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Chiral pions in a magnetic background

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ABSTRACT

We investigate the modification of the pion self-energy at finite temperature due to its interaction with a low-density, isospin-symmetric nuclear medium embedded in a constant magnetic background. To one loop, for fixed temperature and density, we find that the pion effective mass increases with the magnetic field. For the π^-, interestingly, this happens solely due to the trivial Landau quantization shift $\sim |eB|$, since the real part of the self-energy is negative in this case. In a scenario in which other charged particle species are present and undergo an analogous trivial shift, the relevant behavior of the effective mass might be determined essentially by the real part of the self-energy. In this case, we find that the pion mass decreases by $\sim 10\%$ for a magnetic field $|eB| \sim m_\pi^2$, which favors pion condensation at high density and low temperatures.

1. Introduction

The behavior of hadronic matter in a medium under the influence of a strong external magnetic field can be very rich and subtle, and has been the subject of intense investigation in the last few years. In fact, in-medium strong interactions under extreme magnetic fields are of experimental relevance in heavy ion collisions and in astrophysics, exhibit a rich new phenomenology and are amenable to lattice simulations. (For comprehensive reviews, see Ref. [1].)

Even if every model calculation has predicted that large enough magnetic fields, typically of the order of a few times m_π^2, could bring remarkable new features to the thermodynamics of strong interactions, from shifting the chiral and the deconfinement crossover lines in the phase diagram [2–15] to transforming the vacuum into a superconducting medium via ρ-meson condensation [16,17], essentially all models fail to describe coherently the available lattice data [18–21]. The reasons for that are still unclear, although there are some indications that confinement plays a relevant role [15,22], which is not captured in the usual low-energy effective chiral models of QCD [23]. In any case, the situation calls for theoretical investigations in more controlled setups, with less freedom and parameters to adjust. This approach has proved to be fruitful in the large-N_c [22] and perturbative [24] limits of QCD: in the former, the behavior of the critical temperature for deconfinement was found to be in qualitative agreement with lattice data; in the latter, a trivial chiral limit for the two-loop contribution to the QCD pressure in a strong magnetic background was revealed.

Following this line of action, a natural extension is the study of hadronic matter in the complementary, low-energy sector, in the presence of a strong magnetic field, in a controlled setup. Thus, since we are interested in the low-density, low-temperature sector of the phase diagram of nuclear matter, we adopt the framework of chiral perturbation theory, which represents a powerful tool to study the low-energy regime of the pion–nucleon physics [25].

It is the purpose of this work to investigate some properties of isospin-symmetric nuclear matter in the limit of low density and temperature, embedded in a strong magnetic background. In particular, we study the modifications of the spectrum of the lowest energy degree of freedom, the pion, due to the interaction with nucleons and the constant magnetic field. More specifically, we compute the pion effective mass in the presence of a constant magnetic field to one loop. (Even if we do not address the phase diagram here, it should be mentioned that the inclusion of nucleons, and pion–nucleon interactions, proved to be necessary for a satisfactory description of the behavior of the deconfinement critical temperature as a function of the pion mass and isospin [26].) For this purpose, we consider fully relativistic chiral perturbation theory as a framework for our computation. This is needed to define consistently the fermion propagators in a magnetic background. At the same time, this work extends a previous treatment

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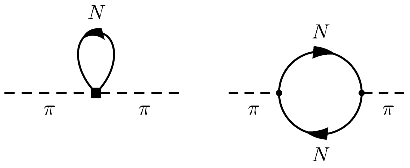

Fig. 1. Diagrams contributing to the lowest-order in-medium pion self-energy. The small dotted vertex corresponds to the one-pion exchange part of the Lagrangian in Eq. (2), while the squared one to the two-pion exchange in the Weinberg–Tomozawa term.

on the calculation of the fermion self-energy in relativistic chiral perturbation theory [27].

In-medium pion properties have been extensively investigated, both in finite systems, *i.e.* pionic atoms [28,29], and in infinite nuclear matter. In the latter, an interesting aspect of pion phenomenology is represented by pion condensation at high densities, introduced by Migdal [30], which is a consequence of the fact that at high density the electron chemical potential grows until it is favorable for a neutron on the top of the Fermi sea to turn into a proton and a (negatively charged) pion. On other hand, the interaction of the pion with the background matter can enhance its self-energy and consequently the pion condensation threshold density. This issue is still open and requires more investigation because of its implications in the context of compact stars phenomenology [31–33]. We shall see in the sequel that the in-medium modification of the (negatively charged) pion self-energy due to the presence of a strong magnetic background might lead to relevant phenomenological consequences.

The Letter is organized as follows. In Section 2, we consider the relativistic formulation of the theory, since in this framework it is possible to define the Green's function of the theory in the presence of a constant magnetic background in a consistent fashion. In Section 3, we compute the lowest-order pion self-energy for the three charge eigenstates in isospin-symmetric nuclear matter. In Section 4, we compute the in-medium effective mass of the pion and its dependence on the value of the applied magnetic field. Finally, in Section 5, we summarize our conclusions. We use natural units $\hbar = c = k_B = 1$. Four-vectors are denoted by capital letters, for instance $P^\mu = (p_0, \mathbf{p})$.

2. Reminder of the pion effective mass

The low-energy phenomenology of pions in nuclear matter is well described in terms of a chirally invariant pion–nucleon interaction Lagrangian, expanded in powers of the low-energy scale of the theory, *i.e.* the ratio of the pion momentum or mass over (4π times) the pion decay constant:

$$\mathcal{L}_{\pi N} = \mathcal{L}^{(1)}_{\pi N} + \mathcal{L}^{(2)}_{\pi N} + \cdots \tag{1}$$

where the leading order, $\mathcal{L}^{(1)}_{\pi N}$, reads [25]

$$\begin{aligned}\mathcal{L}^{(1)}_{\pi N} = -\bar{\Psi}\biggl[&\frac{g_A}{2f_\pi}\gamma^\mu\gamma_5\boldsymbol{\tau}\cdot\partial_\mu\boldsymbol{\pi}\\ &+\frac{1}{4f_\pi^2}\gamma^\mu\boldsymbol{\tau}\cdot(\boldsymbol{\pi}\times\partial_\mu\boldsymbol{\pi})\biggr]\Psi.\end{aligned} \tag{2}$$

Here $\boldsymbol{\tau}$ is the vector of Pauli matrices in isospin space, $\boldsymbol{\pi}$ is the isotriplet of pions, f_π the pion decay constant and g_A is the axial-vector coupling.

The diagrams contributing to the pion self-energy from the Lagrangian (2) are shown in Fig. 1. The former is obtained from the Weinberg–Tomozawa term, while the latter comes from the one-pion exchange Lagrangian. Due to the coupling of the charge to the vector potential, in the case of a constant magnetic background we need to compute separately those diagrams for different pion and nucleon charge eigenstates.

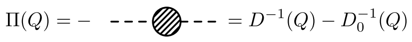

Fig. 2. Pion Schwinger–Dyson equation. Here D_0 is the free pion propagator and D is the full one. The diagram in the previous equation denotes the sum of all one-particle irreducible (1PI) diagrams. $Q^\mu = (\omega, \mathbf{q})$ is the pion four momentum.

Formally, the self-energy can be defined from the pion Schwinger–Dyson equation [34], displayed in Fig. 2.

Pionic modes of excitation in nuclear matter are obtained as solutions $\omega(\mathbf{q})$ of the following equation

$$\omega^2 - \mathbf{q}^2 - m_\pi^2 + \Pi(\omega, \mathbf{q}) = 0, \tag{3}$$

and in the limit of vanishing momenta this solution corresponds to the effective pion mass

$$m_\pi^{*2} = m_\pi^2 - \operatorname{Re}\Pi\bigl(m_\pi^*, \mathbf{q} = 0\bigr). \tag{4}$$

In absence of a magnetic background, it can be shown that the lowest-order (LO) contribution to the effective mass in (4) vanishes in isospin-symmetric nuclear matter [35].

In asymmetric nuclear matter, the LO self-energy of the (negatively charged) pion receives a contribution from the Weinberg–Tomozawa diagram, given by [36]

$$\Pi^{\mathrm{WT}}(\omega, \mathbf{q} = 0) = \frac{\omega}{2f_\pi^2}(\rho_p - \rho_n). \tag{5}$$

In the presence of a magnetic background, the pion charge eigenstates Eq. (4) have to be modified (due to the Landau level quantization) to

$$m_\pi^{*2} = m_\pi^2 - \operatorname{Re}\Pi\bigl(m_\pi^{*2}, \mathbf{q} = 0; \mathbf{B}\bigr) + (2n+1)|eB|, \tag{6}$$

where $\mathbf{B}$ is the magnetic field and n is the index of the Landau level.

In what follows we focus on the case of symmetric nuclear matter in the presence of a constant magnetic background. Thus, any deviation from zero of the LO pion self-energy will give a contribution to the effective pion mass in a magnetic background. Since we are dealing with dilute nuclear matter at low temperatures, we neglect the contribution of anti-nucleons. Moreover, we choose the x_3-axis to be parallel to the magnetic field and $|eB| = eB$, e being the proton electric charge. In order to simplify the calculation, we assume the regime of strong magnetic fields, in which one can apply the lowest-Landau-level (LLL) approximation to simplify the propagators. We neglect the effect of the anomalous magnetic moment of protons and neutrons. The calculation is carried out in the Landau gauge.

3. Pion self-energy in a constant magnetic field

For the negatively charged pion, the first diagram in Fig. 1 leads to the following contribution:

$$\Pi^{\mathrm{WT}}(Q) = \Pi_p^{\mathrm{WT}}(Q) - \Pi_n^{\mathrm{WT}}(Q), \tag{7}$$

where the first term is the proton loop contribution, which by using the Furry representation at finite temperature for the proton propagator [37] reads

$$\Pi_p^{\mathrm{WT}}(Q) = \frac{1}{f_\pi^2}\frac{|eB|}{4\pi^2}\int_{-\infty}^{+\infty} dp_3\, n_F(E_3 - \mu)\frac{\mathbf{p}_L\cdot\mathbf{q}_L}{2E_3}, \tag{8}$$