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# The price of an electroweak monopole

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ABSTRACT

In a recent paper, Cho, Kim and Yoon (CKY) have proposed a version of the $SU(2) \times U(1)$ Standard Model with finite-energy monopole and dyon solutions. The CKY model postulates that the effective U(1) gauge coupling $\to \infty$ very rapidly as the Englert–Brout–Higgs vacuum expectation value $\to 0$, but in a way that is incompatible with LHC measurements of the Higgs boson $H \to \gamma\gamma$ decay rate. We construct generalisations of the CKY model that are compatible with the $H \to \gamma\gamma$ constraint, and calculate the corresponding values of the monopole and dyon masses. We find that the monopole mass could be $< 5.5$ TeV, so that it could be pair-produced at the LHC and accessible to the MoEDAL experiment.



## 1. Introduction

Ever since Dirac first considered the possible existence of monopoles in QED [1], and Schwinger extended his considerations to dyons [2], theorists have explored the possible existence of finite-energy monopoles and dyons, and tried to estimate their masses. As pointed out by 't Hooft [3] and Polyakov [4], one very plausible scenario is that QED is embedded in a semi-simple unified group with coupling $g_U$, in which case the core of the monopole/dyon is regularised and its mass is finite and $\mathcal{O}(V)/g_U$, where $V$ is the vev of an Englert–Brout–Higgs field that breaks the unified group into pieces including a U(1) factor with a $U(1)_{EM}$ component.

However, physics at the electroweak scale is very well described by the Standard Model, which has an $SU(2) \times U(1)$ group structure that does not admit a finite-energy monopole or dyon solution unless its structure is modified [5,6], and there is no sign of an underlying semi-simple unified group that might be broken down to the Standard Model at any accessible energy scale. The question therefore arises whether there is any modification of the Standard Model that might contain a monopole or dyon solution with a mass $\mathcal{O}(v)/g$, where $g$ is a Standard Model gauge coupling and $v$ the vev of the Standard Model Englert–Brout–Higgs field.

Cho, Kim and Yoon (CKY) [7] have recently proposed a scenario for modifying the Standard Model that includes a non-minimal coupling of its Englert–Brout–Higgs field to the square of its U(1) gauge coupling strength: $\mathcal{L} \ni (-1/4)\epsilon(|H|/v)B_{\mu\nu}B^{\mu\nu}$. The coupling function is normalised so that $\epsilon(|H|/v) \to 1$ as $|H| \to v$, in order to restore the conventional normalisation of the U(1) gauge field in the standard electroweak vacuum. Also, in order to have a finite-energy dyon solution, the coupling function should vanish as $|H| \to 0$ like $|H|^n : n > 4 + 2\sqrt{3} \simeq 7.46$, so as to regularise the energy integral at the origin. Effectively, CKY create the possibility of a finite-energy dyon by postulating that the effective U(1) gauge coupling $\to \infty$ sufficiently rapidly as $|H| \to 0$.

CKY do not discuss an ultraviolet completion of the Standard Model that might lead to such behaviour, and nor do we. Our interest is limited to the question whether, in principle, the monopole mass could be regularised with a value low enough for it to be pair-produced at the LHC, and hence accessible to the MoEDAL experiment [8].

In their original model, CKY postulated a simple power law for the coupling function: $\epsilon(|H|/v) \propto (|H|/v)^8$, and calculated a dyon mass $M_D \simeq 0.65 \times (4\pi/e^2)M_W \simeq 7.2$ TeV. There is, however, an experimental problem with this simple power-law Ansatz, since it leads to an effective $H\gamma\gamma$ coupling that is much larger than is allowed by LHC measurements [9]. In the Standard Model, the $H\gamma\gamma$ vertex is generated by loop diagrams (principally those involving $W$ bosons and $t$ quarks), and hence is $\mathcal{O}(\alpha_{EM}/4\pi)$. The data from CMS and ATLAS on the $H \to \gamma\gamma$ decay rate [9] are quite consistent with this Standard Model calculation, so they constrain any additional contribution to be $\mathcal{O}(10^{-3})$: see [10], for example. This implies that, if one expands the coupling function $\epsilon(|H|/v)$ around

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the standard electroweak vacuum with $|H| = v$, the linear term in the expansion, i.e., $\epsilon'(|H|/v)|_{|H|=v}$, should be $\mathcal{O}(10^{-3})$.[1] This condition is manifestly not satisfied if $\epsilon(|H|/v)$ is a simple power of $|H|/v$, but could be satisfied if $\epsilon(|H|/v)$ has a more complicated functional form.

We consider in this paper forms for $\epsilon(|H|/v)$ that contain various combinations of powers $(|H|/v)^n : n \geq 8$, imposing the normalisation condition $\epsilon(1) = 1$ and the LHC condition $\epsilon'(1) = \mathcal{O}(10^{-3})$. If the form of $\epsilon(|H|/v)$ contains just two terms with different powers $n$, their coefficients can be determined using these two conditions, and one can use the classical equations of the Standard Model to calculate the energy (mass) of the lowest-lying monopole configuration. However, if the form of $\epsilon(|H|/v)$ includes more terms, the coefficients cannot be determined. Instead, we use as an additional constraint the Principle of Maximum Entropy (PME) [11], namely that the quantity

$$S(\epsilon) = -\int_0^1 dx\, \epsilon(x) \ln \epsilon(x) \tag{1.1}$$

should be maximised in the space of possible coefficients. Once $S(\epsilon)$ is maximised, one can again use the classical equations of the Standard Model to calculate the energy (mass) of the lowest-lying monopole configuration for the corresponding form of the coupling function $\epsilon(x)$.

We consider several possible functional forms for $\epsilon(|H|/v)$, and calculate the corresponding values of the monopole mass $\mathcal{M}$. For a combination of $(|H|/v)^{10}$ and $(|H|/v)^{12}$ consistent with the LHC $H \to \gamma\gamma$ decay rate, we find $\mathcal{M} = 6.2$ TeV, increasing to 6.6 TeV for a combination of $(|H|/v)^8$ and $(|H|/v)^{10}$, with no further reduction for the maximum-entropy combination of $(|H|/v)^8$, $(|H|/v)^{10}$ and $(|H|/v)^{12}$. On the other hand, forms of $\epsilon(|H|/v)$ combining higher powers $n = 14$ and 16 (with a logarithmic correction) yield lower monopole masses $\sim 5.7$ (5.4) TeV. We conclude that the CKY monopole could indeed weigh $< 5.5$ TeV, so that pair-production at the LHC is an open possibility, opening up interesting perspectives for the MoEDAL experiment [8].

## 2. Review of the Cho–Maison monopole solution

Before discussing the CKY construction [7] of a finite-energy monopole solution in the electroweak theory, we first review the structure of the (infinite-energy) Cho–Maison monopole solution. The Cho–Maison electroweak monopole [5] is a numerical solution of the Weinberg–Salam theory.[2] However, it suffers from a divergence in the energy due to a singularity at the centre of the configuration, $r \to 0$, where $r$ is the radial coordinate. As such, it cannot be considered as physical in the absence of a suitable ultraviolet completion. CKY [7] proposed a mechanism for rendering integrable the divergence at the monopole core, yielding a finite-energy solution that would be physical.

The starting-point of Cho and Maison and CKY is the Lagrangian describing the bosonic sector of the Weinberg–Salam theory,

$$\begin{aligned}\mathcal{L} &= -|D_\mu H|^2 - \frac{\lambda}{2}\left(H^\dagger H - \frac{\mu^2}{\lambda}\right)^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} \\ &= -\frac{1}{2}(\partial_\mu \rho)^2 - \frac{\rho^2}{2}|D_\mu \xi|^2 - \frac{\lambda}{8}\left(\rho^2 - \rho_0^2\right)^2 \\ &\quad - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}\,,\end{aligned} \tag{2.1}$$

where the $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ gauge-covariant derivative is defined as

$$D_\mu \equiv \partial_\mu - i\frac{g}{2}\tau^a A^a_\mu - i\frac{g'}{2}B_\mu\,,$$

and $H$ is the Englert–Brout–Higgs doublet. In the second line of (2.1) this is written as $H = \frac{1}{\sqrt{2}}\rho\xi$, where $\xi^\dagger\xi = 1$, and we define $\rho_0 = \sqrt{2}\mu^2/\lambda = \sqrt{2}v$. The $U(1)_Y$ coupling of $\xi$ is essential for its interpretation as a $CP^1$ field with non-trivial second homotopy, making possible a topologically-stable monopole solution of the equations of motion [5].

Choosing the following Ansatz for the fields in spherical coordinates $(t, r, \theta, \phi)$,

$$\begin{aligned}\rho &= \rho(r), \qquad \xi = i\begin{pmatrix}\sin(\theta/2)\, e^{-i\varphi} \\ -\cos(\theta/2)\end{pmatrix}, \\ \vec{A}_\mu &= \frac{1}{g}A(r)\partial_\mu t\, \hat{r} + \frac{1}{g}(f(r)-1)\,\hat{r}\times\partial_\mu\hat{r}, \\ B_\mu &= \frac{1}{g'}B(r)\partial_\mu t - \frac{1}{g'}(1-\cos\theta)\partial_\mu\varphi,\end{aligned} \tag{2.2}$$

one can find spherically-symmetric field configurations corresponding to electroweak monopoles and dyons.[3] With this Ansatz, the equations of motion take the form

$$\begin{aligned}\ddot{\rho} + \frac{2}{r}\dot{\rho} - \frac{f^2}{2r^2}\rho &= -\frac{1}{4}(A-B)^2\rho + \lambda\left(\frac{\rho^2}{2} - \frac{\mu^2}{\lambda}\right)\rho\,, \\ \ddot{f} - \frac{f^2-1}{r^2}f &= \left(\frac{g^2}{4}\rho^2 - A^2\right)f\,, \\ \ddot{A} + \frac{2}{r}\dot{A} - \frac{2f^2}{r^2}A &= \frac{g^2}{4}\rho^2(A-B)\,, \\ \ddot{B} + \frac{2}{r}\dot{B} &= -\frac{g'^2}{4}\rho^2(A-B)\,.\end{aligned} \tag{2.3}$$

After an appropriate unitary gauge transformation $U$ such that $\xi \to U\xi = \begin{pmatrix}0\\1\end{pmatrix}$, one may obtain the physical gauge fields by rotating through the electroweak mixing angle $\theta_W$,

$$\begin{aligned}W_\mu &= \frac{i}{g}\frac{f(r)}{\sqrt{2}}e^{i\varphi}(\partial_\mu\theta + i\sin\theta_W\,\partial_\mu\varphi), \\ A^{\mathrm{EM}}_\mu &= e\left(\frac{1}{g^2}A(r) + \frac{1}{g'^2}B(r)\right)\partial_\mu t - \frac{1}{e}(1-\cos\theta_W)\partial_\mu\varphi, \\ Z_\mu &= \frac{e}{gg'}\big(A(r) - B(r)\big)\partial_\mu t,\end{aligned} \tag{2.4}$$

where the electric charge $e = g\sin\theta_W = g'\cos\theta_W$. The simplest non-trivial solution to the equations of motion with $A(r) = B(r) = f(r) = 0$ and $\rho = \rho_0 \equiv \sqrt{2}\mu/\sqrt{\lambda}$ describes a charge $4\pi/e$ point monopole with

$$A^{\mathrm{EM}}_\mu = -\frac{1}{e}(1-\cos\theta)\partial_\mu\varphi\,.$$

More general electroweak dyon solutions may be obtained for non-zero $A$, $B$ and $f$. For example, with the boundary conditions

[1] We revisit this constraint more quantitatively in the following, but the precise value is not very important for our estimate of the possible monopole mass.

[2] An analytical existence theorem for such monopole solutions can be established by appropriately adopting arguments by Yang [12].

[3] We emphasise that the $\mathrm{U}(1)_Y$ gauge symmetry is essential for permitting the spherically-symmetric Ansatz (2.2), because spherical symmetry for the gauge field involves embedding the radial isotropy group SO(2) into the gauge group, which requires the Higgs field to be invariant under the U(1) subgroup of SU(2). This is possible with a Higgs triplet, but not with a Higgs doublet [13]. In fact, in the absence of the $\mathrm{U}(1)_Y$ degree of freedom, the above Ansatz describes the SU(2) sphaleron, which is not spherically symmetric [14].