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Non-equilibrium time evolution of higher order cumulants of conserved charges and event-by-event analysis

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ABSTRACT

We investigate the time evolution of higher order cumulants of conserved charges in a volume with the diffusion master equation. Applying the result to the diffusion of non-Gaussian fluctuations in the hadronic stage of relativistic heavy ion collisions, we show that the fourth-order cumulant of net-electric charge at LHC energy is suppressed compared with the recently observed second-order cumulant at ALICE, if the higher order cumulants at hadronization are suppressed compared with their values in the hadron phase in equilibrium. The significance of the experimental information on the rapidity window dependence of various cumulants in investigating the history of the dynamical evolution of the hot medium created in relativistic heavy ion collisions is emphasized.

1. Introduction

Statistical mechanics tells us that observables are fluctuating even in equilibrated medium. Because the fluctuations are determined by the microscopic nature of the medium and sensitive to critical phenomena, they can be exploited to reveal and characterize properties of the medium. In experimental attempts to map the global nature of QCD phase transition at nonzero baryon density in relativistic heavy ion collisions, fluctuation observables, especially those of conserved charges, are believed to be promising observables to diagnose the property of the hot medium [1–8]. Active investigation in heavy ion collisions by event-by-event analyses has recently been performed at the Relativistic Heavy Ion Collider (RHIC) and the Large Hadron Collider (LHC) [9–12]. Numerical analyses of higher order cumulants in equilibrium have been also carried out in lattice QCD Monte Carlo simulations [13].

As an experiment in which fluctuations are measured, heavy ion collisions have several notable features. First, higher order cumulants, which characterize the non-Gaussian nature of fluctuations, have been experimentally observed with good statistics up to the fourth order [10,11]. The measurement is possible because the system observed in the experiments is not large; the observed particle number is at most of order 10^3, while the event-by-event statistics exceed 10^7. Second, the fluctuations observed in experiments, especially those of conserved charges, are not necessarily the same as those in an equilibrated medium, because of the dynamical nature of the hot medium created by heavy ion collisions. Because of these properties, an appropriate description of the dynamical evolution of non-Gaussian fluctuations is required in order to understand the experimental results on higher order cumulants.

Concerning the second point, we remark that the recent experimental result on the net-electric charge fluctuation, $\langle(\delta N_{\rm Q}^{(\rm net)})^2\rangle$, by ALICE Collaboration at LHC [12] supports the non-thermal nature of the observed fluctuation. The value of $\langle(\delta N_{\rm Q}^{(\rm net)})^2\rangle$ in this experiment is suppressed compared with the one in the equilibrated hadronic medium which has been calculated by lattice QCD simulations [13] and the hadron resonance gas (HRG) model [14]. Moreover, the dependence of $\langle(\delta N_{\rm Q}^{(\rm net)})^2\rangle$ on the size of the rapidity window, $\Delta\eta$, shows that the suppression of $\langle(\delta N_{\rm Q}^{(\rm net)})^2\rangle$ is more pronounced for larger $\Delta\eta$. These experimental results are reasonably explained if one attributes the suppression to the survival of fluctuations generated in the primordial deconfined medium [1,3,4,15]. In Refs. [3,4], it is argued that $\langle(\delta N_{\rm Q}^{(\rm net)})^2\rangle$ per unit rapidity normalized by an extensive conserved quantity, such as entropy, takes 2–3 times smaller value in the deconfined medium than the hadronic one. After hadronization, this small fluctuation approaches the equilibrated value in the hadronic medium. Since the variation of the local density of a conserved charge is achieved only through diffusion, the approach of the fluctuation to the equilibrated value becomes slower as the volume where the charge is counted becomes larger. $\langle(\delta N_{\rm Q}^{(\rm net)})^2\rangle$ thus takes a smaller value as

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$\Delta\eta$ becomes larger, which is consistent with the experimental result at ALICE.

On the other hand, the value of $\langle(\delta N_Q^{(\mathrm{net})})^2\rangle$ observed at RHIC energy is consistent with the one in the equilibrated hadronic medium [22,23]. The difference between RHIC and LHC energies indicates that the evolution of the fluctuation in the hot medium is qualitatively different between these energies.

In order to confirm the validity of the above explanation on the fluctuation measured at ALICE and clarify the origin of the qualitative difference between the experimental results at RHIC and LHC energies, it should be instructive to measure the $\Delta\eta$ dependences of various other fluctuation observables in addition to $\langle(\delta N_Q^{(\mathrm{net})})^2\rangle$ in these experiments. For example, the net-baryon number fluctuation, $\langle(\delta N_B^{(\mathrm{net})})^2\rangle$, is an experimentally-observable conserved charge fluctuation [16,17], although neutral baryons are not directly observable. Because the diffusion of the baryon number in the hadronic phase is slower than that of the electric charge due to the large mass of its carriers, baryons, if the origin of the suppression of $\langle(\delta N_Q^{(\mathrm{net})})^2\rangle$ at ALICE is indeed traced back to the smallness of the primordial fluctuations, $\langle(\delta N_B^{(\mathrm{net})})^2\rangle$ must have steeper suppression as a function of $\Delta\eta$ than $\langle(\delta N_Q^{(\mathrm{net})})^2\rangle$.

In event-by-event analyses, one can also measure higher order cumulants of conserved charges [10,11] such as the fourth-order ones $\langle(N_Q^{(\mathrm{net})})^4\rangle_c$ and $\langle(N_B^{(\mathrm{net})})^4\rangle_c$. The experimental analysis of these observables as functions of $\Delta\eta$ can obviously provide us more information on the time evolution of fluctuations in the hot medium. So far, however, systematic studies on the dynamical evolution of higher order cumulants in heavy ion collisions, whose results can be compared with their experimental observation, have not been carried out to the best of the authors' knowledge. The purpose of the present Letter is to make the first investigation on this issue using a simple but theoretically lucid model, and to make a prediction on the $\Delta\eta$ dependence of higher order cumulants in relativistic heavy ion collisions.

2. Stochastic formalism to describe diffusive systems

In relativistic heavy ion collisions with sufficiently large collision energy per nucleon, $\sqrt{s_{\mathrm{NN}}}$, the hot medium created at mid-rapidity has an approximate boost invariance. Useful coordinates to describe such a system are the coordinate-space rapidity η and proper time τ. We denote the net number of a conserved charge per unit coordinate-space rapidity as $n(\eta,\tau)$. In a class of experiments, event-by-event fluctuations of the charge at kinetic freezeout in a phase space corresponding to the rapidity window determined by the experiment are observed. The phase space approximately corresponds to a finite coordinate-space rapidity interval [1]. Assuming that the kinetic freezeout takes place at a certain proper time τ_{fo}, the experimentally-observed conserved charge number at mid-rapidity at RHIC and LHC is given by

$$Q(\tau) = \int_{-\Delta\eta/2}^{\Delta\eta/2} d\eta\, n(\eta,\tau) \tag{1}$$

at $\tau = \tau_{\mathrm{fo}}$ with the rapidity window to count the particle number $\Delta\eta$. In the following, we investigate the time evolution of higher order cumulants of $Q(\tau)$.

In a sufficiently large space–time scale where hydrodynamic equations at first order are applicable, the average of $n(\eta,\tau)$ follows the diffusion equation

$$\partial_\tau n(\eta,\tau) = D\partial_\eta^2 n(\eta,\tau), \tag{2}$$

where D is the diffusion constant in this coordinate system. In a boost-invariantly expanding system, D receives a factor τ^{-2} compared with the diffusion constant in Cartesian coordinate. In order to describe fluctuations around the solution of Eq. (2), one may employ a stochastic model, in which the time evolution of the deterministic part satisfies Eq. (2).

A choice of such stochastic models is the theory of hydrodynamic fluctuations [18,19], in which the hydrodynamic equations are promoted to Langevin equations with stochastic terms representing fast random forces arising from microscopic interactions. In the equation corresponding to the conservation law of a charge, Eq. (2), the derivative of the stochastic force, $\partial_\eta\xi(\eta,\tau)$, is added to the right-hand side of Eq. (2) [15]. The equation is referred to as *stochastic diffusion equation*. Up to Gaussian fluctuations, property of $\xi(\eta,\tau)$ is completely determined by the fluctuation–dissipation relation, which is obtained from the locality of $\xi(\eta,\tau)$ and large time behavior of $n(\eta,\tau)$ [18]. It is known that the stochastic equation determined in this way well describes Gaussian fluctuations in fluids [18]. However, extension of this formalism to treat higher order fluctuations is nontrivial. There is no unique generalization of the fluctuation–dissipation relation to higher orders, or no a priori justification of such extensions.

Concerning the difficulty in the description of non-Gaussian fluctuations, it is worthwhile to note a theorem on Markov process, which states that stochastic forces in a Langevin equation for Markov process are of Gaussian when the equation describes stochastic variables which are continuous and vary continuously[1] [20]. Since the standard theory of hydrodynamic fluctuations describes a Markov process and the hydrodynamic variables are continuous, the theorem demands that $\xi(\eta,\tau)$ be of Gaussian; to allow for nonzero higher order correlations of $\xi(\eta,\tau)$, one must relax at least one of the two conditions, i.e. Markovian and the continuity.

Without the higher order correlation of $\xi(\eta,\tau)$, all higher order cumulants of $Q(\tau)$ vanish in equilibrium unless D explicitly depends on n. Even in such a formalism, the relaxation of non-Gaussianity starting from a particular initial condition can be described. In the physics of fluctuations in relativistic heavy ion collisions, however, nonzero higher order cumulants in equilibrium play a crucial role. First, the experimental results obtained so far report nonzero higher order cumulants near the equilibrated values [10–12]. Second, higher order cumulants are expected to *increase* toward the equilibrated values in the hadronic medium [3,4]. To reproduce these features, the stochastic model must obviously have nonzero higher order cumulants in equilibrium.

In the present study, instead of directly extending the theory of hydrodynamic fluctuations, we investigate the time evolution of higher order cumulants starting from a microscopic model. In this exploratory analysis, as such a model we consider a simple one-dimensional system composed of Brownian particles. Instead of tracking the motion of each Brownian particle separately, however, we represent the system as follows (see, Fig. 1). First, the coordinate η is divided into discrete cells with an equal length a. Second, we consider a single species of particle for the moment, and denote the number of particles in each cell, labeled by an integer m, as n_m, and the probability that each cell contains n_m

[1] This condition needs a brief explanation. For simplicity, we here assume that the stochastic variable is one-dimensional, denoted by x and y. Let $P(x,t+\Delta t|y,t)$ be the conditional probability of x at time $t+\Delta t$ given the system was at y at time t. When the stochastic variable is continuous, it is shown that $P(x,t+\Delta t|y,t)$ satisfies the Lindeberg condition $\lim_{\Delta t\to 0}\frac{1}{\Delta t}\int_{|x-y|>\varepsilon}dx\,P(x,t+\Delta t|y,t)=0$, for arbitrary $\varepsilon>0$ [20]. This condition is used effectively in the proof of the Gaussianity. Note that, when the stochastic variables are discrete, the Lindeberg condition is obviously violated.