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Spin–spin interactions in massive gravity and higher derivative gravity theories

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ABSTRACT

We show that, in the weak field limit, at large separations, in sharp contrast to General Relativity (GR), all massive gravity theories predict distance-dependent spin alignments for spinning objects. For all separations GR requires anti-parallel spin orientations with spins pointing along the line joining the sources. Hence total spin is minimized in GR. On the other hand, while massive gravity at small separations ($m_g r \leqslant 1.62$) gives the same result as GR, for large separations ($m_g r > 1.62$) the spins become parallel to each other and perpendicular to the line joining the objects. Namely, the potential energy is minimized when the total spin is maximized in massive gravity for large separations. We also compute the spin–spin interactions in quadratic gravity theories and find that while at large separations GR result is intact, at small separations, spins become perpendicular to the line joining sources and anti-parallel to each other.

1. Introduction

Consider two widely separated spinning massive objects (for example two galaxies or galaxy clusters) that interact via gravity: What is the minimum energy configuration for their spin orientations, and how does the result depend on whether the graviton is massive or not? In this work we will compute the spin–spin interactions of point-like objects in massive gravity. We will show that introducing a small graviton mass gives the highly unexpected result of changing the spin orientations of sources from the one predicted in GR. Arguably, massive gravity is the most natural modification of GR that has implications in the overall dynamics – accelerated expansion – of the universe and hence a detailed study of gravitomagnetic effects such as the one done in this work is needed.

Before we give a detailed derivation of the results in the next section in D-dimensional spacetimes and higher curvature theories, let us summarize our findings here for the case of $D = 3 + 1$ for GR and massive gravity. Consider two localized spinning point-like sources described with the components of the energy-momentum tensor

$$T_{00} = m_a \delta^{(3)}(\vec{x} - \vec{x}_a),$$
$$T^i{}_0 = -\frac{1}{2} J_a^k \epsilon^{ikj} \partial_j \delta^{(3)}(\vec{x} - \vec{x}_a), \tag{1}$$

where $a = 1, 2$. Here m_a is the mass and $\vec{J}_a$ is the spin of the particle. Then, working in a flat background, from the tree-level diagram of one graviton exchange, we can calculate the potential energy as

$$U = -\frac{4\pi G}{t} \int d^4x\, d^4x'\, T^{\mu\nu}(x) G_{\mu\nu\alpha\beta}(x, x') T^{\alpha\beta}(x'), \tag{2}$$

where $G_{\mu\nu\alpha\beta}(x, x')$ is the Green's function of the theory at hand and t is a large time that will drop at the end. In GR this computation gives

$$U_{GR} = -\frac{G m_1 m_2}{r} - \frac{G}{r^3}\left[\vec{J}_1 \cdot \vec{J}_2 - 3 \vec{J}_1 \cdot \hat{r}\, \vec{J}_2 \cdot \hat{r}\right], \tag{3}$$

where $\vec{r} = r\hat{r}$ is the distance between the two sources. Spin–spin part can be attractive or repulsive depending on the spin orientations. Maximum value of $\vec{J}_1 \cdot \vec{J}_2 - 3 \vec{J}_1 \cdot \hat{r}\, \vec{J}_2 \cdot \hat{r}$, that is the minimum of the potential energy is achieved when $\vec{J}_1$ and $\vec{J}_2$ are anti-parallel and point along $\hat{r}$ as depicted in Fig. 1. That means in GR, for *any given* r, potential energy is minimized for anti-parallel spin orientations, if we neglect the tidal and orbital angular momentum effects. (The computation here is of course not a good approximation for close binary systems, such as two neutron stars *etc.*, but it is a valid approximation for two widely separated galaxies or galaxy clusters.) Let us give the results of the same computation

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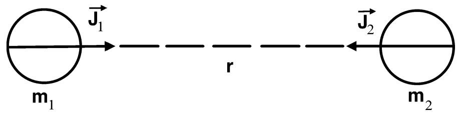

Fig. 1. Minimum energy configuration in GR, as long as weak field limit is applicable.

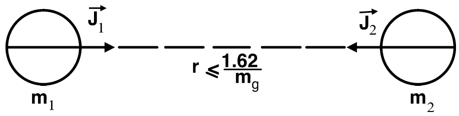

Fig. 2. Minimum energy configuration in massive gravity for $m_g r \leqslant 1.62$.

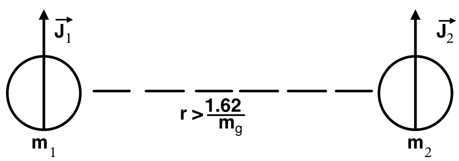

Fig. 3. In massive gravity, at large separations, the potential energy is minimized when the spins are perpendicular to the line joining the sources.

in massive gravity. At this point one might worry about which massive gravity to use. The crucial point is that in the weak field limit around flat space, any viable (non-linear, ghost-free) massive gravity theory reduces to the Fierz–Pauli (FP) theory that describes 5 degrees of freedom. Hence the following computation is a universal, weak field, large distance, prediction of *all* massive gravity theories built to describe 5 degrees of freedom around flat space. The Lagrangian density of the linear massive gravity is

$$\mathcal{L}_{\text{FP}} = \frac{1}{16\pi G}\left[R - \frac{m_g^2}{4}\left(h_{\mu\nu}^2 - h^2\right)\right] + \mathcal{L}_{\text{matter}}, \tag{4}$$

where m_g is the mass of the graviton, we found that at the lowest order the potential energy is

$$U_{\text{FP}} = -\frac{4}{3}Gm_1m_2\frac{e^{-m_g r}}{r} - \frac{Ge^{-m_g r}(1+m_g r+m_g^2r^2)}{r^3} \times \left[\vec{J}_1\cdot\vec{J}_2 - 3\vec{J}_1\cdot\hat{r}\vec{J}_2\cdot\hat{r}\frac{(1+m_g r+\frac{1}{3}m_g^2r^2)}{(1+m_g r+m_g^2r^2)}\right]. \tag{5}$$

It is clear that, in contrast to the GR result, in massive gravity depending on the distance between the sources, spin–spin part of the potential energy is minimized for different spin orientations determined by the maximization of the function (see Appendix A for details)

$$f(\theta,\varphi_1,\varphi_2) = \cos(\theta) - 3\frac{(1+x+\frac{1}{3}x^2)}{(1+x+x^2)}\cos(\varphi_1)\cos(\varphi_2), \tag{6}$$

where $x = m_g r$ and θ is the angle between the spins and φ_i is the angle between $\vec{J}_i$ and $\vec{r}$. Maximization of (6) yields: anti-parallel spins for $x \leqslant \frac{1+\sqrt{5}}{2} \approx 1.62$ as in the case of GR depicted in Fig. 2. On the other hand, for $x > \frac{1+\sqrt{5}}{2} \approx 1.62$, one gets parallel spins which are perpendicular to the line joining the sources as in Fig. 3.

The important conclusion one learns is that while in GR minimal potential energy is realized for minimum total spin at all separations, in massive gravity potential energy is minimized for maximum total spin for $m_g r > 1.62$.[1]

[1] We would like to thank A. Dane whose simulation of the spin–spin interaction led us to realize this point where spins suddenly change orientations. Note that the same point that is the "Golden Number" arises when one considers stable circular orbits in the Newtonian theory with a Yukawa potential. Namely, stable circular orbits exist for $x \leqslant \frac{1+\sqrt{5}}{2}$. We thank F. Öktem for this point.

2. Derivation of the results

To derive the above results and their D-dimensional generalizations in GR, massive gravity and quadratic gravity, it is somewhat more convenient to use the propagator found in [1] to represent (2). In order to avoid repeating the computations of all three theories let us consider the most general theory which includes these theories:

$$\begin{aligned} S = &\int d^Dx\sqrt{-g}\left\{\frac{1}{\kappa}R - \frac{2\Lambda_0}{\kappa} + \alpha R^2 + \beta R_{\mu\nu}^2 \right.\\ &\left. + \gamma\left(R_{\mu\nu\sigma\rho}^2 - 4R_{\mu\nu}^2 + R^2\right)\right\} \\ &+ \int d^Dx\sqrt{-g}\left\{-\frac{m_g^2}{4\kappa}\left(h_{\mu\nu}^2 - h^2\right) + \mathcal{L}_{\text{matter}}\right\}. \end{aligned} \tag{7}$$

In [1], we computed the scattering amplitude ($A = Ut$) corresponding to a graviton exchange in this theory and presented it with sufficient detail, hence we quote here the result:

$$\begin{aligned} 4A = {} & 2T'_{\mu\nu}\left\{(\beta\bar{\Box} + a)\left(\Delta_L^{(2)} - \frac{4\Lambda}{D-2}\right) + \frac{m_g^2}{\kappa}\right\}^{-1}T^{\mu\nu} \\ & + \frac{2}{D-1}T'\left\{(\beta\bar{\Box} + a)\left(\bar{\Box} + \frac{4\Lambda}{D-2}\right) - \frac{m_g^2}{\kappa}\right\}^{-1}T \\ & - \frac{4\Lambda}{(D-2)(D-1)^2}T'\left\{(\beta\bar{\Box} + a)\left(\bar{\Box} + \frac{4\Lambda}{D-2}\right) - \frac{m_g^2}{\kappa}\right\}^{-1} \\ & \times\left\{\bar{\Box} + \frac{2\Lambda D}{(D-2)(D-1)}\right\}^{-1}T \\ & + \frac{2}{(D-2)(D-1)}T'\left\{\frac{1}{\kappa} + 4\Lambda f - c\bar{\Box} - \frac{m_g^2}{2\kappa\Lambda}(D-1)\right\}^{-1} \\ & \times\left\{\bar{\Box} + \frac{2\Lambda D}{(D-2)(D-1)}\right\}^{-1}T, \end{aligned} \tag{8}$$

where we have dropped the integral signs not to clutter the notation and also to properly account all those theories in the corresponding limits, we have provisionally introduced an effective cosmological constant which is determined via the quadratic equation $\frac{\Lambda-\Lambda_0}{2\kappa} + f\Lambda^2 = 0$. The other parameters that appear above are defined as

$$f \equiv (D\alpha + \beta)\frac{(D-4)}{(D-2)^2} + \gamma\frac{(D-3)(D-4)}{(D-1)(D-2)}, \tag{9}$$

$$a \equiv \frac{1}{\kappa} + \frac{4\Lambda D}{D-2}\alpha + \frac{4\Lambda}{D-1}\beta + \frac{4\Lambda(D-3)(D-4)}{(D-1)(D-2)}\gamma, \tag{10}$$

$$c = \frac{4(D-1)\alpha + D\beta}{D-2}. \tag{11}$$

With all these parameters at hand, one covers all the three theories that we are interested in. For example the result for General Relativity follows from $m_g^2 = \alpha = \beta = \gamma = 0$ which yield $a = \frac{1}{\kappa}$ and $f = c = 0$. For flat backgrounds one has

$$4A = -2\kappa T'_{\mu\nu}\left(\partial^2\right)^{-1}T^{\mu\nu} + \frac{2\kappa}{D-2}T'\left(\partial^2\right)^{-1}T. \tag{12}$$

More explicitly the last equation is

$$\begin{aligned} 4A = {} & -2\kappa\int d^Dx\int d^Dx'\,T_{\mu\nu}(x')G(x,x')T^{\mu\nu}(x) \\ & + \frac{2\kappa}{(D-2)}\int d^Dx\int d^Dx'\,T(x')G(x,x')T(x), \end{aligned} \tag{13}$$