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Characteristic size and mass of galaxies in the Bose–Einstein condensate dark matter model

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ABSTRACT

We study the characteristic length scale of galactic halos in the Bose–Einstein condensate (or scalar field) dark matter model. Considering the evolution of the density perturbation we show that the average background matter density determines the quantum Jeans mass and hence the spatial size of galaxies at a given epoch. In this model the minimum size of galaxies increases while the minimum mass of the galaxies decreases as the universe expands. The observed values of the mass and the size of the dwarf galaxies are successfully reproduced with the dark matter particle mass $m \simeq 5 \times 10^{-22}$ eV. The minimum size is about $6 \times 10^{-3}\sqrt{m/H}\lambda_c$ and the typical rotation velocity of the dwarf galaxies is $O(\sqrt{H/m})$ c, where H is the Hubble parameter and λ_c is the Compton wave length of the particle. We also suggest that ultra compact dwarf galaxies are the remnants of the dwarf galaxies formed in the early universe.

One of the long standing questions in astronomy is what determines the size of galaxies. In this paper we show that the Bose–Einstein condensate (BEC) dark matter (DM) or the scalar field dark matter (SFDM) can explain the minimum size and the mass of galaxies in a unified way.

DM remains a great mystery in astrophysics, particle physics and cosmology. The cold dark matter (CDM) model is very successful in explaining the large scale structures in the universe, but has many problems in explaining galactic structures. For example, one of the early evidences for the DM presence is the flatness of galactic rotation curves [1], however the CDM is not so successful in explaining the rotation curves in galaxy cores. Numerical studies with ΛCDM model predict a cusped halo central density and many subhalos, which are also in discord with observational data [2–5]. On the other hand, the BEC/SFDM [6–9] can be a good alternative to the CDM, because the BEC/SFDM plays the role of the CDM at super-galactic scales and suppresses sub-galactic structures. In this model the DM is a BEC of the scalar particles with the ultra-light mass $m \simeq 5 \times 10^{-22}$ eV, whose quantum nature prevents the formation of the structures smaller than a galaxy due to the long Compton wavelength $\lambda_c = 2\pi\hbar/mc \simeq 0.08$ pc.

There are two other difficulties the CDM models encounter. First, the studies on satellite dwarf spheroidal (dSph) galaxies of the Milky Way [10,11] indicate that a typical dSph never has a size < kpc, and that the mass enclosed within the radius of 300 pc in dwarf galaxies is approximately constant ($\sim 10^7 M_\odot$) regardless of their luminosity [12]. This result implies the existence of a minimum mass scale in addition to the minimum length scale for DM dominated objects [10,13]. However, without introducing the roles of visible matter the CDM models usually predict DM dominated structures down to $10^{-6}M_\odot$. Second, the observations [14–16] of the size evolution of the most massive galaxies imply that these galaxies rapidly grow their size about 5 times since $z \sim 2$ while in the CDM models we expect compact early galaxies having smaller masses.

In Ref. [17] we showed that BEC/SFDM can explain the minimum mass of dwarf galaxies, if there is a minimum length scale. We also proposed that the size evolution of the massive galaxies can be attributed to the evolution of a length scale ξ of BEC DM [18]. In these works we considered the various length scales for ξ such as λ_c, a thermal de Broglie wavelength or a self-interaction scale. For all galaxies $\xi \gg \lambda_c$, and we need to find the exact physical origin of this long scale, which is the main subject of this paper.

The conjecture that DM is in BEC has a long history. (See Refs. [19–24] for a review.) Baldeschi et al. [6] studied the galactic halos of self-gravitating bosons, and Membrado et al. [7] calculated the rotation curves of self-gravitating boson halos. Sin [8]

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suggested that the halos are like atoms made of ultra-light BEC DM. Lee and Koh [9] suggested that the DM halos are the giant boson stars described by the relativistic scalar field theory. Similar ideas were suggested by many authors [25–43]. In literature it has been shown that BEC/SFDM could explain the many observed aspects such as rotation curves [28,44–46], the large scale structures of the universe [47], the cosmic background radiation, and spiral arms [48].

In this paper, we show that BEC/SFDM has the natural length scale and the mass scale determined only by background matter density and the DM particle mass m. In the BEC DM model [8] a galactic DM halo is described with the wave function $\psi(\mathbf{r})$, which is the solution of the Gross–Pitaevskii equation (GPE)

$$i\hbar\partial_t\psi(\mathbf{r},t) = -\frac{\hbar^2}{2m}\nabla^2\psi(\mathbf{r},t) + m\Phi\psi(\mathbf{r},t) \tag{1}$$

with a self-gravitation potential Φ. This equation could be obtained from the mean field approximation of a BEC Hamiltonian or the non-relativistic approximation of SFDM action [9]. For simplicity, we consider the spherical symmetric case with

$$\Phi(r) = \int_0^r dr'\frac{1}{r'^2}\int_0^{r'} dr'' 4\pi r''^2 (GmM|\psi(r)|^2 + \rho_v), \tag{2}$$

where M is the mass of the halo, and ρ_v is the mass density of visible matter. We do not consider a particle self-interaction term in this paper.

The Madelung representation [20,23]

$$\psi(r,t) = \sqrt{A(r,t)}e^{iS(r,t)} \tag{3}$$

is useful for studying the cosmological structure formation in the fluid approach. Here the amplitude A and DM density have a relation $\rho = mA$. Substituting Eq. (3) into GPE, one can obtain a continuity equation

$$\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\mathbf{v}) = 0 \tag{4}$$

and a modified Euler equation

$$\frac{\partial\mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v} + \nabla\Phi + \frac{\nabla p}{\rho} - \frac{\nabla Q}{m} = 0 \tag{5}$$

with a quantum potential $Q \equiv \frac{\hbar^2}{2m}\frac{\Delta\sqrt{\rho}}{\sqrt{\rho}}$, a fluid velocity $\mathbf{v} \equiv \nabla S/2m$, and the pressure from a self-interaction pressure p (if there is). Here, Δ is the Laplacian. The quantum pressure term $\nabla Q/m$ is the key difference between the CDM and the BEC DM. Perturbing the equations (4) and (5) around $\rho = \bar{\rho}$, $\mathbf{v} = 0$, and $\Phi = 0$ and then combining the two perturbed equations gives a differential equation for density perturbation $\delta\rho \equiv \rho - \bar{\rho}$,

$$\frac{\partial^2\delta\rho}{\partial t^2} + \frac{\hbar^2}{4m^2}\nabla^2(\nabla^2\delta\rho) - c_s^2\nabla^2\delta\rho - 4\pi G\bar{\rho}\delta\rho = 0, \tag{6}$$

where c_s is the sound velocity from p, and $\bar{\rho}$ is the average background matter density (see, for example, Ref. [49] for details). We have ignored the effect of the cosmic expansion in this equation for simplicity. We can rewrite this equation into the Fourier transformed equation of the density contrast $\delta \equiv \delta\rho/\bar{\rho} = \delta_k e^{ik\cdot r}$ with a wave vector k,

$$\frac{d^2\delta_k}{dt^2} + \left[(c_q^2 + c_s^2)k^2 - 4\pi G\bar{\rho}\right]\delta_k = 0, \tag{7}$$

where $c_q = \hbar k/2m$ is a quantum velocity. Note that the k^4 dependent term (the c_q dependent term) came from the perturbation of the quantum pressure term. From this equation we can see that the BEC DM behaves like the CDM for a small k (for a large scale) while for a large k (at a small scale) the quantum pressure disturbs the structure formation. If the self-interaction is negligible we can ignore the c_s term. Equating $c_q^2k^2$ with $4\pi G\bar{\rho}$ defines the time dependent quantum Jeans length scale [30],

$$\begin{aligned}\lambda_Q(z) &= \frac{2\pi}{k} = \left(\frac{\pi^3\hbar^2}{m^2G\bar{\rho}(z)}\right)^{1/4}\\ &\simeq 55\,593\left(\frac{\rho_b}{m_{22}^2\Omega_m h^2\bar{\rho}(z)}\right)^{1/4}\ \text{pc},\end{aligned} \tag{8}$$

where the current matter density $\rho_b = 2.775\times10^{11}\Omega_m h^2\ M_\odot/\text{Mpc}^3$, the (dark + visible) matter density parameter $\Omega_m = 0.315$ [50], $h = 0.673$ and $m_{22} = m/10^{-22}$ eV. The quantum Jeans mass can be defined as

$$M_J(z) = \frac{4\pi}{3}\bar{\rho}(z)\lambda_Q^3 = \frac{4}{3}\pi^{\frac{13}{4}}\left(\frac{\hbar}{G^{\frac{1}{2}}m}\right)^{\frac{3}{2}}\bar{\rho}(z)^{\frac{1}{4}}, \tag{9}$$

which is the minimum mass of the DM structures at z. Note that the only time dependent term in the righthand side is the average density.

Though λ_Q is related to the minimum length scale of DM dominated objects [51,52], λ_Q alone does not determine the actual size of galaxies. Usually, $\lambda_Q > \xi > \lambda_c$. We need a governing equation for stable configurations of the DM dominated objects. To find the characteristic length ξ we study the ground state of the GPE. In the BEC/SFDM model, $\xi \sim \hbar/m\Delta v$ due to the uncertainty principle, where Δv is the velocity dispersion of DM in a halo. However, we have not been able to derive Δv from any theory so far. From Eq. (1) the energy E of the halo can be approximated as

$$E(\xi) \simeq \frac{\hbar^2}{2m\xi^2} + \int_0^{\xi} dr'\frac{Gm}{r'^2}\int_0^{r'} dr'' 4\pi r''^2(\rho(r'') + \rho_v(r'')), \tag{10}$$

as a function of the halo length scale ξ. The ground state can be found by extremizing it by ξ [53];

$$\frac{dE(\xi)}{d\xi} \simeq -\frac{\hbar^2}{m\xi^3} + \frac{GMm}{\xi^2} = 0, \tag{11}$$

where, $M \equiv \int_0^{\xi} dr'' 4\pi r''^2(\rho(r'') + \rho_v(r''))$ is the total mass within ξ. Solving Eq. (11) gives [8,53]

$$\xi = \frac{\hbar^2}{GMm^2} = \frac{c^2\lambda_c^2}{4\pi^2 GM}. \tag{12}$$

The quantum Jeans mass represents the smallest amount of the DM having enough self-gravity to overcome the quantum velocity, so M_J in Eq. (9) can be identified to be M of the smallest galaxies. Therefore, from Eq. (9) the smallest galaxy formed at z has a size (the gravitational Bohr radius)

$$\xi(z) = \frac{\hbar^2}{GM_J(z)m^2} = \frac{3\hbar^{1/2}}{4\pi^{13/4}(Gm^2\bar{\rho}(z))^{1/4}} \propto \bar{\rho}(z)^{-1/4}, \tag{13}$$

which is a quantum mechanical relation absent in the CDM models. Therefore, $M_J(z)$ and $\xi(z)$ represent the time dependent mass and size of the smallest galaxies at the redshift z. Recall that M (and M_J) is the total mass including DM and visible matter, which explains the universal minimum mass of dwarf galaxies independent of visible matter fraction [18].

Once we fix one of ξ and M_J, the other is fixed automatically. In the previous works it was uncertain which one comes first, and