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Global monopoles can change Universe's topology

Anja Marunović *, Tomislav Prokopec

Institute for Theoretical Physics, Spinoza Institute and EMMEΦ, Utrecht University, Leuvenlaan 4, 3584 CE Utrecht, The Netherlands

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ABSTRACT

If the Universe undergoes a phase transition, at which global monopoles are created or destroyed, topology of its spatial sections can change. More specifically, by making use of Myers' theorem, we show that, after a transition in which global monopoles form, spatial sections of a spatially flat, infinite Universe becomes finite and closed. This implies that global monopoles can change the topology of Universe's spatial sections (from infinite and open to finite and closed). Global monopoles cannot alter the topology of the space-time manifold.

1. Introduction

The question of global properties (topology) of our Universe is a fascinating one, and it has been attracting attention for a long time. Yet only as-of-recently the data have been good enough to put meaningful observational constraints on the Universe's topology. While Einstein's equations uniquely specify local properties of space-time (characterized by the metric tensor), they fail to determine its global (topological) properties. Friedmann, Robertson and Walker (FRW) were first who observed that the most general solution corresponding to spatially homogeneous Universe with constant curvature κ of its spatial sections is the following FLRW metric (L stands for Lemaitre),

$$ds^2 = -c^2dt^2 + \frac{a^2(t)dr^2}{1-\kappa r^2} + a^2(t)r^2[d\theta^2 + \sin^2(\theta)d\phi^2], \qquad (1)$$

where c is the speed of light and $0 \leq r < \infty$, $0 \leq \theta \leq \pi$, $0 \leq \phi < 2\pi$ are spherical coordinates. Recent cosmic microwave background and large scale structure observations tell us that, at large scales the metric (1) describes quite accurately our Universe. When κ in (1) is

1. negative ($\kappa < 0$), then spatial sections of the Universe are hyperbolic,
2. zero ($\kappa = 0$), then spatial sections are flat;
3. positive ($\kappa > 0$), then the spatial sections are positively curved and they are locally homeomorphic to the geometry of the three dimensional sphere.

Older literature typically assumes that $\kappa \leq 0$ implies infinite spatial sections, while when $\kappa > 0$, spatial sections are compact. While the latter statement is correct, recent advancements in our understanding of (topology of) three dimensional manifolds tell us that we must be much more careful when drawing conclusions from the observational fact that the metric describing our observable Universe is well approximated by the FLRW metric (1). Namely, various boundary conditions could be imposed on the Universe's spatial sections [1], giving as a result a large number of possible three dimensional manifolds, only one of which corresponds to that of our Universe.

Let us now briefly recall the relevant observational facts. The first observational evidence that supports that we live in a (nearly) flat universe ($\kappa \approx 0$) was presented in 2000 by the balloon experiments Boomerang [2] and Maxima [3]. A recent bound on κ [4] is obtained when observations of BAOs (Baryon Acoustic Oscillations) are combined with the Planck data [5] and the polarization data from the WMAP satellite (WP),

$$\Omega_\kappa = -0.003 \pm 0.003, \qquad \Omega_\kappa = -\frac{\kappa c^2}{H_0^2}, \qquad (2)$$

where $H_0 \simeq 68$ km/s/Mpc (when the BAO data are dropped, one obtains $0.006 > \Omega_\kappa > -0.086$ [5]). Eq. (2) implies a large lower bound on the curvature radius of spatial sections, $R_c = 1/\sqrt{|\kappa|} \geq 60$ Gpc. The bound (2) implies the following robust conclusion: "our [observable] Universe is spatially flat to an accuracy of better than a percent" (cited from page 42 of Ref. [5]).

Even if the Universe is spatially flat, it can be made finite by imposing suitable periodic boundary conditions; the precise nature of periodic conditions determines Universe's global topology [1]. Although different scenarios have been considered in literature (good reviews are given in Refs. [1,6–10]), so far no evidence has been

* Corresponding author.
E-mail addresses: a.marunovic@uu.nl (A. Marunović), t.prokopec@uu.nl (T. Prokopec).

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found that would favor any of the proposed models. For example, extensive mining of the CMB data has been performed [11–13] in order to find pairs of circles, which are a telltale signature for non-trivial large-scale topology of the Universe, but so far no convincing signature has been found.

The above considerations make an implicit assumption that spatial curvature of the Universe is given and that it cannot be changed throughout the history of our Universe. In this letter we argue that this assumption ought to be relaxed, and we propose a dynamical mechanism:

formation of global monopoles at an early universe phase transition,

by which the (average, measured) spatial curvature of the Universe can change in the sense that it will become positive if it starts slightly negative or zero. Strictly speaking this is true provided the Universe was before the transition non-compact, *i.e.* it was created with no periodic boundary conditions imposed on it.

This claim will leave many readers with a queasy feeling since, when κ changes from $\kappa \le 0$ to $\kappa > 0$, spatial sections could change from infinite (hyperbolic or parabolic) to finite (elliptic), thus changing the topology of spatial sections. One should keep in mind that all this happens at *space-like* hyper-surfaces of constant time, and hence it is not in contradiction with any laws of causality. And yet it does leave us with an uncomfortable feeling that 'somewhere there' distant spatial sections of the Universe are *reconnecting*, thereby changing them from infinite to finite and periodic. This will be the case provided all spatial dimensions are equally affected, which is the case in the mechanism considered in this letter. Even though not directly observable today, this process can have direct consequences for our future. Indeed, when an observer in that reconnected Universe sends a (light) signal, it will eventually arrive from the opposite direction. Furthermore, the future of a spatially finite (compact) universe can change from infinite and uneventful to finite and singular (namely, if cosmological constant is zero such a universe will end up in a Big Crunch singularity). Because of all of these reasons, a tacit consensus has emerged that no topology change is possible in our Universe (albeit strictly speaking measurements constrain the Universe's spatial topology only after recombination). We argue in this work that this consensus needs to be reassessed.

In fact, the idea that the curvature of spatial sections could change can be traced back to the work of Krasinski [14] based on Stephani's exact solution [15] to Einstein's equations. Even though Krasinski has argued that the curvature of spatial sections could dynamically change, he has not offered any mechanism by which such a change could occur [16]. In this letter we provide such a dynamical mechanism.

A particularly instructive case to consider is the maximally symmetric de Sitter space, whose geometry can be clearly visualized from its five dimensional (flat, Minkowskian) embedding (see Fig. 1),

$$dS^2 = -dT^2 + dX_1^2 + dX_2^2 + dX_3^2 + dX_4^2\,,$$
$$R_H^2 = T^2 - R^2\,,\ R^2 = X_1^2 + X_2^2 + X_3^2 + X_4^2\,. \qquad (3)$$

Thus de Sitter space is geometrically a four dimensional hyperboloid $\mathbb{H}^4$, and its symmetry is the five dimensional Lorentz group, $SO(1,4)$, which has – just like the Poincaré group of the symmetries of Minkowski space – 10 symmetry generators. This means that de Sitter space also has 10 global symmetries (Killing vectors). Common coordinates on de Sitter space (3) are those of constant curvature of its spatial sections, and they include: (a) closed (global) coordinates have $\kappa > 0$; (b) flat (Euclidean) coordinates (Poincaré patch) have $\kappa = 0$ and (c) open coordinates (hyperbolic sections) have $\kappa < 0$. Krasinski has, however, pointed out that there are also de Sitter coordinates in which κ changes in time. Both cases, when κ changes from negative to positive, and *v.v.* are possible. An example of the metric when $\kappa(t)$ changes from negative to positive can be easily inferred from [14],

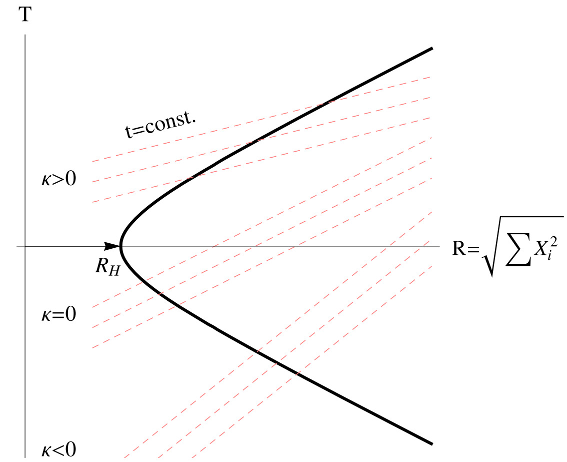

Fig. 1. Hypersurfaces of constant time of de Sitter $\mathbb{H}^4$ with a time dependent κ.

$$ds^2 = -\frac{c^2(r/r_0)^4}{[1+ctr^2/r_0^3]^2[(Hr_0/c)^2 - ct/r_0]}dt^2 + \frac{1}{[1+ctr^2/r_0^3]^2}\Big[dr^2 + r^2 d\theta^2 + \sin^2(\theta)d\phi^2\Big]\,, \qquad (4)$$

where r_0, c, H are constants. That this is a de Sitter space can be checked, for example, by evaluating the Riemann tensor. One finds

$$R_{\mu\nu\alpha\beta} = (R/12)(g_{\mu\alpha}g_{\nu\beta} - g_{\mu\beta}g_{\nu\alpha})\,, \qquad (5)$$

where $R = 12H^2/c^2 \equiv 12/R_H^2$ is the Ricci curvature scalar, $H =$ const. is the Hubble parameter and $R_H = c/H$ is the Hubble radius. Relation (5) holds uniquely for maximally symmetric spaces such as de Sitter space. The curvature of spatial sections of de Sitter in (4) can be inferred from the Riemann curvature of spatial sections,

$$^{(3)}R_{ijkl} = \frac{^{(3)}R}{6}(g_{ik}g_{jl} - g_{il}g_{jk})\,,\quad {}^{(3)}R = \frac{24ct}{r_0^3} \equiv \frac{6\kappa(t)}{a^2(t)}\,, \qquad (6)$$

from which we infer,

$$\kappa(t) = \frac{4cta^2(t)}{r_0^2}\,,\quad \text{with}\quad a(t) = e^{Ht}\,, \qquad (7)$$

which means that $\kappa < 0$ for $t < 0$, $\kappa = 0$ for $t = 0$, and $\kappa > 0$ for $t > 0$. Note that topology of spatial sections changes at $t = 0$. For $t < 0$ the sections are three dimensional hyperboloids, with a time dependent (physical) throat radius $r_c(t) = r_0^{3/2}/\sqrt{-ct}$, for $t = 0$ they are paraboloids and for $t > 0$ they are three-spheres with a (time-dependent) radius, $r_c = r_0^{3/2}/\sqrt{ct}$ (see Fig. 1). Consequently, topology of spatial sections changes at $t = 0$, as can be seen in Fig. 1 [17]. A similar (albeit inhomogeneous) construction is possible on FLRW space-times. While this shows that there are observers for which topology of spatial sections of an expanding space-time changes, it does not tell us how to realize such a change, and whether such a change is possible in a realistic setting. This is what we address next.