}$. Notice that:

$$P_{\bar\alpha\bar\beta} = P_{\alpha\beta}(\delta_{\mathrm{CP}} \to -\delta_{\mathrm{CP}}), \qquad (4)$$

$$P_{\beta\alpha} = P_{\alpha\beta}(\delta_{\mathrm{CP}} \to -\delta_{\mathrm{CP}}), \quad \alpha, \beta = e, \mu, \tau. \qquad (5)$$

As it is well known, $P_{\mu e}$ is mainly dependent of θ_{13} and δ whereas $P_{\mu\mu}$ is recognized to be more sensitive to the atmospheric parameters; although the dependence on δ is suppressed by the small r, the approximation adopted shows that θ_{13} already appears at leading order. We then expect that flavour models with different parameter spaces, that is with the mixing parameters living in different regions, are also characterized by different transition probabilities that, extracted from the experimental data, can help in distinguishing among them. In our numerical computations we consider the mixing angles to vary within the 2σ intervals taken from [12]:

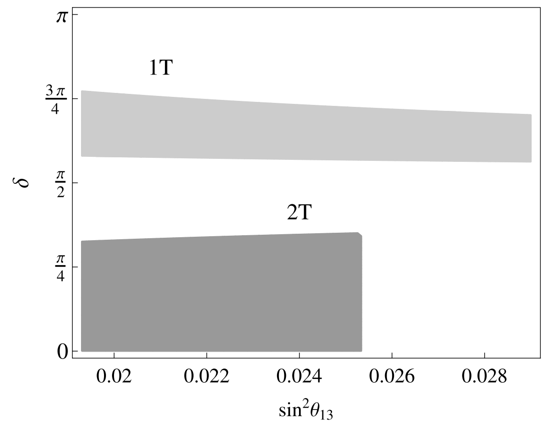

Fig. 1. Allowed values of δ as a function of $\sin^2\theta_{13}$ as derived imposing the correlations among the mixing parameters, Eqs. (7)–(8) for the model $1T$ and Eqs. (9)–(10) for $2T$.

$$
\begin{aligned}
\sin^2\theta_{23} &= 0.386^{+0.062}_{-0.038} \\
\sin^2\theta_{13} &= 0.0241^{+0.0049}_{-0.0048} \\
\sin^2\theta_{12} &= 0.307^{+0.035}_{-0.032}, \qquad (6)
\end{aligned}
$$

whereas the CP phase is left free to vary in the whole $[0, 2\pi)$ range. We consider the mass differences as constant quantities, $\Delta m^2_{31} = 2.4 \times 10^{-3}$ eV^2, $\Delta m^2_{21} = 7.5 \times 10^{-5}$ eV^2, since the models studied in this Letter do not give any information on the neutrino masses.

2.2. A summary of the models $1T$ and $2T$

In this section we recall the main features of the correlations arising from two different models discussed in [6], of which we also follow the nomenclature. Both of them have $T_\alpha = T_e$. The first model, called $1T$, uses the generator $S_1 = \mathrm{Diag}(1, -1, -1)$ and the pair of values $(p, m) = (4, 3)$, which corresponds to the group S_4. The obtained relations among the mixing angles are:

$$\cos^2\theta_{12} = \frac{2}{3\cos^2\theta_{13}} \qquad (7)$$

and

$$\tan 2\theta_{23} = -\frac{1 - 5s^2_{13}}{2\cos\delta s_{13}\sqrt{2(1 - 3s^2_{13})}}, \qquad (8)$$

also obtained in the explicit model of Ref. [13] and further studied in [14].

For any values of θ_{13}, the first relation always gives an acceptable value of the solar angle, within the 2σ bounds quoted in Eq. (6), so this relation does not set any restriction on the reactor angle. It has to be noted, however, that the dependence on the cosines function forces $\sin^2\theta_{12}$ to be around 0.31–0.32, very close to the current central value. On the other hand, Eq. (8) imposes a constraint on the possible pairs of (θ_{13}, δ) needed to fulfill the bounds for θ_{23} in Eq. (6); in particular, the value of the CP phase can never be maximal in this model and, in order to have an atmospheric angle in the first octant, the relation $\cos\delta > \pi/2$ must hold. The exact bounds in the (θ_{13}, δ)-plane can be derived numerically from Eq. (8) and are shown in Fig. 1 where, as expected,