Physics Letters B 758 (2016) 118–124

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Heavy quarkonium in a holographic basis

Yang Li [a,*], Pieter Maris [a], Xingbo Zhao [b], James P. Vary [a]

[a] Department of Physics and Astronomy, Iowa State University, Ames, IA 50011, USA
[b] Institute of Modern Physics, Chinese Academy of Sciences, Lanzhou 730000, China

ARTICLE INFO

Article history:
Received 7 October 2015
Received in revised form 11 April 2016
Accepted 30 April 2016
Available online 4 May 2016
Editor: J.-P. Blaizot

Keywords:
Heavy quarkonium
Light front
Holographic QCD
Spectroscopy
Decay constant
Form factor

ABSTRACT

We study the heavy quarkonium within the basis light-front quantization approach. We implement the one-gluon exchange interaction and a confining potential inspired by light-front holography. We adopt the holographic light-front wavefunction (LFWF) as our basis function and solve the non-perturbative dynamics by diagonalizing the Hamiltonian matrix. We obtain the mass spectrum for charmonium and bottomonium. With the obtained LFWFs, we also compute the decay constants and the charge form factors for selected eigenstates. The results are compared with the experimental measurements and with other established methods.

1. Introduction

Describing hadrons from quantum chromodynamics (QCD) remains a fundamental challenge in nuclear physics. Inspired by the discovery of a remarkable gauge/string duality [1], holographic QCD models, most notably the AdS/QCD [2], have been proposed as analytic semi-classical approximations to QCD (for a recent review, see Ref. [3]). In light of these phenomenological successes, as well as the recent progress in the ab initio nuclear structure calculations [4–7], the basis light-front quantization (BLFQ) [8] has been developed as a non-perturbative approach to address QCD bound-state problems from first principles.

BLFQ is based on the Hamiltonian formalism in light-front dynamics (LFD, [9]) in Minkowski space. The central task of the Hamiltonian approach is to diagonalize the QCD Hamiltonian operator,

$$P^+\hat{P}^-|\psi_h\rangle = M_h^2|\psi_h\rangle. \quad (1)$$

Here $P^\pm = P^0 \pm P^3$ is the longitudinal momentum and the light-front quantized Hamiltonian operator, respectively. The eigenvalues directly produce the invariant-mass spectrum. The eigenfunctions, known as the light-front wavefunctions (LFWFs), play a pivotal role in the study of the hadron structures in deep inelastic scattering (DIS) [10] and deeply virtually Compton scattering (DVCS) [11]. In the Fock space expansion, Eq. (1) becomes a relativistic quantum many-body problem and can be solved by constructing and diagonalizing the many-body Hamiltonian matrix (see, e.g., [12] for a review).

The advantages of LFD are made explicit by BLFQ which can employ an arbitrary single-particle basis subject to completeness and orthonormality. By adopting a single-particle AdS/QCD basis, BLFQ naturally extends the AdS/QCD LFWFs to the multi-particle Fock sectors [8]. Furthermore, this basis preserves all the kinematical symmetries of the full Hamiltonian [13,14]. Such choice is in parallel with the no-core shell model (NCSM) used in non-relativistic quantum many-body theory [5]. State-of-the-art computational tools developed in the many-body theory can be used to address the QCD eigenvalue problem [15]. BLFQ has been applied successfully to a range of non-perturbative problems, including the electron anomalous magnetic moment [16,17], non-linear Compton scattering [18,19] and the positronium spectrum [20,21]. In this paper, we apply the BLFQ approach to the heavy quarkonium.

Working with the full QCD Hamiltonian is a formidable task. In practice, we truncate the Fock space to a finite number of particles. The leading-order truncation $|q\bar{q}\rangle + |q\bar{q}g\rangle$ introduces the one-gluon exchange which produces correct short-distance physics as well as the spin-dependent interaction needed for the fine and hyperfine structures. The Abelian version of this interaction was extensively used in the literature [20,22–25] to calculate the QED bound-state

* Corresponding author.
E-mail addresses: leeyoung@iastate.edu (Y. Li), pmaris@iastate.edu (P. Maris), xbzhao@impcas.ac.cn (X. Zhao), jvary@iastate.edu (J.P. Vary).

http://dx.doi.org/10.1016/j.physletb.2016.04.065

spectrum in LFD. However, the one-gluon exchange itself is not sufficient to reproduce the hadron spectrum since confinement is also needed. Holographic QCD provides an appealing approximation to confinement.

Heavy quarkonium is an ideal laboratory for studying non-perturbative aspects of QCD and their interplay with the perturbative physics [26]. Conventional theoretical tools include the non-relativistic potential models (NRPMs) [27,28], non-relativistic QCD (NRQCD) [29], heavy quark effective field theory [30], Dyson–Schwinger Equations (DSE) [31–34], and Lattice QCD [35]. The recent discoveries of tetraquark [36] and pentaquark [37] states have renewed interests in the theoretical investigation of heavy quarkonium. Extensive data on heavy quarkonium have been produced by experimental facilities, such as Belle, CLEO and LHC.

Numerous light-front phenomenologies have been developed for heavy quarkonium (see e.g. [38–44] and the references therein). Our approach shares some similarity with these models. Yet, there are also major differences. First of all, our approach employs holographic QCD (confining interaction) and realistic LFQCD (one-gluon exchange). Secondly and most importantly, we solve quarkonium as a two-body bound-state problem using a Hamiltonian method that is applicable to arbitrary many-body bound states, once the (effective) Hamiltonian and the basis space are specified. We exploit the fact that BLFQ is developed as a flexible computational platform for relativistic strong interaction many-body bound-state problems [8,15], designed to deal with general Hamiltonians, realistic or phenomenological.

Our goal in this work can be simply stated: we aim to improve the light-front holographic QCD results [45] by including a realistic one-gluon exchange interaction. Computationally, we intend to lay the foundation for the extension to higher Fock sectors.

2. Effective Hamiltonian

2.1. Phenomenological confinement

Our effective Hamiltonian consists of the holographic QCD Hamiltonian and the one-gluon exchange. We adopt the light-front AdS/QCD soft-wall (SW) Hamiltonian for the first part [46]. This simple model gives a reasonable description of the hadron spectrum and structures (see Ref. [45] for a review). Its effective "light-cone" Hamiltonian reads,

$$H_{\rm SW} \equiv P^+\hat{P}^-_{\rm SW} - \boldsymbol{P}^2_\perp = \frac{\boldsymbol{k}^2_\perp}{x(1-x)} + \kappa^4 x(1-x)\boldsymbol{r}^2_\perp, \quad (2)$$

where, $x = p^+_q/P^+$ is the longitudinal momentum fraction of the quark, $\boldsymbol{k}_\perp = \boldsymbol{p}_{q\perp} - x\boldsymbol{P}_\perp$ is the relative transverse momentum, and $\boldsymbol{r}_\perp$ is the transverse separation of the partons. κ is the strength of the confining potential. Note that the "light-cone Hamiltonian" has mass squared dimension, whose eigenvalues are the squared invariant masses. Following Brodsky and de Téramond [46], it is convenient to introduce the holographic coordinate $\boldsymbol{\zeta}_\perp = \sqrt{x(1-x)}\boldsymbol{r}_\perp$, and its conjugate $\boldsymbol{q}_\perp = \boldsymbol{k}_\perp/\sqrt{x(1-x)} \equiv -i\nabla_{\zeta_\perp}$. In light-front holography, $\zeta_\perp$ is mapped to the fifth coordinate z of the AdS space. In these coordinates, $H_{\rm SW}$ is a harmonic oscillator (HO),

$$H_{\rm SW} = \boldsymbol{q}^2_\perp + \kappa^4\boldsymbol{\zeta}^2_\perp. \quad (3)$$

Its eigenvalues follow the Regge trajectory $M^2 = 2\kappa^2(2n+|m|+1)$. Its eigenfunctions are 2D HO functions in the holographic variables,

$$\phi_{nm}(\boldsymbol{q}_\perp) = e^{im\theta}\left(\frac{q_\perp}{\kappa}\right)^{|m|} e^{-q^2_\perp/(2\kappa^2)} L_n^{|m|}(q^2_\perp/\kappa^2). \quad (4)$$

Here $q_\perp = |\boldsymbol{q}_\perp|$, $\theta = \arg \boldsymbol{q}_\perp$, and $L^m_n(z)$ is the associated Laguerre polynomial. We adopt these functions as our basis. This basis has the advantage that in the many-body sector, it allows the exact factorization of the center-of-mass motion in the single-particle coordinates. This is a very valuable property, because the boson/fermion symmetrization/anti-symmetrization in the relative coordinates quickly becomes intractable, as the number of identical particles increases [13,14]. For this work, however, we do not have identical particles in the $q\bar{q}$ sector and we will use the relative coordinate. In future extensions, as sea quarks and gluons are added, it may be more advantageous to adopt single-particle coordinates.

The soft-wall Hamiltonian Eq. (3) is designed for massless quarks, and it is inherently 2-dimensional. For the heavy quarkonium systems, it should be modified to incorporate the quark masses and the longitudinal dynamics,

$$H_{\rm SW} \to H_{\rm CON} = \boldsymbol{q}^2_\perp + \kappa^4\boldsymbol{\zeta}^2_\perp + \frac{m_q^2}{x} + \frac{m_{\bar{q}}^2}{1-x} + V_L(x). \quad (5)$$

Here V_L is a longitudinal confining potential. Several longitudinal confining potentials have been proposed [47–49]. Here we propose a new longitudinal confinement which shares features with others proposed,

$$V_L(x) = -\frac{\kappa^4}{(m_q+m_{\bar{q}})^2}\partial_x\big(x(1-x)\partial_x\big), \quad (6)$$

where $\partial_x \equiv (\partial/\partial x)_{\zeta_\perp}$. This term combined with the mass term from the kinetic energy forms a Sturm–Liouville problem,

$$-\partial_x\big(x(1-x)\partial_x\chi_l(x)\big) + \frac{1}{4}\Big(\frac{\alpha^2}{1-x} + \frac{\beta^2}{x}\Big)\chi_l(x) = \lambda_l^{(\alpha,\beta)}\chi_l(x), \quad (7)$$

where $\alpha = 2m_{\bar{q}}(m_q+m_{\bar{q}})/\kappa^2$, $\beta = 2m_q(m_q+m_{\bar{q}})/\kappa^2$. The solutions of Eq. (7) are analytically known in terms of the Jacobi polynomial $P_l^{(a,b)}(z)$,

$$\chi_l(x) = x^{\frac{1}{2}\alpha}(1-x)^{\frac{1}{2}\beta} P_l^{(\alpha,\beta)}(2x-1) \quad (8)$$

and form a complete orthogonal basis on the interval $[0,1]$. The corresponding eigenvalues are

$$\lambda_l^{(\alpha,\beta)} = (l+\tfrac{1}{2}(\alpha+\beta))(l+\tfrac{1}{2}(\alpha+\beta)+1), \quad (l=0,1,2,\cdots). \quad (9)$$

Comparing to other forms of longitudinal confinement, our proposal implements several attractive features. First, the basis functions resemble the known asymptotic parton distribution $\sim x^\alpha(1-x)^\beta$ with $\alpha, \beta > 0$ [50]. This is our primary motivation for adopting the longitudinal confinement Eq. (6). Second, the basis function is also analytically known, which brings numerical convenience. Third, in the non-relativistic limit $\min\{m_q, m_{\bar{q}}\} \gg \kappa$, the longitudinal confinement sits on equal footing with the transverse confinement, where together, they form a 3D harmonic oscillator potential,

$$V_{\rm con} = \frac{m_q m_{\bar{q}}}{(m_q+m_{\bar{q}})^2}\kappa^4\boldsymbol{r}^2, \quad (10)$$

and rotational symmetry is manifest. This non-relativistic reduction also provides us an order-of-magnitude estimate of the model parameters for our heavy quarkonium application. Fourth, in the massless limit $\max\{m_q, m_{\bar{q}}\} \ll \kappa$, the longitudinal mode stays in the ground state and the longitudinal wavefunction $\chi_0(x) = \text{const}$. Thus we restore the massless model of Brodsky and de Téramond.[1]

[1] Note that in our normalization convention, the LFWFs differ from Brodsky et al.'s [45] by a factor $\sqrt{x(1-x)}$.