



# Relativistic soliton collisions of axion type dark matter



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## ABSTRACT

Axion-like scalar fields and the Lane–Emden (LE) truncation of their periodic potential are analyzed as a toy model of dark matter halos. Then, collisions of the well-known kinks in (1 + 1) spacetime dimensions can be mapped to those of localized lumps of the LE equation. Here, we generalize this mapping to (2 + 1)D or even (3 + 1)D and discuss a challenging intrinsic inelastic effect during relativistic soliton collisions.

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## 1. Introduction

Head-on collisions of massive galaxy clusters, like those occurring in the so-called *bullet cluster* [8] are a challenge for the cold dark matter (CDM) paradigm, inasmuch as either the individual galaxies or the hot X-ray emitting interstellar gas are only partially anchored to the DM lumps.

Here, we are going to probe axion-like particles [35–37] as dark matter candidates. In particular, we continue our investigations [4, 5] whether or not two dark matter halos pulling towards one another can be modeled via soliton-type collisions, but now in a more realistic setting of (2 + 1) or even 4 spacetime dimensions.

Since solitons or lumps appear to be rather stable entities, they behave effectively like colliding particles, i.e. after leaving the interaction region where they may deform due to a temporally inelastic mechanism studied here in a relativistic setting, they ultimately return to their original shapes and velocities.

Recently, the previous proposal of Kolb and Tkachev [16,17] that DM may be composed of a gas of ‘axion mini clusters’ or mini axion stars has been adopted and developed further [21,22, 2]. Moreover, for such Bose–Einstein (BE) type condensates one needs some self-interaction and, in view of the self-similarity of solitons, one would end up with much larger configurations resembling lump-type halos to be considered here.

## 2. Nonlinear Klein–Gordon equation

As one of the most representative relativistic models, the Klein–Gordon equation has found applications in many areas: Besides the nonlinear version that results in the known  $\phi^4$  model and higher-order field theories, it is used to describe successive phase transitions on realistic systems. Such processes are known to occur in ferroelastic and ferroelectric crystals and in meson physics. Some of these models present different kind of solitons; topological and nontopological, kinks, lump and breather type.

In the Klein–Gordon case, we depart from the wave operator

$$\square := \nabla \cdot \nabla - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \quad (1)$$

of Jean-Baptiste le Rond d'Alembert, still being linear in its derivatives. Thus it is invariant under the general Lorentz transformations (boosts)

$$ct \rightarrow \gamma(ct - \vec{v} \cdot \vec{x}/c) \quad (2)$$

$$\vec{x} \rightarrow \vec{x} + \frac{\gamma - 1}{v^2} (\vec{v} \cdot \vec{x}) \vec{v} - \gamma \vec{v}t, \quad (3)$$

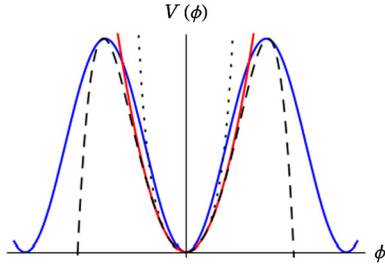
where  $\vec{x} := (x, y, z)$  is the radius vector of an event and

$$\gamma = 1/\sqrt{1 - \vec{v} \cdot \vec{v}/c^2} \quad (4)$$

the Lorentz factor, cf. [29].

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**Fig. 1.** (Color online.) Nonlinear potentials: sine-Gordon (blue),  $\phi^2$  (red),  $\phi^4$  (dotted) and  $\phi^6$  (dashed).

The challenge comes in the election of the potential  $V = V(\phi)$  in the nonlinear Klein–Gordon (NLKG) equation

$$\square\phi = \frac{\partial V(\phi)}{\partial\phi}. \quad (5)$$

For a start, let us consider another well known model, the sine-Gordon equation: The exact and complete potential has received many applications, and different truncations of (7) may approximate rather well other models: The first term of the Taylor series corresponds to the mass in the original Klein–Gordon equation; the next one would give rise to the famous  $\phi^4$ -theory. Combining the first and third term can be reckoned as the modified  $\phi^6$ -theory of Lane–Emden, and so on [14]. By plotting some of the potentials we obtained an approximated view of the field-theoretical ‘vacuum’ at the center, see Fig. 1.

In particular, we are interested in the LE form of  $\phi^6$ -theory, a model which had been use before in astrophysics and has the mathematical virtue that some analytic expressions are available for 2 till 4 spacetime dimensions.

So far, the study of such solitons has had many advances in 1 + 1 dimensional systems, but higher dimensions remain in many cases unsolvable. One of the hardest problems arise due to the lack of integrability, which, at times, is fully or partially lost.

Since the DM distributions in the bullet cluster appear not to be affected during merging, we postulate an axion-like scalar component of galaxies and clusters and analyze its solitary wave behavior, or as in our previous 2D toy model [4], its Lane–Emden (LE) truncation [23]

$$V_{\text{LE}}(\phi) = \frac{m^2}{2}\phi^2(1 - \chi\phi^4). \quad (6)$$

Although globally unbounded due to  $\chi = -\lambda^2/(360m^4)$ , the LE truncation (6) has the advantage that, in 4D, it admits exact spherically symmetric static solutions [23] which model quite well [25] DM halos of individual galaxies. Moreover, more than a century ago<sup>1</sup> it was considered as approximate models for the density of the sun as well as for the distribution of stars in globular clusters.

### 3. Soliton collisions in (1 + 1)D

The *axions* of Quantum Chromodynamics (QCD) with inertial mass  $m$  are self-interacting via the *effective* [15,30] periodic potential

$$V_a(\phi) = \frac{m^4}{\lambda} \left[ 1 - \cos\left(\frac{\sqrt{\lambda}}{m}\phi\right) \right]$$

<sup>1</sup> Jonathan Homer Lane: “On the theoretical temperature of the Sun under the hypothesis of a gaseous mass maintaining its volume by its internal heat and depending on the Laws of Gases known to Terrestrial experiment”. The American Journal of Science and Arts. 2 50 (1870) 57–74.

$$\simeq V_{\text{LE}}(\phi) - \frac{\lambda}{4!}\phi^4 - \dots \quad (7)$$

which deviates from the LE potential at higher order of  $|\phi|$ .

When several solitons collide, one expects that they recuperate their initial shapes and velocities after some time has passed. Remnants from crossing the scattering region may be the concomitant phase shifts or displacements of the centers of the individual solitary waves due the nonlinear, partially inelastic interaction.

For constructing multi-solitons, the well-established *Bäcklund transformation* (BT), cf. Refs. [38,32,13] is employed which may also bridge between different types of nonlinear equations.

Let us depart from the sine-Gordon (sG) equation [31] which in dimensionless light-cone coordinates  $\xi := \frac{1}{2}(\tilde{x} + c\tilde{t})$  and  $\eta := \frac{1}{2}(\tilde{x} - c\tilde{t})$  acquires the form

$$\theta_{\xi\eta} = \sin\theta \quad (8)$$

and is *CPT* invariant. In a moving frame, it has the exact kink solution

$$\theta = 4C \arctan[\exp\gamma(\tilde{x} - u\tilde{t})], \quad (9)$$

for  $C = 1$  and anti-kink for  $C = -1$ . Since its spatial derivative

$$\theta_{\tilde{x}} = 2\gamma C \operatorname{sech}[\gamma(\tilde{x} - u\tilde{t})] \quad (10)$$

becomes localized and square-integrable, its absolute value will facilitate a subsequent comparison with the scattering behavior of solitons or lumps regarded as Bose–Einstein condensates [24,28] of DM.

Due to Bianchi’s permutability theorem of BTs, there results the ‘nonlinear superposition’ principle

$$\tan[(\theta_3 - \theta_0)/4] = \mathbb{B} \tan[(\theta_1 - \theta_2)/4], \quad (11)$$

where  $\mathbb{B}$  is a common or ‘average’ relativistic velocity of the superposed solitons. This allows us to algebraically construct multi-kink solutions of the sG equation a la Perring and Skyrme [26,41].

The *CPT* invariance of our relativistic KG equation, will allow us to distinguish solitons from anti-solitons:

Here we focus on the collision of two kinks (instead of a collision of a kink and its CP odd anti-kink, as in Ref. [4]) and obtain from the trivial seed solution  $\theta_0 = 0$  the exact solution

$$\theta_{\text{kk}} = 4 \arctan[K(\zeta_1, \zeta_2)], \quad (12)$$

where the kinetic factor

$$\begin{aligned} K(\zeta_1, \zeta_2) &:= \mathbb{B} \frac{\exp(\zeta_1) + C \exp(\zeta_2)}{\exp(\zeta_1 + \zeta_2) - C} \\ &= \frac{\exp(\zeta_1 + \gamma_1 \delta_1) + C \exp(\zeta_2 + \gamma_2 \delta_2)}{\exp(\zeta_1 + \zeta_2) - C} \\ &\simeq \exp(\zeta_1 + \gamma_1 \delta_1) + C \exp(\zeta_2 + \gamma_2 \delta_2) \end{aligned} \quad (13)$$

depends on the initial velocities and the inverse Lorentz transformations  $\zeta_i := \gamma_i(\tilde{x} + u_i \tilde{t})$ . The former is also known as Hirota’s formula [11,12].

At large separations from the interaction region, cf. Fig. 2, the solution (12) clearly decouples asymptotically into a (non-interacting) kink–kink or kink–antikink pair [7] distinguished by the sign  $C = \pm 1$  of the topological charge.

#### 3.1. Collisions of Lane–Emden lumps

For the Lane–Emden equation of interest in astrophysics, an exact auto-Bäcklund transformation has yet not been found. Instead a generalized transformation or *mapping* will serve us as a guide in constructing multi-lump solution. Consider the Lane–Emden potential (6) as a truncation of (7), then the corresponding non-linear KG equation in light-cone coordinates simplifies to

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