Physics Letters B 756 (2016) 200–204

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Information metric and Euclidean Janus correspondence

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ARTICLE INFO

Article history:
Received 31 December 2015
Accepted 3 March 2016
Available online 7 March 2016
Editor: M. Cvetič

ABSTRACT

We consider the quantum information metric of a family of CFTs perturbed by an exactly marginal operator, which has the dual description of the Euclidean Janus geometries. We first clarify its two dimensional case dual to the three dimensional Janus geometry, which recently appeared in arXiv:1507.07555 [2]. We generalize this correspondence to higher dimensions and get a precise agreement between the both sides. We also show that the mixed-state information metric of the same family of CFTs has a dual description in the Euclidean version of the Janus time-dependent black hole geometry.

1. Introduction

There has been a lot of progress in understanding the AdS/CFT correspondence [1]. However the details of the dictionary in application of decoding general gravity backgrounds are far from complete. The recent proposal [2] of quantum information metrics and their dual gravity descriptions (see also [3] for the related works) is one such example of new dictionary which may shed new light on this issue.

Information metric or fidelity susceptibility of quantum system measures the distance between two infinitesimally different quantum states $|\psi(\lambda)\rangle$ and $|\psi(\lambda+\delta\lambda)\rangle$ where λ is a parameter describing a family of perturbed quantum system (see [4] and the review [5]). The fidelity of the perturbation is defined by the inner product of these states,

$$\mathcal{F}(\lambda, \lambda+\delta\lambda) \equiv |\langle\psi(\lambda)|\psi(\lambda+\delta\lambda)\rangle| = 1 - G_{\lambda\lambda}\delta\lambda^2 + O(\delta\lambda^3) \tag{1.1}$$

measuring the overlap of the two states where the quantum information metric $G_{\lambda\lambda}$ is defined as the coefficient of $-\delta\lambda^2$. This has an application in understanding the quantum phase transitions or, in general, characteristic response of quantum system under some parametric perturbation. One of the prime examples of this correspondence in [2] involves two dimensional CFTs perturbed by an exactly marginal scalar operator and their gravity dual given by the Euclidean three dimensional Janus system. Originally the Janus geometry [6] is dual to an interface CFT where the boundary values of the scalar field dual to an exactly marginal operator jump across the interface and has many other applications including the physics of Janus time-dependent black holes (TDBH) [7,8].

In this note, we would like to extend this correspondence in two ways. First, we generalize the above correspondence to arbitrary dimensions. Namely the information metric of $d+1$ dimensional CFTs perturbed by an exactly marginal scalar operator will be dual to the $d+2$ dimensional Euclidean Janus geometry. Below we shall make this clear by matching the two sides in a rather precise manner. Next we consider the mixed-state information metric [9] that measures an infinitesimal distance between two thermal states labeled by perturbations of the underlying quantum system.[1] In the field theory side, we shall consider again two dimensional CFTs perturbed by an exactly marginal scalar operator but, now, at finite temperature. The thermal state will be described by the thermal density matrix whose trace corresponds to the standard thermal partition function. Its gravity dual will be identified with the Euclidean Janus time-dependent black hole. Indeed the Lorentzian Janus TDBH involves two causally separate boundary spacetimes, on each of which a CFT lives [8]. The values of the coupling of the exactly marginal scalar operator of these CFTs differ from each other. In the field theory side, the system is described by the tensor product state of the two CFTs which is entangled initially in a manner appropriate to describe the mixed state fidelity. From the view point of the one boundary CFT, its time evolution

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[1] See also [10] for the recent discussion of the mixed-state information metric in relation with holography.

http://dx.doi.org/10.1016/j.physletb.2016.03.012
0370-2693/

describes the thermalization of an initial out-of-equilibrium perturbation [8]. We shall argue that the Euclidean version of this Janus TDBH geometry describes the mixed-state information metric if one introduces a regularization in a particular manner. We further generalize this Euclidean Janus TDBH correspondence to higher dimensions where the corresponding CFTs are defined on $S^1 \times \Sigma_d$ with $d > 1$ where Σ_d is the hyperbolic space which can be made to be compact by an appropriate quotient by its symmetry action.

2. Information metric and Euclidean Janus

We begin by first clarifying the analysis in Ref. [2] for the 2D case which has the dual description of the 3D Euclidean Janus geometry. We then generalize this correspondence to higher dimensions. Let $|\Omega_i\rangle$ $(i = 1, 2)$ describe the ground state of $d + 1$ dimensional CFT$_i$ on R^{1+d} whose Lagrangian density and Hamiltonian are denoted by $\mathcal{L}_i(\lambda_i)$ and $H_i(\lambda_i)$ respectively. Then the fidelity is defined by

$$\mathcal{F} = |\langle\Omega_2|\Omega_1\rangle| = \frac{1}{(Z_1 Z_2)^{\frac{1}{2}}} \int \mathcal{D}\Phi\, e^{-\int d^d x\left[\int_0^\infty d\tau\,\mathcal{L}_2 + \int_{-\infty}^0 d\tau\,\mathcal{L}_1\right]} \tag{2.1}$$

where Φ collectively denotes the field content of the underlying field-theory and Z_i is the partition function of the corresponding Euclidean system. Let us assume the perturbation is given by the primary operator as

$$\mathcal{L}_2 - \mathcal{L}_1 = \delta\lambda\, O(\tau, x) \tag{2.2}$$

with $\delta\lambda = \lambda_2 - \lambda_1$ and we would like to compute the regularized inner product $\mathcal{F}_\epsilon = |\langle\Omega_2(\epsilon)|\Omega_1\rangle|$ where the regularized state $|\Omega_2(\epsilon)\rangle$ is defined by

$$|\Omega_2(\epsilon)\rangle = \frac{e^{-\epsilon H_1}|\Omega_2\rangle}{(\langle\Omega_2|e^{-2\epsilon H_1}|\Omega_2\rangle)^{\frac{1}{2}}} \tag{2.3}$$

With this set-up, the information metric is identified in terms of the two-point correlation function as

$$G_{\lambda\lambda} = \frac{1}{2}\int_\epsilon^\infty d\tau_2 \int_{-\infty}^{-\epsilon} d\tau_1 \int d^d x_2 \int d^d x_1 \langle O(\tau_2, x_2) O(\tau_1, x_1)\rangle \tag{2.4}$$

We then specialize to the case of scalar primary operator of dimension Δ for which the two-point function is given by

$$\langle O(\tau, x) O(\tau', x')\rangle = \frac{\mathcal{N}_\Delta}{\left[(\tau-\tau')^2 + (x-x')^2\right]^\Delta} \tag{2.5}$$

In order to match with the gravity description, we shall take the normalization

$$\mathcal{N}_\Delta = \frac{2\eta\, \ell^d\, \Gamma(\Delta+1)}{\pi^{\frac{d+1}{2}}\Gamma(\Delta - \frac{d+1}{2})} \tag{2.6}$$

where $\eta = \frac{1}{16\pi G}$, with the $d + 2$ dimensional Newton's constant G, and ℓ is the AdS radius scale. In the AdS/CFT correspondence, there exists a bulk scalar field dual to the scalar primary operator O of dimension Δ and the above normalization follows from the bulk scalar action whose kinetic term is normalized as $I_\phi = +\eta \int d^{d+1}x\sqrt{g}(\nabla\phi)^2 + \cdots$ [11]. By the straightforward computation, the information metric of the perturbation can be identified with[2]

$$G_{\lambda\lambda} = \frac{\eta\ell^d V_d}{2\sqrt{\pi}} \frac{\Delta\,\Gamma(\Delta - \frac{d}{2} - 1)}{(2\Delta - d - 1)\Gamma(\Delta - \frac{d+1}{2})} \frac{1}{(2\epsilon)^{2\Delta-(d+2)}} \tag{2.7}$$

where V_d is the spatial volume of R^d. We assumed here $2\Delta - (d + 2) > 0$, otherwise one needs further IR cut-off dependence. When the deformation is by an exactly marginal operator of dimension $\Delta = d + 1$, the expression is further reduced to

$$G_{\lambda\lambda} = \frac{\eta\ell^d V_d}{2\sqrt{\pi}} \frac{\Gamma(\frac{d}{2})}{\Gamma(\frac{d+1}{2})} \frac{1}{(2\epsilon)^d} \tag{2.8}$$

which will be compared to the gravity side below. In these expressions, we see that the information metric scales linearly as the spatial volume V_d of the spacetime on which the state is defined.

Let us now turn to the dual gravity side. The dual geometry is described by the AdS Einstein-scalar system described by the action

$$I = -\frac{1}{16\pi G}\int d^{d+2}x\sqrt{g}\left[R - g^{ab}\partial_a\phi\partial_b\phi + \frac{d(d+1)}{\ell^2}\right] \tag{2.9}$$

where ℓ is the AdS radius scale. In three and five dimensions, this system can be consistently embedded into the type IIB supergravity and, hence, via the standard AdS/CFT correspondence, the microscopic understanding of the underlying system is allowed [6,7]. But we shall not discuss this direction any further in this note. The scalar field here corresponds to an exactly marginal scalar operator in the field-theory side, which is further identified as a Lagrange density operator in the above type IIB supergravity examples.

For three dimensional case, we review the computation in Ref. [2] with a slight refinement. The relevant Janus solution is given by

$$ds^2 = \ell^2\left[dy^2 + f(y)\, ds^2_{AdS_2}\right]$$
$$\phi(y) = \phi_0 + \frac{1}{\sqrt{2}}\log\left(\frac{1 + \sqrt{1-2\gamma^2} + \sqrt{2}\gamma\tanh y}{1 + \sqrt{1-2\gamma^2} - \sqrt{2}\gamma\tanh y}\right) \tag{2.10}$$

where $f(y) = \frac{1}{2}(1 + \sqrt{1-2\gamma^2}\cosh 2y)$ with $\gamma < \frac{1}{\sqrt{2}}$ and we choose the AdS$_2$ metric that of the Euclidean Poincaré patch given by $ds_{AdS_2} = \frac{d\xi^2 + dx^2}{\xi^2}$. Since $\phi(\pm\infty) = \phi_0 \pm \frac{1}{\sqrt{2}}\tanh^{-1}\sqrt{2}\gamma$, one finds

$$\delta\lambda = \lambda_2 - \lambda_1 = 2\gamma + O(\gamma^3) \tag{2.11}$$

with the identification $\phi(-\infty) = \lambda_1$ and $\phi(\infty) = \lambda_2$. For the regularization of the on-shell gravity action, we introduce a pure AdS metric given by

$$d\hat{s}^2 = \ell^2\left[d\hat{y}^2 + \frac{1}{2}(1 + \cosh 2\hat{y})\, ds^2_{AdS_2}\right] \tag{2.12}$$

To match the Janus geometry at $\pm y_\infty$ with the above reference metric at $\pm\hat{y}_\infty$, the matching condition can be read off as

$$\sqrt{1-2\gamma^2}\cosh 2y_\infty = \cosh 2\hat{y}_\infty \tag{2.13}$$

With this preparation, the on-shell gravity action is evaluated as

$$I_\gamma = \frac{\ell}{4\pi G} V_{AdS_2}\Gamma_\gamma \tag{2.14}$$

where the factor Γ_γ is given by the integral

$$\Gamma_\gamma = \int_{-y_\infty}^{y_\infty} dy\,\frac{1}{2}(1 + \sqrt{1-2\gamma^2}\cosh 2y)$$
$$= y_\infty + \frac{1}{2}\sqrt{1-2\gamma^2}\sinh 2y_\infty \tag{2.15}$$

[2] The result here agrees with that of Ref. [2] after taking care of the normalization of the two-point correlation function in (2.6).