Physics Letters B 758 (2016) 106–112

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Noncommutative effects of spacetime on holographic superconductors

Debabrata Ghorai [a], Sunandan Gangopadhyay [b,c,*]

[a] S.N. Bose National Centre for Basic Sciences, JD Block, Sector III, Salt Lake, Kolkata 700098, India
[b] Department of Physics, West Bengal State University, Barasat, India
[c] Inter University Centre for Astronomy & Astrophysics, Pune, India

ARTICLE INFO

Article history:
Received 13 March 2016
Received in revised form 14 April 2016
Accepted 3 May 2016
Available online 6 May 2016
Editor: M. Cvetič

ABSTRACT

The Sturm–Liouville eigenvalue method is employed to analytically investigate the properties of holographic superconductors in higher dimensions in the framework of Born–Infeld electrodynamics incorporating the effects of noncommutative spacetime. In the background of pure Einstein gravity in noncommutative spacetime, we obtain the relation between the critical temperature and the charge density. We also obtain the value of the condensation operator and the critical exponent. Our findings suggest that the higher value of noncommutative parameter and Born–Infeld parameter make the condensate harder to form. We also observe that the noncommutative structure of spacetime makes the critical temperature depend on the mass of the black hole and higher value of black hole mass is favourable for the formation of the condensate.

1. Introduction

Holographic superconductors have been studied extensively in recent times. Their importance lies in the fact that they mimic some properties of high T_c superconductors. The interest rose after the demonstration in [1] that an Abelian Higgs model in AdS spacetime leads to a spontaneous symmetry breaking and thus giving rise to a scalar hair near the horizon of the black hole. The important ingredient which goes in the construction of such holographic superconductor models is the correspondence between gravity and gauge theory, namely, the AdS/CFT correspondence [2].

Spacetime noncommutativity has been another prominent area of research in recent years. The idea of noncommutative (NC) spacetime, first formally introduced by Snyder [3] back in 1947 was not considered seriously by other scientists till recently when such a structure emerged naturally from investigations carried out in string theory [4]. It was in this paper that NC field theory was resurrected and rules were given to move from ordinary quantum field theory (QFT) to NC QFT. In more recent times, a noncommutative inspired Schwarzschild metric was obtained in [5,6]. Here the effect of noncommutativity was introduced through a smeared matter source which was then used to solved Einstein's equation of general relativity. An important aspect of this black hole solution was the removal of black hole singularity. The thermodynamics of this black hole solution was investigated in details in [7].

In this paper, we want to investigate the role of NC geometry on the AdS/CFT duality, in particular to study its effect on holographic superconductor models in higher dimensions. Such a study had been carried out earlier in [8] in 4-dimensions. Here we generalize this analysis to arbitrary dimensions by considering the d-dimensional generalization of the NC Schwarzschild black hole. We also present expressions for the critical temperature which is more accurate than the one given in [8] as will be clear in the subsequent discussion. Further we consider Born–Infeld (BI) electrodynamics thereby including the effect of non-linearity in the analysis. There are quite a few reasons which make it worthwhile to study the effect of BI electrodynamics. First of all, it is the only non-linear theory that remains invariant under electromagnetic duality. Another intriguing feature is that it has a nice weak field limit [9–12]. It also finds remarkable application in string theory [13,14]. We would like to mention that the technique that we have adopted in this paper to obtain the relation between the critical temperature and the charge density is the Sturm–Liouville (SL) eigenvalue approach.

This paper is organized as follows. In section 2, we show the basic holographic set up in noncommutative spacetime in the background of electrically charged black hole in arbitrary dimension. In section 3, taking into account the effect of the Born–Infeld electrodynamics, we have derived the relation between critical

* Corresponding author.
E-mail addresses: debanuphy123@gmail.com, debabrataghorai@bose.res.in (D. Ghorai), sunandan.gangopadhyay@gmail.com (S. Gangopadhyay).

http://dx.doi.org/10.1016/j.physletb.2016.05.011
0370-2693/

temperature and charge density using the Sturm–Liouville eigenvalue problem. In section 4, we analytically obtain an expression for the condensation operator in d-dimension near the critical temperature. We conclude finally in section 5.

2. Set up in noncommutative spacetime

We by considering a noncommutative charged Schwarzschild-AdS_d black hole whose metric is given by [6]

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2 h_{ij}dx^i dx^j$$
$$f(r) = K + \frac{r^2}{L^2} - \frac{2MG_d}{r^{d-3}\Gamma(\frac{d-1}{2})}\gamma\left(\frac{d-1}{2}, \frac{r^2}{4\theta}\right) \quad (1)$$

where $h_{ij}dx^i dx^j$ denotes the line element of a $(d-2)$-dimensional hypersurface with zero curvature and

$$\gamma(s,x) = \int_0^x t^{s-1}e^{-t}dt \quad (2)$$

is the lower incomplete Gamma function and K represents the curvature. As $\theta \to 0$, the noncommutative metric $f(r)$ gives back the commutative Schwarzschild metric in d-dimensions. We also work in the probe limit which basically implies that the backreaction on the spacetime metric $f(r)$ is not taken into account.

The Hawking temperature of this black hole, which is interpreted as the temperature of the conformal field theory on the boundary, is given by

$$T_H = \frac{f'(r_+)}{4\pi} \quad (3)$$

where r_+ is the radius of the horizon of the black hole.

Since the construction of the holographic s-wave superconductor requires a planar symmetry, we set $K = 0$ which implies that

$$f(r) = \frac{r^2}{L^2} - \frac{2MG_d}{r^{d-3}\Gamma(\frac{d-1}{2})}\gamma\left(\frac{d-1}{2}, \frac{r^2}{4\theta}\right). \quad (4)$$

Using the fact that metric vanishes at the event horizon, we get the radius of the event horizon r_+ in d-dimensions to be

$$r_+^{d-1} = \frac{2MG_dL^2}{\Gamma(\frac{d-1}{2})}\gamma\left(\frac{d-1}{2}, \frac{r_+^2}{4\theta}\right). \quad (5)$$

For convenience we shall set $L = 1$ in the rest of the analysis. The above relation enables us to write the metric (4) as

$$f(r) = r^2 - \frac{r_+^{d-1}}{r^{d-3}} \cdot \frac{\gamma(\frac{d-1}{2}, \frac{r^2}{4\theta})}{\gamma(\frac{d-1}{2}, \frac{r_+^2}{4\theta})}. \quad (6)$$

From eq.(s) (3) and (6), we obtain the expression for the Hawking temperature of the black hole

$$T_H = \frac{1}{4\pi}\left[(d-1)r_+ - r_+^2 \frac{\gamma'(\frac{d-1}{2}, \frac{r_+^2}{4\theta})}{\gamma(\frac{d-1}{2}, \frac{r_+^2}{4\theta})}.\right] \quad (7)$$

Computing the derivative of the incomplete Gamma function (2), we get

$$T_H = \frac{r_+}{4\pi}\left[(d-1) - \frac{4MG_d}{\Gamma(\frac{d-1}{2})} \cdot \frac{e^{-\frac{r_+^2}{4\theta}}}{(4\theta)^{\frac{d-1}{2}}}\right] \quad (8)$$

The matter Lagrangian density is due to the presence of a gauge field and complex scalar field. This reads

$$\mathcal{L}_{matter} = \frac{1}{b}\left(1 - \sqrt{1 + \frac{b}{2}F^{\mu\nu}F_{\mu\nu}}\right) - (D_\mu\psi)^* D^\mu\psi - m^2\psi^*\psi \quad (9)$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$; $(\mu, \nu = 0, 1, 2, 3, 4)$ is the field strength tensor, $D_\mu\psi = \partial_\mu\psi - iqA_\mu\psi$ is the covariant derivative, A_μ and ψ represent the gauge field and the scalar field respectively.

Considering that the black hole possesses only electric charge, we make the ansatz [15] $A_\mu = (\phi(r), 0, 0, 0)$ and $\psi = \psi(r)$. We can also choose magnetic field but now we only have presented electric field. Using this ansatz, the equations of motion for the scalar fields and electric potential read [16]

$$\psi''(r) + \left(\frac{d-2}{r} + \frac{f'(r)}{f(r)}\right)\psi'(r) + \left(\frac{q^2\phi^2(r)}{f(r)^2} - \frac{m^2}{f(r)}\right)\psi(r) = 0 \quad (10)$$

$$\phi''(r) + \frac{d-2}{r}\phi'(r) - \frac{d-2}{r}b\phi'(r)^3 - \frac{2q^2\phi(r)\psi^2(r)}{f(r)}(1 - b\phi'(r)^2)^{\frac{3}{2}} = 0 \quad (11)$$

where prime denotes derivative with respect to r. The rescalings $\psi \to \psi/q$, $\phi \to \phi/q$ and $\kappa^2 \to q^2\kappa^2$ [17] allow one to set $q = 1$.

To solve the above non-linear coupled differential equations (10)–(11), we must fix the boundary condition for $\phi(r)$ and $\psi(r)$ which are physically acceptable. For regularization, one requires $\phi(r_+) = 0$ and $\psi(r_+)$ to be finite at the horizon.

Near the boundary of the bulk, the asymptotic behaviour of ψ and ϕ are not affected by noncommutativity. This is because near the boundary, r is large and therefore $e^{\frac{-r^2}{4\theta}} \ll 1$ since θ is small. The asymptotic behaviour of the fields can be written as [18]

$$\phi(r) = \mu - \frac{\rho}{r^{d-3}} \quad (12)$$
$$\psi(r) = \frac{\psi_-}{r^{\Delta_-}} + \frac{\psi_+}{r^{\Delta_+}} \quad (13)$$

where

$$\Delta_\pm = \frac{(d-1) \pm \sqrt{(d-1)^2 + 4m^2}}{2}. \quad (14)$$

The gauge/gravity duality allows one to interpret ρ and μ as the charge density and chemical potential of the boundary field theory. For the choice $m^2 = -3$ with the Breitenlohner–Freedman bound [19], we have $\Delta_+ = 3$ $\Delta_- = 1$ for $d = 5$. This allows one to choose ψ_+ or ψ_-. In this paper we shall choose $\psi_- = 0$. This basically means that ψ_+ is dual to the expectation value of the condensation operator J in the absence of the source ψ_-.

Using $z = \frac{r_+}{r}$, the field equations (10)–(11) take the form

$$\psi''(z) + \left(\frac{f'(z)}{f(z)} - \frac{d-4}{z}\right)\psi'(z) + \frac{r_+^2}{z^4}\left(\frac{\phi^2(z)}{f(z)^2} - \frac{m^2}{f(z)}\right)\psi(z) = 0 \quad (15)$$

$$\phi''(z) - \frac{d-4}{z}\phi'(z) + \frac{d-2}{r_+^2}b\phi'(z)^3 z^3 - \frac{2r_+^2\phi(z)\psi^2(z)}{f(z)z^4}\left(1 - \frac{bz^4}{r_+^2}\phi'(z)^2\right)^{\frac{3}{2}} = 0 \quad (16)$$