



Soft asymptotics with mass gap



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ABSTRACT

From the operator product expansion the gluon condensate controls a certain power law correction to the ultraviolet behavior of the gauge theory. This is reflected by the asymptotic behavior of the effective gluon mass function as determined by its Schwinger–Dyson equation. We show that the current state of the art determination of the gluon mass function by Binosi, Ibanez and Papavassiliou points to a vanishing gluon condensate. If this is correct then the vacuum energy also vanishes in massless QCD. This result can be interpreted as a statement about a softness in the ultraviolet behavior and the consistency of this behavior with a mass gap.

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Schwinger–Dyson (SD) equations provide a useful tool for the study of dynamical symmetry breaking by providing information about the momentum dependent dynamical mass functions. The existence of these mass functions signals a mass gap, a distortion of the theory in the infrared. The mass function also specifies power law corrections to the asymptotic ultraviolet behavior of the theory. These corrections in turn are related via the operator product expansion to condensates, vacuum expectation values of local operators. For example in massless QCD the dynamical quark mass function solution of the SD equation has an asymptotic behavior that points to a quark condensate appearing in the operator product expansion of two quark fields. The condensate merely encodes a particular effect that the mass gap has on the ultraviolet behavior of the theory.

In contrast to the quark condensate the gluon condensate does not break a symmetry of the theory, and the result is that the perturbative contribution is sensitive to any dimensionful regulator (a UV cutoff). Such an additional explicit breaking of scale invariance can be avoided with a scale invariant regulator, e.g. dimensional regularization, and in this case the perturbative contribution vanishes at any finite order.¹ But even with the choice of a scale invariant regulator, a resummation of a class of diagrams via the renormalization group leads to the infrared Landau pole at a scale Λ defined by

$$\ln\left(\frac{\Lambda}{\mu}\right) = \int_{g(\mu)}^{\infty} \frac{dx}{\beta(x)}. \quad (1)$$

This dimensional transmutation is taken as a signal of a non-vanishing gluon condensate of order Λ^4 . It is also argued that the gluon condensate is needed to cancel or correct particular ambiguities in perturbation theory (renormalons etc.) that are also related to the Landau pole. All this assumes that the Landau pole is more than just an artifact of resummed perturbation theory. Indeed a variety of approaches [1–10] indicate that the Landau pole does not survive nonperturbative effects that cause the coupling strength to saturate at a finite value in the infrared. Then the Λ determined by (1) vanishes. Lattice studies [11–16] have verified the associated damping of gluonic fluctuations in the infrared as described by an effective gluon mass function. This is the view we adopt here, in which case a different approach to the gluon condensate is needed.

In the same way as for the quark condensate, we may view the gluon condensate as just encoding a particular correction to the ultraviolet behavior due to the presence of the mass gap. We compare the operator product expansions for the quark and gluon mass functions ($\Sigma(p^2)$ and $m^2(p^2)$ respectively) [17,18].

$$\lim_{p^2 \rightarrow \infty} \Sigma(p^2) = c_1(p/\mu)m_\psi(\mu) + \frac{c_2(p/\mu)\langle\bar{\psi}\psi\rangle_\mu}{p^2} + \dots, \quad (2)$$

$$\lim_{\substack{p^2 \rightarrow \infty \\ m_\psi \rightarrow 0}} p^2 m^2(p^2) = a_1(p/\mu)\langle G_{\alpha\beta}G^{\alpha\beta}\rangle_\mu + \frac{a_2(p/\mu)\langle\bar{\psi}\psi\rangle_\mu^2}{p^2} + \dots. \quad (3)$$

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¹ A related example is the vacuum energy in a free massless field theory, which vanishes by Lorentz and scale invariance. Spurious contributions arise unless a scale invariant regulator is chosen. In the same way we avoid spurious contributions to the gluon condensate.

Shown in these expansions are the leading gauge invariant terms in the asymptotic behavior of the mass functions. We comment on possible gauge dependent contributions below. The two condensates are purely nonperturbative and each has a renormalization scale dependence. We see the sense in which the gluon condensate is the analog of m_ψ rather than $\langle\bar{\psi}\psi\rangle$, since m_ψ and the gluon condensate govern the leading term in the respective OPE. Once the bare quark mass is set to zero then $m_\psi(\mu) = 0$ remains consistent with the SD result for $\Sigma(p^2)$. We note that the quark condensate that is generated contributes only to the subleading terms in both expansions.

The similarity of the role of the leading terms of the OPEs also shows up in the trace of the energy-momentum tensor

$$\Theta_\alpha^\alpha = \frac{\beta}{2g} G_{\alpha\beta} G^{\alpha\beta} + m_\psi (1 + \gamma_m) \bar{\psi}\psi. \quad (4)$$

This operator statement gives a relation between the vacuum expectation values $\langle\Theta_\alpha^\alpha\rangle$, $\langle G_{\alpha\beta} G^{\alpha\beta}\rangle_\mu$ and $\langle\bar{\psi}\psi\rangle_\mu$, all of which are taken to vanish in perturbation theory. $\langle\Theta_\alpha^\alpha\rangle$ is a physical quantity independent of μ since it is four times the vacuum energy by Lorentz invariance. The μ dependence of the other two condensates must then be cancelled by the μ dependence of their coefficients in (4). The point is that for $\langle\Theta_\alpha^\alpha\rangle$ to be non-vanishing one or both of the leading terms in the OPEs (2) and (3) (that is $m_\psi(\mu)$ and/or $\langle G_{\alpha\beta} G^{\alpha\beta}\rangle_\mu$) needs to be present.

This property of the OPEs is a statement about the asymptotic behavior of the fundamental fields and so the question of whether vacuum energy vanishes $\langle\Theta_\alpha^\alpha\rangle = 0$ becomes a question of whether a certain ultraviolet boundary condition is satisfied by the theory. Thus for massless QCD ($m_\psi = 0$) it becomes a fundamental question as to whether the leading term of the gluon mass OPE is or is not generated by nonperturbative effects that necessarily go beyond chiral symmetry breaking. This highlights the importance of the gluon mass SD equation, which can in principle be used to determine the asymptotic behavior of the gluon mass function. In other words the SD approach could tell us whether vanishing vacuum energy is compatible with a mass gap.

The SD approach provides a nonperturbative framework (along with the lattice) to define the propagators of the fundamental fields at all momenta. The behaviors of gluon and ghost propagators that are emerging are deepening our understanding of confinement and the mass gap. We note though that any SD analysis involves a truncation of the complete SD equations and this introduces uncertainties, especially in the precise shape of the mass functions at low momentum. For our purposes we can have more confidence in the SD results for the gross features of the asymptotic behavior of the gluon mass function. For example the fact that the asymptotic behavior of the quark mass function in massless QCD is consistent with $m_\psi(\mu) = 0$ in (2) is a robust result of the SD analysis.

Significant progress towards obtaining a more accurate SD equation for the gluon mass has been made [19]. The full SD kernel, which has both one and two loop parts in terms of dressed quantities, has been reduced to a manageable form with the help of the pinch technique [20] and the background field method. Ward identities are maintained to reflect the fact that the gluon mass function should not explicitly break gauge symmetries. This is able to sufficiently specify modified vertices $\Gamma \rightarrow \Gamma' = \Gamma_m + V$ involving “pole vertices” V . The whole analysis takes place in Landau gauge.

The following Euclidean space integral equation for the gluon mass function is obtained [19].

$$m^2(q^2) = -\frac{4\pi\alpha_s C_A}{1 + G(q^2)} \frac{1}{q^2} \int \frac{d^4k}{(2\pi)^4} m^2(k^2)$$

$$\times \Delta_\rho^\mu(k) \Delta^{\nu\rho}(k+q) \mathcal{K}_{\mu\nu}(k, q), \quad (5)$$

$$\mathcal{K}_{\mu\nu}(k, q) = [(k+q)^2 - k^2] \{1 - [Y(k+q) + Y(k)]\} g_{\mu\nu} - [Y(k+q) - Y(k)] (q^2 g_{\mu\nu} - 2q_\nu q_\mu). \quad (6)$$

The gluon propagator is

$$\Delta_{\mu\nu}^{ab}(q) = \delta^{ab} \Delta_{\mu\nu}(q) = \delta^{ab} (g_{\mu\nu} - q_\mu q_\nu / q^2) \Delta(q^2). \quad (7)$$

The factor $1/(1 + G(q^2))$ is identified with $q^2 D(q^2)$ where $D(q^2)$ is the ghost propagator, apparently to good approximation. The quantity $Y(k)$ is a one loop sub-diagram in the two loop contribution to the kernel. If evaluated using tree level propagators and vertices it is [19]

$$Y(k^2) = -\frac{\alpha_s C_A}{4\pi} \frac{15}{16} \log \frac{k^2}{\mu^2}. \quad (8)$$

This introduces a renormalization scale dependence in the SD equation.

The propagators $\Delta(q^2)$ and $D(q^2)$ are obtained from a fit to lattice data. Since the lattice analysis uses a renormalization scale of 4.3 GeV, this is the choice adopted for μ [19]. The result is an integral equation which is linear in $m^2(q^2)$. The latter is obtained numerically and normalized to agree with the lattice gluon propagator at $q^2 = 0$.

The authors in [21] extend these results to the unquenched case by incorporating the quark loop contribution, where the quark propagator used is obtained from the quark SD equation. They start with the gluon mass solution in the quenched case, where lattice results for the quenched propagators are used in (6), and then by incorporating the quark effects via an iterative procedure they obtain a prediction for the modified gluon propagator in the $n_f = 2$ case. The result agrees very well with the lattice $n_f = 2$ result [16] which simulates 2 light dynamical quarks, and so this is an apparent success of their approach. For our purposes the main point is that (6) provides a determination of the unquenched gluon mass function when unquenched lattice results for the propagators are used. Note that the SD equation as derived, being homogeneous in $m^2(q^2)$, is blind to operator mixing between $G_{\alpha\beta} G^{\alpha\beta}$ and $m_\psi \bar{\psi}\psi$ and so its results can only apply to massless QCD, $m_\psi = 0$.

Given that our interest is in the asymptotic behavior of $m^2(q^2)$ we should ensure that the analysis reflects the known asymptotic behavior of QCD as much as possible. In particular since the propagator functions $\Delta(q^2)$ and $D(q^2)$ are input into the SD equation, it is simple to introduce their correct asymptotic behavior. From the renormalization group this is

$$\Delta(q^2) \rightarrow \ln(q^2)^\gamma / q^2, \quad D(q^2) \rightarrow \ln(q^2)^\delta / q^2 \quad (9)$$

with $\gamma = -(13C_A - 4n_f)/(22C_A - 4n_f)$ and $\delta = -9C_A/(44C_A - 8n_f)$ in Landau gauge. $\gamma = -31/58$ and $\delta = -27/116$ for $n_f = 2$ and $C_A = 3$ for $SU(3)$.

We shall implement this asymptotic behavior while fitting the propagator functions to the $n_f = 2$ lattice results, which exist for a range of momenta up to $q^2 \sim \mu^2$. In particular we fit $q^2 \Delta(q^2)$ to the SDE curve on the first plot in Fig. (9) in [21] and $q^2 D(q^2)$ to the green curve on the first plot in Fig. (4) in [16]. We can obtain good fits via the following simple fitting functions, which are only meant to extend down to $\sim 10^{-3}$ GeV².

$$\Delta(q^2)^{-1} = m_0^2 + q^2 [a + b \ln(q^2 + c)]^{-\gamma}, \quad (10)$$

$$D(q^2)^{-1} = q^2 [d + e \ln(q^2 + f)]^{-\delta}, \quad (11)$$

m_0 is set to the $q^2 = 0$ lattice value $m_0 = 0.413$ GeV and

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