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Collapsing objects with the same gravitational trajectory can radiate away different amount of energy

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ABSTRACT

We study radiation emitted during the gravitational collapse from two different types of shells. We assume that one shell is made of dark matter and is completely transparent to the test scalar (for simplicity) field which belongs to the standard model, while the other shell is made of the standard model particles and is totally reflecting to the scalar field. These two shells have exactly the same mass, charge and angular momentum (though we set the charge and angular momentum to zero), and therefore follow the same geodesic trajectory. However, we demonstrate that they radiate away different amount of energy during the collapse. This difference can in principle be used by an asymptotic observer to reconstruct the physical properties of the initial collapsing object other than mass, charge and angular momentum. This result has implications for the information paradox and expands the list of the type of information which can be released from a collapsing object.

1. Introduction

In Einstein–Maxwell theory a stationary black hole solution is generally characterized by its mass, electric charge and angular momentum. In more general theories, some scalar field hairs have also been found [1–3], and they can be considered as generalized (or Noether) charges. All additional information about the initial state of matter that formed the black hole is lost during the collapse. This includes the global charges (e.g. lepton number, baryon number, flavor [4]), angular momentum, charge and energy distributions (as opposed to their total values which are conserved) etc. To recover this information after the black hole is formed seems to be impossible without invoking some exotic physics. Instead of looking at the $t \to \infty$, i.e. an exact Schwarzschild solution in an asymptotically flat space–time, we can take a look at the near horizon region. Information about the initial state might be released during the collapse, since once the collapse is over there is no much one can do. It is well known that during the collapse an object radiates away its higher multipoles and other irregularities in the so-called balding phase before a perfect spherically symmetric horizon is formed. The problem is that these are all gravitational degrees of freedom, and cannot account for other non-gravitational information content. In [5,6], it was shown that gravitational collapse is followed by the so-called pre-Hawking radiation from the very beginning of the collapse, simply because the metric is time dependent. This radiation becomes completely thermal Hawking radiation only in $t \to \infty$ limit when the event horizon is formed. Since the collapsing object has only finite amount of mass, an asymptotic observer would never witness the formation of the horizon at $t \to \infty$. For him, the collapsing shell will slowly get converted into not-quite-thermal radiation before it reaches its own Schwarzschild radius. It was demonstrated in [7] that the evolution is completely unitary in such a setup.

In this paper, we also concentrate on the pre-Hawking radiation, but we are using the standard analysis as defined in [8,9]. We explicitly construct an example in which two shells have exactly the same mass, charge and angular momentum (though we set the charge and angular momentum to zero for simplicity). By construct, they follow the same gravitational trajectory, however they emit different radiation during the collapse. We achieve this by giving different physical properties to the collapsing shells, other than mass, charge and angular momentum. In particular, one of the shells is completely transparent to radiation, and the other is totally reflecting. This is for example the situation where one of the shells is made of dark matter and the other of the standard model particles. If one studies emission of the standard model particles

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from these shells, then the dark matter shell will be completely transparent to radiation, and the standard model shell will be totally or partially reflecting. Of course, there is a whole continuum of cases between the totally reflecting and totally transparent shells, but for the purpose of illustration, these two extremes will suffice. For simplicity, we use a spherically symmetric falling shell. In this case only s-wave scalar field is relevant, and therefore the radiation field is chosen to be a scalar field. In the realistic standard model, one could use any other field. We show that the flux of energy and power spectra of radiation emitting from these two shells is notably different, though in the limit of $t \to \infty$ the fluxes become identical. Thus, an observer studying the flux of the standard model particles from a collapsing shell could in principle tell if the shell is made of the dark or ordinary matter.

2. The trajectory of the collapsing shell

For our purpose, we consider a freely falling massive spherical shell. The time dependent radius of the shell is $R(\tau)$, where τ is the proper time of the observer located on the shell. The geometry outside the shell is Schwarzschild

$$ds^2 = -\left(1 - \frac{2M}{r}\right)dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 d\Omega \quad (1)$$

$$d\Omega = d\theta^2 + \sin^2\theta d\phi^2. \quad (2)$$

The geometry inside the shell is by the Birkhoff theorem flat Minkowski space

$$ds^2 = -dT^2 + dr^2 + r^2 d\Omega \quad (3)$$

The motion of the shell can be found by matching the geometry inside and outside the shell [10]. The equation of motion is given in terms of the conserved quantity μ, which is just the rest mass of the shell.

$$\mu = -R\left[(1 - \frac{2M}{R} + \dot{R}^2)^{\frac{1}{2}} - (1 + \dot{R}^2)^{\frac{1}{2}}\right]. \quad (4)$$

Here, $\dot{R} = \frac{dR}{d\tau}$. From Eq. (4) we have

$$\dot{R} = \left(\frac{M^2}{\mu^2} - 1 + \frac{M}{R} + \frac{\mu^2}{4R^2}\right)^{\frac{1}{2}} \quad (5)$$

Then, the proper time on the shell is given by

$$\tau = \int d\tau = \int \frac{dR}{\dot{R}} \quad (6)$$

The time coordinate of an asymptotic observer on the shell is

$$t = \int dt = \int \frac{\left(1 + \frac{\dot{R}^2}{1 - \frac{2M}{R}}\right)^{\frac{1}{2}}}{\left(1 - \frac{2M}{R}\right)^{\frac{1}{2}}} d\tau \quad (7)$$

The time coordinate of an observer on the shell is

$$T = \int dT = \int \left(1 + \dot{R}^2\right)^{\frac{1}{2}} d\tau \quad (8)$$

3. Reflecting and transparent shells

We are set to study whether two massive shells with the same gravitational trajectory can have different pre-Hawking radiation. To achieve this we consider two shells of equal mass, but one is completely transparent to a scalar field that propagates in this background, while the other one reflects the scalar field totally.

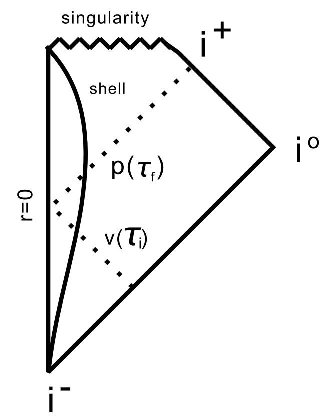

Fig. 1. Penrose diagram for the transparent collapsing shell. The mode crosses the shell at some initial time τ_i, passes through the center, and crosses again at some final time τ_f.

The evolution of the scalar field in a curved background outside the shell is described by

$$\Box\phi = 0 \quad (9)$$

where the $\Box$ operator is covariant. Inside the shell, the $\Box$ operator is Minkowski. Because of the spherical symmetry, as usual, we simplify the discussion and focus on a $1 + 1$ dimensional scalar field, $\phi(t, r)$, which satisfies the wave equation

$$\partial_t^2\phi - \partial_{r^*}^2\phi = 0, \text{ for } r > R \quad (10)$$

$$\partial_T^2\phi - \partial_r^2\phi = 0, \text{ for } r < R \quad (11)$$

Here $r^* = \int \frac{dr}{1-\frac{2M}{r}}$ is the usual tortoise coordinate. The trajectory of the spherical shell is given by Eq. (5), and it is the same for both shells since they have the same mass (and carry no charge nor angular momentum). There are two types of solutions to the wave equation for $r > R$, i.e. $f(t \pm r^*)$. The function $f(t - r^*)$ represents a wave moving to the right, while $f(t + r^*)$ represents a wave moving to the left. When a plane wave is propagating inward toward the origin, it is considered as an ingoing mode and can be written as

$$\phi_{\text{in}} \sim \exp(-i\omega v) \quad (12)$$

where we defined the ingoing and outgoing null coordinates $v = t + r^*$ and $u = t - r^*$. When the ingoing mode passes through the center, it starts propagating outward (away from the center), and it becomes an outgoing mode. The form of wave function is the same as before, but its argument must be a function of an outgoing coordinate $f(u)$, i.e.

$$\phi_{\text{out}} \sim \exp(-i\omega p(u)), \quad (13)$$

where $p(u)$ is a function of the coordinate u.

The shells in our discussion here are massive, which is different from the massless shells discussed in usual cases. The massless scalar field is moving faster than the shell and will pass through or be reflected by the matter on the shell.

We now consider the transparent shell first. While the shell is collapsing, the incoming scalar field mode passes through the shell and reaches the center of the shell. Once it passes through the center, it becomes an outgoing mode. As shown in Fig. 1, the mode crosses the shell at some initial time τ_i, passes through the center, and crosses again at some final time τ_f. Since the field moves at the speed of light, τ_i and τ_f must satisfy the condition

$$R(\tau_f) + R(\tau_i) = T(\tau_f) - T(\tau_i). \quad (14)$$