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The Logotropic Dark Fluid as a unification of dark matter and dark energy

Pierre-Henri Chavanis

Laboratoire de Physique Théorique, Université Paul Sabatier, 118 route de Narbonne, 31062 Toulouse, France

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ABSTRACT

We propose a heuristic unification of dark matter and dark energy in terms of a single "dark fluid" with a logotropic equation of state $P = A\ln(\rho/\rho_P)$, where ρ is the rest-mass density, $\rho_P = 5.16 \times 10^{99}\ \mathrm{g\,m^{-3}}$ is the Planck density, and A is the logotropic temperature. The energy density ϵ is the sum of a rest-mass energy term $\rho c^2 \propto a^{-3}$ mimicking dark matter and an internal energy term $u(\rho) = -P(\rho) - A = 3A\ln a + C$ mimicking dark energy (a is the scale factor). The logotropic temperature is approximately given by $A \simeq \rho_\Lambda c^2/\ln(\rho_P/\rho_\Lambda) \simeq \rho_\Lambda c^2/[123\ln(10)]$, where $\rho_\Lambda = 6.72 \times 10^{-24}\ \mathrm{g\,m^{-3}}$ is the cosmological density and 123 is the famous number appearing in the ratio $\rho_P/\rho_\Lambda \sim 10^{123}$ between the Planck density and the cosmological density. More precisely, we obtain $A = 2.13 \times 10^{-9}\ \mathrm{g\,m^{-1}\,s^{-2}}$ that we interpret as a fundamental constant. At the cosmological scale, our model fulfills the same observational constraints as the ΛCDM model (they will differ in about 25 Gyrs when the logotropic universe becomes phantom). However, the logotropic dark fluid has a nonzero speed of sound and a nonzero Jeans length which, at the beginning of the matter era, is about $\lambda_J = 40.4$ pc, in agreement with the minimum size of the dark matter halos observed in the universe. The existence of a nonzero Jeans length may solve the missing satellite problem. At the galactic scale, the logotropic pressure balances the gravitational attraction, providing halo cores instead of cusps. This may solve the cusp problem. The logotropic equation of state generates a universal rotation curve that agrees with the empirical Burkert profile of dark matter halos up to the halo radius. In addition, it implies that all the dark matter halos have the same surface density $\Sigma_0 = \rho_0 r_h = 141\ M_\odot/\mathrm{pc}^2$ and that the mass of dwarf galaxies enclosed within a sphere of fixed radius $r_u = 300$ pc has the same value $M_{300} = 1.93 \times 10^7\ M_\odot$, in remarkable agreement with the observations [Donato et al. [10], Strigari et al. [13]]. It also implies the Tully–Fisher relation $M_b/v_h^4 = 44\ M_\odot\,\mathrm{km^{-4}\,s^4}$. We stress that our model has no free parameter.

1. Introduction

The nature of dark matter (DM) and dark energy (DE) is still unknown and remains one of the greatest mysteries of modern cosmology. DM has been introduced in astrophysics to account for the missing mass of the galaxies inferred from the virial theorem [1] and to explain their flat rotation curves [2]. DE has been introduced in cosmology to account for the present acceleration of the expansion of the universe [3]. In the standard cold dark matter (ΛCDM) model, DM is represented by a pressureless fluid and DE is ascribed to the cosmological constant Λ introduced by Einstein [4]. The ΛCDM model works remarkably well at the cosmological scale and is consistent with ever improving measurements of the cosmic microwave background (CMB) from WMAP and Planck missions [5,6]. However, it encounters serious problems at the galactic scale. In particular, it predicts that DM halos should be cuspy [7] while observations reveal that they have a flat core [8]. On the other hand, the ΛCDM model predicts an over-abundance of small-scale structures (subhalos/satellites), much more than what is observed around the Milky Way [9]. These problems are referred to as the "cusp problem" and "missing satellite problem". The expression "small-scale crisis of CDM" has been coined.

There are also unexplained important observational results. For example, it is an empirical fact that the surface density of galaxies has the same value $\Sigma_0 = \rho_0 r_h = 141^{+83}_{-52}\ M_\odot/\mathrm{pc}^2$ even if their sizes and masses vary by several orders of magnitude (up to 14 orders of magnitude in luminosity) [10]. On the other hand, it is known that the asymptotic circular velocity of the galaxies is related to

E-mail address: chavanis@irsamc.ups-tlse.fr.

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their baryonic mass by the Tully–Fisher (TF) relation $M_b/v_h^4 = 47 \pm 6\ M_\odot\,\mathrm{km}^{-4}\,\mathrm{s}^4$ [11,12]. Finally, Strigari et al. [13] have shown that all dwarf spheroidal galaxies (dSphs) of the Milky Way have the same total DM mass contained within a radius of $r_u = 300$ pc. From the observations, they obtained $\log(M_{300}/M_\odot) = 7.0^{+0.3}_{-0.4}$. To our knowledge, there is no theoretical explanation of these observational results.

The small scale problems of the ΛCDM model are related to the assumption that DM is pressureless. This assumption is valid if DM is made of weakly interacting massive particles (WIMPs) with a mass in the GeV–TeV range. These particles freeze out from thermal equilibrium in the early universe and, as a consequence of this decoupling, cool off rapidly as the universe expands. In order to solve the small-scale crisis of CDM, some authors have developed alternative models of DM. For example, it has been proposed that DM halos are made of fermions (such as sterile neutrinos) with a mass in the keV range [14,15], or bosons (such as axions) in the form of Bose–Einstein condensates (BECs) with a mass ranging from 10^{-2} eV to 10^{-22} eV depending whether the bosons interact or not [16,17]. In these models, the quantum pressure prevents gravitational collapse and leads to cores instead of cusps.[1] These models sometimes provide a good fit of the rotation curves of galaxies but they do not explain the universality (and the values) of Σ_0, M_b/v_h^4, and M_{300}.

On the other hand, at the cosmological scale, despite its success at explaining many observations, the ΛCDM model has to face two theoretical problems. The first one is the cosmic coincidence problem, namely why the ratio of DE and DM is of order unity today if they are two different entities [19]. The second one is the cosmological constant problem [20]. The cosmological constant Λ is equivalent to a constant energy density $\epsilon_\Lambda = \rho_\Lambda c^2 = \Lambda c^2/8\pi G$ associated with an equation of state $P = -\epsilon$ involving a negative pressure. Some authors [21] have proposed to interpret the cosmological constant in terms of the vacuum energy. Cosmological observations lead to the value $\rho_\Lambda = \Lambda/8\pi G = 6.72 \times 10^{-24}\ \mathrm{g\,m^{-3}}$ of the cosmological density (DE). However, particle physics and quantum field theory predict that the vacuum energy should be of the order of the Planck density $\rho_P = c^5/\hbar G^2 = 5.16 \times 10^{99}\ \mathrm{g\,m^{-3}}$. The ratio between the Planck density ρ_P and the cosmological density ρ_Λ is

$$\frac{\rho_P}{\rho_\Lambda} \sim 10^{123}, \tag{1}$$

so these quantities differ by 123 orders of magnitude! This is the origin of the cosmological constant problem.[2] To circumvent this problem, some authors have proposed to abandon the cosmological constant Λ and to explain the acceleration of the universe in terms of a dark energy with a time-varying density associated with a scalar field called "quintessence" [22]. As an alternative to quintessence, Kamenshchik et al. [23] have proposed a heuristic unification of DM and DE in terms of an exotic fluid with an equation of state $P = -A/\epsilon$ called the Chaplygin gas. This equation of state provides a model of universe that behaves as a pressureless fluid (DM) at early times, and as a fluid with a constant energy density (DE) at late times, yielding an exponential acceleration similar to the effect of the cosmological constant. However, in the intermediate regime of interest, this model does not give a good agreement with the observations [24] so that various generalizations of the Chaplygin gas model have been considered. In this Letter, we propose a new unification of DM and DE based on a logotropic equation of state [25]. This model is consistent with cosmological (large scales) and astrophysical (small scales) observations. In particular, it predicts the correct values of Σ_0, M_b/v_h^4, and M_{300} with a remarkable accuracy, and without free parameter.

[1] In the context of the classical CDM model, dark matter cusps may be erased by baryonic feedback [18].

[2] Actually, the vacuum energy that is of the order of the cut-off in the quantum field theory ranges from the TeV scale (for SUSY, large extra dimensions) to the Planck scale. This reduces the discrepancy from 123 to ∼60 orders of magnitude.

2. Logotropic cosmology

2.1. The logotropic dark fluid

The Friedmann equations for a flat universe without cosmological constant are [26]:

$$\frac{d\epsilon}{dt} + 3\frac{\dot a}{a}(\epsilon + P) = 0, \quad H^2 = \left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G}{3c^2}\epsilon, \tag{2}$$

where $\epsilon(t)$ is the energy density, $P(t)$ is the pressure, $a(t)$ is the scale factor, and $H = \dot a/a$ is the Hubble parameter.

For a relativistic fluid at $T = 0$, or for an adiabatic evolution (which is the case for a perfect fluid), the first law of thermodynamics reduces to [26]:

$$d\epsilon = \frac{P + \epsilon}{\rho} d\rho, \tag{3}$$

where ρ is the rest-mass density. Combined with the equation of continuity (2), we get

$$\frac{d\rho}{dt} + 3\frac{\dot a}{a}\rho = 0 \Rightarrow \rho = \frac{\rho_0}{a^3}, \tag{4}$$

where ρ_0 is the present value of the rest-mass density, and the present value of the scale factor is taken to be $a_0 = 1$. This equation, which expresses the conservation of the rest-mass, is valid for an arbitrary equation of state.

For an equation of state specified under the form $P = P(\rho)$, Eq. (3) can be integrated to obtain the relation between the energy density ϵ and the rest-mass density. We obtain

$$\epsilon = \rho c^2 + \rho \int^\rho \frac{P(\rho')}{\rho'^2} d\rho' = \rho c^2 + u(\rho), \tag{5}$$

where the constant of integration is set equal to zero. We note that $u(\rho)$ can be interpreted as an internal energy density. Therefore, the energy density ϵ is the sum of the rest-mass energy ρc^2 and the internal energy $u(\rho)$. The rest-mass energy is positive while the internal energy can be positive or negative. Of course, the total energy $\epsilon = \rho c^2 + u(\rho)$ is always positive.

We assume that the universe is filled with a single dark fluid (DF) described by the logotropic equation of state

$$P = A \ln\left(\frac{\rho}{\rho_P}\right). \tag{6}$$

It will be called the Logotropic Dark Fluid (LDF). A priori, we have two unknown parameters in our model: a reference energy density A (logotropic temperature) and a reference mass density ρ_P. Using Eqs. (5) and (6), the relation between the energy density and the rest-mass density is

$$\epsilon = \rho c^2 - A \ln\left(\frac{\rho}{\rho_P}\right) - A = \rho c^2 + u(\rho). \tag{7}$$

The energy density is the sum of two terms: a rest-mass energy term $\rho c^2 \propto a^{-3}$ that mimics DM and an internal energy term $u(\rho) = -P(\rho) - A = 3A \ln a - A \ln(\rho_0/\rho_P) - A$ that mimics DE. This decomposition leads to a natural, and physical, unification of DM and DE and elucidates their mysterious nature.