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Spinning gauged boson stars in anti-de Sitter spacetime

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ABSTRACT

We study axially symmetric solutions of the Einstein–Maxwell–Klein–Gordon equations describing spinning gauged boson stars in a $(3+1)$-dimensional asymptotically AdS spacetime. These smooth horizonless solutions possess an electric charge and a magnetic dipole moment, their angular momentum being proportional to the electric charge. A special class of solutions with a self-interacting scalar field, corresponding to static axially symmetric solitons with a nonzero magnetic dipole moment, is also investigated.

1. Introduction

Recently there has been a lot of interest in solutions of the general relativity with a negative cosmological constant Λ coupled to a Maxwell field and a charged scalar with mass M and gauge coupling constant q. Working in four spacetime dimensions, this system is usually described by the action

$$I=\int_{\mathcal{M}} d^4x\sqrt{-g}\bigg[\frac{1}{16\pi G}(R-2\Lambda)-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} -\frac{1}{2}g^{\mu\nu}\big(D_\mu\psi^* D_\nu\psi+D_\nu\psi^* D_\mu\psi\big)-U\big(|\psi|\big)\bigg], \quad (1.1)$$

where G is the gravitational constant, R is the Ricci scalar associated with the spacetime metric $g_{\mu\nu}$, $F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu$ is the U(1) field strength tensor and $D_\mu\psi=\partial_\mu\psi+iqA_\mu\psi$ is the covariant derivative. $U(|\psi|)$ denotes the potential of the scalar field ψ, whose mass is defined by $M^2=\frac{1}{2}\frac{\partial^2 U}{\partial|\psi|^2}|_{\psi=0}$.

Although (1.1) does not correspond to a consistent truncation of a more fundamental theory (unless ψ is vanishing), it can be viewed however, as a simple toy model for the charged scalar dynamics of systems that appear in concrete examples of the AdS/CFT correspondence. The interest in this system (sometimes called the gravitating Abelian Higgs model) has been boosted due to Gubser's observation [1] that the Einstein–Maxwell black holes can become unstable to forming scalar hair at low temperatures. This instability results in new branches of solutions with scalar hair, which are thermodynamically favored over the Reissner–Nordström-AdS black holes. The charged black branes which are asymptotically AdS in a Poincaré coordinate patch are of main interest, their physics being recently exploited to obtain a dual gravitational description of important phenomena in condensed matter physics (in particular superconductivity and phase transitions) in a three-dimensional flat spacetime $R_t\times R^2$. A discussion of these aspects can be found e.g. in [2], together with a large set of references.

However, another natural arena to study solutions of the model (1.1) is to consider instead a globally AdS background, with a line element $ds^2=-(1+\frac{r^2}{\ell^2})dt^2+\frac{dr^2}{1+\frac{r^2}{\ell^2}}+r^2(d\theta^2+\sin^2\theta\, d\varphi^2)$ (where t is the time coordinate and r, θ, φ are the spherical coordinates with the usual range and $\Lambda=-3/\ell^2$), the conformal boundary being a static Einstein universe $R_t\times S^2$. In this case, the simplest non-vacuum solutions of the model (1.1) are the boson stars with a vanishing gauge field, $F_{\mu\nu}=0$. These are smooth horizonless configurations representing gravitational bound states of a complex scalar field with a harmonic time dependence, which provide us with the simplest model of a relativistic star.[1] Such solutions have been extensively studied for $\Lambda=0$, i.e. an asymptotically flat spacetime background, starting with the early work of Kaup [3] and Ruffini and Bonazzola [4]. They have found interesting physical applications, being proposed as candidates for dark matter halos and as dark alternatives to astrophysical black hole candidates; also, they may help explain galaxy rotation curves – see the review work [5].

[1] One should remark that in contrast to ordinary stars or neutron stars, generically the boson stars do not display a sharp edge. In this case, the matter is not confined in a finite region of space and the boson stars possess only an *effective* radius [5] (see however, the compact boson stars in [6]).

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The boson starts with AdS asymptotics are believed to play an important role in holographic gauge theories through the AdS/CFT correspondence.[2] While the early studies restricted to spherically symmetric configurations [7,8] (see also [9,10]) recently there has been some progress on including the effects of rotation. Spinning AdS boson stars in $d = 4$ spacetime dimensions have been discussed in [11]. These stationary localized configurations possess a finite mass and angular momentum, their angular momentum being quantized, $J = nQ$ (with n an integer and Q the Noether charge), the energy density exhibiting a toroidal distribution. A particular set of higher-dimensional[3] spinning boson stars possessing equal angular momenta has been considered in [12,13], for $d = 2k + 1 \geqslant 5$ and a special multiplet scalar fields ansatz [14].

An interesting question to address in this context is the issue of rotating horizonless solutions within the full model (1.1), and, in particular, how an electric charge would affect their properties. For $\Lambda = 0$, this problem has been addressed in Ref. [16], which gave numerical evidence for the existence of spinning gauged boson stars in a Minkowski spacetime background. Similar to a Kerr–Newman black hole, these solutions possess a nonzero electric charge and a magnetic dipole moment. However, their pattern is rather similar to that of the ungauged boson stars discussed in [17–19]; in particular, one finds again a limited range for the allowed scalar field frequencies.

In this work we show that the spinning gauged boson stars in [16] can be generalized for an AdS background. In some sense, these globally regular configurations can be regarded as regularized Kerr–Newman-AdS solutions, the event horizon and the singularity disappearing due to the supplementary interaction with a complex scalar field. These gauged boson stars can also be viewed as axially symmetric generalizations of the spherically symmetric gauged solutions in [8,20–22]. Similar to that case, our results show that they exist also up to a maximal value of the gauge coupling constant. Their angular momentum is quantized, being proportional to the electric charge.

This Letter is organized as follows. In the next Section we formulate a numerical approach of the problem based on a specific ansatz. The numerical results are given in Section 3, where we exhibit the physical properties of the gauged spinning boson star solutions. We conclude in Section 4 with some further remarks.

2. The problem

2.1. The equations and the ansatz

The configurations investigated in this work are solutions of the coupled Einstein–Maxwell-scalar field equations

$$E_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} - 8\pi G T_{\mu\nu} = 0, \tag{2.2}$$

$$D_\mu D^\mu \psi = \frac{\partial U}{\partial |\psi|^2}\psi,$$
$$\nabla_\mu F^{\mu\nu} = iq\big[\big(D^\nu\psi^*\big)\psi - \psi^*\big(D^\nu\psi\big)\big] \equiv qj^\nu, \tag{2.3}$$

where $T_{\mu\nu}$ is the stress-energy tensor

$$T_{\mu\nu} = F_{\mu\alpha}F_{\nu\beta}g^{\alpha\beta} - \frac{1}{4}g_{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}$$
$$+ \big(D_\mu\psi^* D_\nu\psi + D_\nu\psi^* D_\mu\psi\big)$$
$$- g_{\mu\nu}\left[\frac{1}{2}g^{\alpha\beta}\big(D_\alpha\psi^* D_\beta\psi + D_\beta\psi^* D_\alpha\psi\big) + U\big(|\psi|\big)\right]. \tag{2.4}$$

This model is invariant under the local U(1) gauge transformation

$$\psi \to \psi e^{-iq\alpha}, \qquad A_\mu \to A_\mu + \partial_\mu\alpha, \tag{2.5}$$

with α a real function.

We are interested in stationary axially symmetric configurations, with a spacetime geometry admitting two Killing vectors ∂_t and ∂_φ, in a system of adapted coordinates. Then the most general line element is written as $ds^2 = G_{tt}(x)\,dt^2 + 2G_{t\varphi}(x)\,dt\,d\varphi + G_{\varphi\varphi}(x)\,d\varphi^2 + h_{ij}(x)\,dx^i\,dx^j$, with $x^i = (r, \theta)$. In the numerics, it is convenient to choose a metric gauge with $(1 + \frac{r^2}{\ell^2})h_{rr} = h_{\theta\theta}/r^2$ and $h_{r\theta} = 0$. This leads to a metric ansatz with four unknown functions, a convenient form being

$$ds^2 = -F_0(r,\theta)\left(1 + \frac{r^2}{\ell^2}\right)dt^2 + F_1(r,\theta)\left(\frac{dr^2}{1 + \frac{r^2}{\ell^2}} + r^2\,d\theta^2\right)$$
$$+ F_2(r,\theta)r^2\sin^2\theta\left(d\varphi - \frac{W(r,\theta)}{r}\,dt\right)^2, \tag{2.6}$$

with (F_i, W) (where $i = 0, 1, 2$) being smooth in r, θ.

For the scalar field, we adopt the stationary ansatz [17–19]:

$$\psi(t, r, \theta, \varphi) = \phi(r,\theta)e^{i(n\varphi - \omega t)}, \tag{2.7}$$

where $\phi(r,\theta)$ is a real function, and ω and n are real constants. Single-valuedness of the scalar field requires $\psi(\varphi) = \psi(2\pi + \varphi)$; thus the constant n must be an integer, i.e., $n = 0, \pm1, \pm2, \ldots$. In what follows, we shall take $n \geqslant 0$ and $\omega \geqslant 0$, without any loss of generality.

A consistent ansatz for the U(1) gauge field reads

$$\mathcal{A} = A_\mu\,dx^\mu = A_t(r,\theta)\,dt + A_\varphi(r,\theta)\sin\theta\left(d\varphi - \frac{W}{r}\,dt\right). \tag{2.8}$$

Substituting (2.6), (2.7), (2.8) in the field equations (2.2), (2.3) results[4] in a set of seven coupled non-linear PDEs of the form $\nabla^2\mathcal{F}_a = \mathcal{J}_a$ where $\mathcal{F}_a = (F_0, F_1, F_2, W; \phi; A_\varphi, A_t)$, $\mathcal{J}_a$ are 'source' terms depending on the functions $\mathcal{F}_a$ and their first derivatives, while ∇^2 is the Laplace operator associated with the auxiliary space $d\sigma^2 = dr^2/(1 + \frac{r^2}{\ell^2}) + r^2\,d\theta^2$.

Solutions of this model with a vanishing gauge field $A_t = A_\varphi = 0$ have been discussed in [11], generalizing for the AdS spacetime the asymptotically flat rotating boson stars in [17–19]. Note that, in contrast to that case, the (t,φ)-dependence of the scalar field ψ can now be gauged away by applying the local U(1) symmetry (2.5) with $\alpha = (n\varphi - \omega t)/q$. However, this would also change the gauge field as $A_t \to A_t - \omega/q$, $A_\varphi \to A_\varphi + n/q$, so that it would become singular in the $q \to 0$ limit. Therefore, in order to be able to consider this limit, we prefer to keep the (t,φ)-dependence in the scalar field ansatz and to fix the corresponding gauge freedom by setting $A_t = A_\varphi = 0$ at infinity.

We note also that the spherically symmetric limit is found for $n = 0$, in which case the functions F_0, F_1, F_2 and ϕ, A_t depend only on r, with $F_1 = F_2$ and $W = A_\varphi = 0$.

[2] However, note that the boson starts do not have natural counterparts in a Poincaré coordinate patch.

[3] For completeness, we mention that spinning boson stars with an AdS_3 background have been studied in [15]. However, these solutions have rather special properties.

[4] Note that the Einstein equations $E_\theta^r = 0$, $E_r^r - E_\theta^\theta = 0$ are not automatically satisfied, yielding two constraints. However, following [23], one can show that these constraints are satisfied as a consequence of the identities $E^\nu_{\mu;\nu} = 0$ plus the set of chosen boundary conditions. In practice, the constraint equations E_θ^r and $E_r^r - E_\theta^\theta$ are used to monitor the numerical accuracy of the solutions.