Physics Letters B 756 (2016) 371–375

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

On the effect of time-dependent inhomogeneous magnetic fields in electron–positron pair production

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ARTICLE INFO

Article history:
Received 11 February 2016
Accepted 10 March 2016
Available online 15 March 2016
Editor: A. Ringwald

Keywords:
Electron–positron pair production
QED in strong fields
Kinetic theory
Wigner formalism

ABSTRACT

Electron–positron pair production in space- and time-dependent electromagnetic fields is investigated. Especially, the influence of a time-dependent, inhomogeneous magnetic field on the particle momenta and the total particle yield is analyzed for the first time. The role of the Lorentz invariant $\mathbf{E}^2 - \mathbf{B}^2$, including its sign and local values, in the pair creation process is emphasized.

1. Introduction

Although already predicted in the first half of the last century [1] electron–positron pair production attracted renewed attention over the last decade. This interest is strengthened by experiments verifying the possibility of creating matter by light–light scattering [2]. Upcoming laser facilities, *e.g.*, ELI [3,4] and XFEL [5,6], as well as newly proposed experiments [7] are expected to deepen our understanding of matter creation from fields.

Note that in the very special case of constant and homogeneous fields the Lorentz invariants

$$\mathcal{F} = \frac{1}{2}\left(\mathbf{E}^2 - \mathbf{B}^2\right), \quad \mathcal{G} = \mathbf{E}\cdot\mathbf{B} \tag{1}$$

determine the particle production rate [8]. In constant crossed fields $\mathcal{G}$ vanishes which highlights then the role of the action density $\mathcal{F}$ in pair production.

Although electric and magnetic fields appear in equal magnitude in the quantity $\mathcal{F}$ magnetic fields are usually ignored in theoretical investigations of pair production. This may be motivated by the fact that for perfect settings the magnetic field vanishes in the overlapping region of two colliding laser beams [9]. Hence, the majority of studies on pair production have examined this process for time-dependent electric fields only [10,11]. (NB: Configurations with an additional constant magnetic field have been investigated in [12].)

But in studies of pair production by electric fields it turns out that exactly the time-dependence of the fields is most influential, and depending on it one observes different mechanisms behind pair production [13,14]. In a first, almost superficial, way one can distinguish multi-photon pair production [15,16] from the Schwinger effect [17,18]. Employing multi-timescale fields, however, a rich phenomenology opens up. Hereby, *e.g.*, the dynamically-assisted Schwinger effect [10,19–21] is only one, although the most prominent, example.

Given this situation, and in view of realistic possibilities of an experimental verification, it is an unsatisfactory situation that so little is known about pair production in non-constant magnetic fields [16,22]. The clarification of potential, currently unknown phenomena associated with time-dependent magnetic fields is one of the required next steps if theoretical results on Schwinger pair production shall be put to the scrutiny of experiment.

Among worldline [23,24] and WKB-like formalism [25], the introduction of quantum kinetic theory [26] has helped to understand pair production in homogeneous, but time-dependent electric fields. (NB: For recent developments concerning quantum kinetic theory see, *e.g.*, Refs. [14,27–32].) However, to accurately describe pair production in laser fields one has to take into account spatial inhomogeneities [23,33–36] as well as magnetic fields [16]. In this letter, we will discuss the results of our

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http://dx.doi.org/10.1016/j.physletb.2016.03.027

exploratory study on the influence of time-dependent, spatially inhomogeneous magnetic fields on the particle production rate using still a relatively simple model for the gauge potential. To put these results into perspective, we will also compare the outcome of these calculations with a field configuration not fulfilling the homogeneous Maxwell equations. Our results are based upon the Dirac–Heisenberg–Wigner (DHW) approach [37], which was successfully employed for spatially inhomogeneous electric fields only recently [33,34].

2. Formalism

Throughout this article the convention $c = \hbar = m = 1$ will be used. The theoretical approach employed here is based on the fundament laid by Refs. [37].

The fundamental quantity in the DHW approach is the covariant Wigner operator

$$\hat{\mathcal{W}}_{\alpha\beta}(r,p) = \frac{1}{2}\int d^4s\, e^{ips}\,\mathcal{U}(A,r,s)\;\mathcal{C}_{\alpha\beta}(r,s), \tag{2}$$

where we have introduced the density operator

$$\mathcal{C}_{\alpha\beta}(r,s) = \left[\bar{\psi}_\beta(r-s/2),\psi_\alpha(r+s/2)\right] \tag{3}$$

and the Wilson line factor

$$\mathcal{U}(A,r,s) = \exp\left(\mathrm{i}e\int_{-1/2}^{1/2} d\psi\; A(r+\psi s)\; s\right). \tag{4}$$

The vector potential A is given in mean-field approximation, r and s denote center-of-mass and relative coordinates, respectively. Taking the vacuum expectation value of the Wigner operator and projecting on equal time (*i.e.*, performing an integral $\int dp_0$) yields the single-time Wigner function $\mathcal{W}(\mathbf{x},\mathbf{p},t)$.

The simplest way to incorporate inhomogeneous magnetic fields is to investigate pair production in the xz-plane. However, there are in total three different ways of defining the basis matrices for a DHW calculation with only two spatial dimensions: one representation using 4-spinors and two representations using 2-spinors. Generally, the 4-spinor formulation contains all information on the pair production process, while the results from a 2-spinor formulation are spin-dependent (one describes electrons with spin up and positrons with spin down [38] and the other describes the spin-reversed particles).

To simplify the calculations we use a 2-spinor representation. Hence, we decompose the Wigner function into Dirac bilinears:

$$\mathcal{W}(\mathbf{x},\mathbf{p},t) = \frac{1}{2}\left(\mathbb{1}\,\mathbb{s} + \gamma_\mu \mathbb{v}^\mu\right). \tag{5}$$

Following Refs. [37] we are able to identify $\mathbb{s}$ as mass density and $\mathbb{v}^\mu$ as charge and current densities.

We can reduce the corresponding equations of motions for the Wigner coefficients $\mathbb{s}$ and $\mathbb{v}^\mu = (\mathbb{v}_0, \mathbb{v}^1, \mathbb{v}^3)$ to the form (see, *e.g.*, Ref. [14]):

$$D_t\mathbb{v}_0 + \mathbf{D}\cdot\mathbb{v} = 0, \tag{6}$$

$$D_t\mathbb{s} \quad -2\left(\Pi_x\cdot\mathbb{v}^3 - \Pi_z\cdot\mathbb{v}^1\right) = 0, \tag{7}$$

$$D_t\mathbb{v}^1 + D_x\cdot\mathbb{v}_0 - 2\Pi_z\cdot\mathbb{s} = -2\mathbb{v}^3, \tag{8}$$

$$D_t\mathbb{v}^3 + D_z\cdot\mathbb{v}_0 + 2\Pi_x\cdot\mathbb{s} = 2\mathbb{v}^1, \tag{9}$$

with the pseudo-differential operators

$$D_t = \partial_t \;+ e\int d\xi \quad \mathbf{E}\left(\mathbf{x}+\mathrm{i}\xi\nabla_p,t\right)\cdot\nabla_p, \tag{10}$$

$$\mathbf{D} \;= \nabla_x + e\int d\xi \quad \mathbf{B}\left(\mathbf{x}+\mathrm{i}\xi\nabla_p,t\right)\times\nabla_p, \tag{11}$$

$$\mathbf{\Pi} \;= \mathbf{p} \quad -\mathrm{i}e\int d\xi\,\xi\;\mathbf{B}\left(\mathbf{x}+\mathrm{i}\xi\nabla_p,t\right)\times\nabla_p. \tag{12}$$

The vacuum initial conditions are given by

$$\mathbb{s}_{vac}(\boldsymbol{p}) = -\frac{2}{\sqrt{1+\boldsymbol{p}^2}}, \quad \mathbb{v}^{1,3}_{vac}(\boldsymbol{p}) = -\frac{2\boldsymbol{p}}{\sqrt{1+\boldsymbol{p}^2}}. \tag{13}$$

For later use we explicitly subtract the vacuum terms by defining

$$\mathbb{w}^v = \mathbb{w} - \mathbb{w}_{vac}, \tag{14}$$

with $\mathbb{w} = \mathbb{v}_0,\ \mathbb{s},\ \mathbb{v}^1$ and $\mathbb{v}^3$, respectively [17]. The particle number density in momentum space is given by

$$n(p_x,p_z) = \int dz\,\frac{\mathbb{s}^v + p_x\mathbb{v}^{v,1} + p_z\mathbb{v}^{v,3}}{\sqrt{1+\boldsymbol{p}^2}}. \tag{15}$$

When evaluated at asymptotic times, this quantity gives the particle momentum spectrum. Subsequently, the particle yield per unit volume element is obtained via $N = \int dp_x\, dp_z\, n(p_x,p_z)$.

In the following we will discuss pair production for one specific 2-spinor representation. The results for particles with opposite spin can be obtained performing $p_z \to -p_z$.

3. Solution strategies

As momentum derivatives appear as arguments of $\mathbf{E}$ and $\mathbf{B}$ we Taylor-expand the pseudo-differential operators in (10)–(12) up to fourth order [14]. To increase numerical stability canonical momenta are used:

$$\boldsymbol{q} = \boldsymbol{p} + e\boldsymbol{A}(\boldsymbol{x},t). \tag{16}$$

In order to solve eqs. (6)–(9) numerically, spatial and momentum directions are equidistantly discretized, and additionally we set $\mathbb{w}^v(\mathbf{x}_0) = \mathbb{w}^v\left(\mathbf{x}_{N_\mathbf{x}}\right)$ as well as $\mathbb{w}^v(\mathbf{p}_0) = \mathbb{w}^v\left(\mathbf{p}_{N_\mathbf{p}}\right)$. We further demand Dirichlet boundary conditions

$$\mathbb{w}^v\left(\mathbf{x}_{k_i},\mathbf{p}_{k_j}\right) = 0 \quad \text{if} \quad k_i = 0 \text{ or } k_j = 0. \tag{17}$$

The derivatives are then calculated using pseudospectral methods in Fourier basis [39]. The time integration was performed using a Dormand–Prince Runge–Kutta integrator of order 8(5, 3) [40].

4. Model for the fields

For our studies of pair production in electromagnetic fields, we choose a vector potential of the form

$$\boldsymbol{A}(z,t) = \varepsilon\,\tau\left(\tanh\left(\frac{t+\tau}{\tau}\right) - \tanh\left(\frac{t-\tau}{\tau}\right)\right) \times \exp\left(-\frac{z^2}{2\lambda^2}\right)\boldsymbol{e}_x. \tag{18}$$

If not stated otherwise, the electric and magnetic field are derived from this expression. Note that the field configuration obeys $\mathcal{G} = \boldsymbol{E}\cdot\boldsymbol{B} = 0$. Moreover, the homogeneous Maxwell equations are automatically fulfilled and additionally $\nabla\cdot\boldsymbol{E} = 0$ holds.

The electric field is antisymmetric in time exhibiting a double peak structure with ε denoting the field strength. The field strength of the magnetic field, however, is suppressed relative to