Physics Letters B 728 (2014) 380–385

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Nonperturbative infrared enhancement of non-Gaussian correlators in de Sitter space

J. Serreau

APC, AstroParticule et Cosmologie, Université Paris Diderot, CNRS/IN2P3, CEA/Irfu, Observatoire de Paris, Sorbonne Paris Cité, 10, rue Alice Domon et Léonie Duquet, 75205 Paris Cedex 13, France

ARTICLE INFO

Article history:
Received 7 September 2013
Received in revised form 17 November 2013
Accepted 1 December 2013
Available online 4 December 2013
Editor: L. Alvarez-Gaumé

Keywords:
Quantum field theory
De Sitter space
Infrared effects
$1/N$-expansion

ABSTRACT

We compute the four-point correlation function of a light $O(N)$ scalar field in de Sitter space in the large-N limit. For superhorizon momentum modes, infrared effects strongly enhance the size of loop contributions. We find that in the deep infrared limit, the latter are of the same order as the tree-level one. The tree-level momentum structure, characteristic of a contact term, gets renormalized by a factor of order unity. In addition loop contributions give rise to a new momentum structure, characteristic of an exchange diagram, corresponding to the exchange of an effective composite scalar degree of freedom.

1. Introduction

Quantum field theory in curved spaces is a topic of great interest with a long history [1]. The case of de Sitter space has attracted a lot of attention both because of its large degree of symmetry and because of its phenomenological relevance for the early inflationary era and for the current accelerated expansion of the universe. Specific phenomena such as gravitational redshift or particle creation imposes one to rethink much of what is known in Minkowski space, starting from the basic notions of particle and vacuum state, even for free fields [2]. At present, free gauge fields, such as the photon or the graviton, are still the subjects of debates [3].

Interacting fields can be studied by means of perturbation theory [4–9]. They pose practical and conceptual issues. An example is the trans-Planckian problem [10], i.e., the question of the effective decoupling between infrared and ultraviolet physics, which underlies the very concept of quantum field theory on de Sitter space. They also reveal novel specific features as compared to the flat space case. For instance, scalar fields of sufficiently large mass – in units of the expansion rate – are fundamentally unstable and can decay to themselves [11]. Light fields, which have no Minkowski analog, are also of great interest because of their phenomenological relevance, e.g., for inflationary cosmology. They exhibit strong semi-classical fluctuations for superhorizon modes and turn out to be essentially nonperturbative, even at weak coupling, due to large infrared effects [5,12]. In recent years, various methods inspired from flat space techniques have been developed to deal with infrared issues in de Sitter space. Results are still rather scarce but the nonperturbative aspects of light scalar fields are being unravelled [13–23].

A typical example is the phenomenon of dynamical mass generation: a field with vanishing tree-level mass develops an effective mass due to its self-interactions [12,24]. This lifts the flat tree-level potential and regulates possible infrared divergences. Incidentally, this results in nonanalytic coupling dependences of physical observables. A similar phenomenon has been demonstrated for an $O(N)$ scalar field in the large-N limit in the case where the tree-level potential shows spontaneous symmetry breaking [16]. Strong infrared fluctuations restore the symmetry, as anticipated in [25], and lead to nonperturbatively enhanced loop contributions [21].

Immediate phenomenological implications of nontrivial field interactions in the inflationary universe are possible quantum corrections to standard inflationary observables [6,26], or the possibility of non-Gaussian features of primordial density fluctuations [27]. As a first step towards the understanding of the actual cosmological (curvature) perturbations, it often proves useful to consider the simpler case of test scalar fields on a de Sitter background. In this context, it has been pointed out that infrared effects may lead to parametrically enhanced non-Gaussianities at tree-level both for

E-mail address: serreau@apc.univ-paris7.fr.

0370-2693 © 2013 The Author. Published by Elsevier B.V. Open access under CC BY license.
http://dx.doi.org/10.1016/j.physletb.2013.12.004

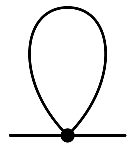

Fig. 1. The self-energy in the limit $N \to \infty$; see Eq. (10). The internal line corresponds to the propagator G itself, hence the nonperturbative character of this limit.

light (massless) fields [13] and for the case of a negative tree-level square mass [16].

The calculation of Ref. [13] is based on estimating the four-point correlator of an $O(N)$ scalar field by including loop corrections to the external legs propagators but keeping a simple tree-level interaction vertex. In this Letter, we extend on this and consider loop corrections to the four-point vertex as well. We show that the corresponding contributions to the four-point correlator are also amplified by infrared/secular effects and eventually contribute the same order in coupling as the tree-level contribution. We consider an $O(N)$ theory with quartic self-interactions in the large-N limit. This sums up infinitely many loop diagrams and enables us to capture genuine nonperturbative effects. Using the expressions for the field propagator and four-point vertex function recently obtained in Refs. [16,21], we compute the equal time four-point correlation function for superhorizon modes, which we obtain in closed analytical form. This allows us to analyze the loop contributions in detail and to show that a perturbative treatment fails for superhorizon momenta. We find that radiative corrections give an order one contribution to the tree-level contact term and give rise to an additional momentum structure, characteristic of an exchange diagram.

2. General setting

Consider the $O(N)$-symmetric scalar field theory with classical action (a sum over $a = 1, \dots, N$ is implied)

$$\mathcal{S}[\varphi] = \int_x \left\{ \frac{1}{2}\varphi_a\left(\Box - m_{\text{dS}}^2\right)\varphi_a - \frac{\lambda}{4!N}(\varphi_a\varphi_a)^2 \right\}, \tag{1}$$

with the invariant measure $\int_x \equiv \int d^{d+1}x\sqrt{-g}$, on the expanding Poincaré patch of a $d+1$-dimensional de Sitter space. In terms of comoving spatial coordinates $\mathbf{X}$ and conformal time $-\infty < \eta < 0$, the line-element reads (we choose the Hubble scale $H = 1$)

$$ds^2 = \eta^{-2}\left(-d\eta^2 + d\mathbf{X}\cdot d\mathbf{X}\right). \tag{2}$$

In Eq. (1), the mass term $m_{\text{dS}}^2 = m^2 + \xi\mathcal{R}$ includes a possible coupling to the Ricci scalar $\mathcal{R} = d(d+1)$ and $\Box$ is the appropriate Laplace operator.

In the following we consider the n-point correlation and vertex functions of the conformally rescaled fields $\phi_a(x) = (-\eta)^{\frac{1-d}{2}}\varphi_a(x)$ in the (interacting) Bunch Davies vacuum state. The latter are conveniently expressed in terms of time-ordered products of field operators along a closed contour in (conformal) time; see, e.g., [20]. For instance the two-point function $G_{ab}(x,x') = \langle T_\mathcal{C}\phi_a(x)\phi_b(x')\rangle$, where $T_\mathcal{C}$ denotes time-ordering along the contour $\mathcal{C}$, encodes both the statistical and spectral correlators $F_{ab}(x,x') = \frac{1}{2}\langle\{\phi_a(x),\phi_b(x')\}\rangle$ and $\rho_{ab}(x,x') = i\langle[\phi_a(x),\phi_b(x')]\rangle$:

$$G_{ab}\left(x,x'\right) = F_{ab}\left(x,x'\right) - \frac{i}{2}\text{sign}_\mathcal{C}\left(x^0 - x'^0\right)\rho_{ab}\left(x,x'\right), \tag{3}$$

where the sign function is to be understood on the contour $\mathcal{C}$. It was shown in [16] that, in the large-N limit, the system only admits $O(N)$-symmetric solutions. We thus have $\langle\phi_a\rangle = 0$ and $G_{ab} = \delta_{ab}G$.

In the symmetric phase, the four-point correlation and vertex functions $G^{(4)}$ and $\Gamma^{(4)}$ are related by

$$G^{(4)}_{ABCD} = G_{AA'}G_{BB'}G_{CC'}G_{DD'}i\Gamma^{(4)}_{A'B'C'D'} \tag{4}$$

where capital letter indices collectively denote space–time variables and $O(N)$ indices and an appropriate integral/summation over repeated indices is understood. Here, we are interested in computing the equal-time four-point correlator in comoving momentum space $G^{(4)}(\eta, \mathbf{K}_1, \dots, \mathbf{K}_4)$ for superhorizon physical momenta, $-K_i\eta \lesssim 1$, where $K_i = |\mathbf{K}_i|$. Both the propagator G and the vertex $\Gamma^{(4)}$ have been computed recently in the infrared regime in the limit $N \to \infty$ [16,21]. Let us briefly review the results relevant for our present purposes.

In comoving momentum space, the propagator has the free-field-like expression, for $\text{sign}_\mathcal{C}(\eta - \eta') = 1$,

$$G\left(\eta,\eta',K\right) = \frac{\pi}{4}\sqrt{\eta\eta'}H_\nu(-K\eta)H_\nu^*\left(-K\eta'\right) \tag{5}$$

where $H_\nu(z)$ is the Hankel function of the first kind and $\nu = \sqrt{d^2/4 - M^2}$. Here, M a self-consistent, dynamically generated mass, to be discussed shortly. In the cases of interest below, $M \ll 1$ and it is convenient to introduce the small parameter $\varepsilon = d/2 - \nu \approx M^2/d$. For superhorizon modes, the statistical and spectral two-point function read

$$F_{\text{IR}}\left(\eta,\eta',K\right) = \sqrt{\eta\eta'}\frac{F_\nu}{(K^2\eta\eta')^\nu}, \tag{6}$$

$$\rho_{\text{IR}}\left(\eta,\eta',K\right) = -\sqrt{\eta\eta'}\,\mathcal{P}_\nu^0\left(\ln\frac{\eta}{\eta'}\right), \tag{7}$$

where $F_\nu = [2^\nu\Gamma(\nu)]^2/4\pi$ and we introduced the function

$$\mathcal{P}_a^b(x) = \frac{\sinh(ax)}{a}e^{-b|x|}. \tag{8}$$

The self-consistent mass M satisfies the gap equation

$$M^2 = m_{\text{dS}}^2 + \sigma \tag{9}$$

where the constant σ is given by the tadpole diagram of Fig. 1. Retaining only the dominant infrared contribution in the loop (see [16] for a complete treatment), one gets

$$\sigma = \frac{\lambda}{6N}\left\langle\varphi^2(x)\right\rangle \approx \frac{\lambda_{\text{eff}}}{\varepsilon}, \tag{10}$$

where we introduced $\lambda_{\text{eff}} = \lambda F_\nu\Omega_d/12(2\pi)^d$ and $\Omega_d = 2\pi^{d/2}/\Gamma(d/2)$. Eq. (9) is solved as

$$M^2 = \frac{m_{\text{dS}}^2}{2} + \sqrt{\frac{(m_{\text{dS}}^2)^2}{4} + d\lambda_{\text{eff}}}. \tag{11}$$

This produces the known [6,12,14–16] result $M^2 \propto \sqrt{\lambda}$ in the case of light (massless) fields $m_{\text{dS}}^2 \ll \lambda$. The nonanalytic coupling dependence reflects the nonperturbative infrared character of the phenomenon of mass generation.

3. Four-point correlator

The four-point vertex function can be written as [21]

$$\Gamma^{(4)}_{abcd}(\eta_i,\mathbf{K}_i) = [\eta_1\cdots\eta_4]^{\frac{d-3}{4}} \times \left\{\delta_{ab}\delta_{cd}\,\delta_\mathcal{C}(\eta_1 - \eta_2)\delta_\mathcal{C}(\eta_3 - \eta_4)iD(\eta_1,\eta_3,K_{12}) + \text{perm.}\right\}, \tag{12}$$

where $\delta_\mathcal{C}(\eta - \eta')$ is a Dirac delta function on the contour, $K_{ij} = |\mathbf{K}_i + \mathbf{K}_j|$ and 'perm.' denotes the two permutations needed to